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Velocity Observer Design of Space Tether System using Immersion and Invariance technique

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Contents

The Research Background

Fig. 1 Image of **TISS-1**
(USA & Canada)

Fig. 2 Image of **STARS**
（Japan）

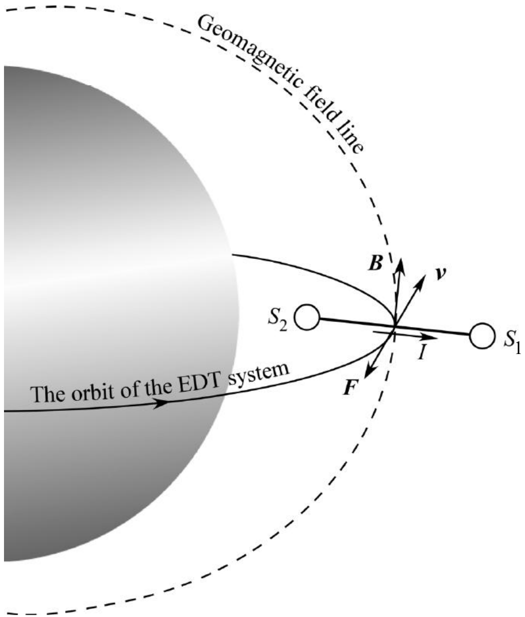


Fig. 3 Scheme of tethered spacecraft system

The Research Background

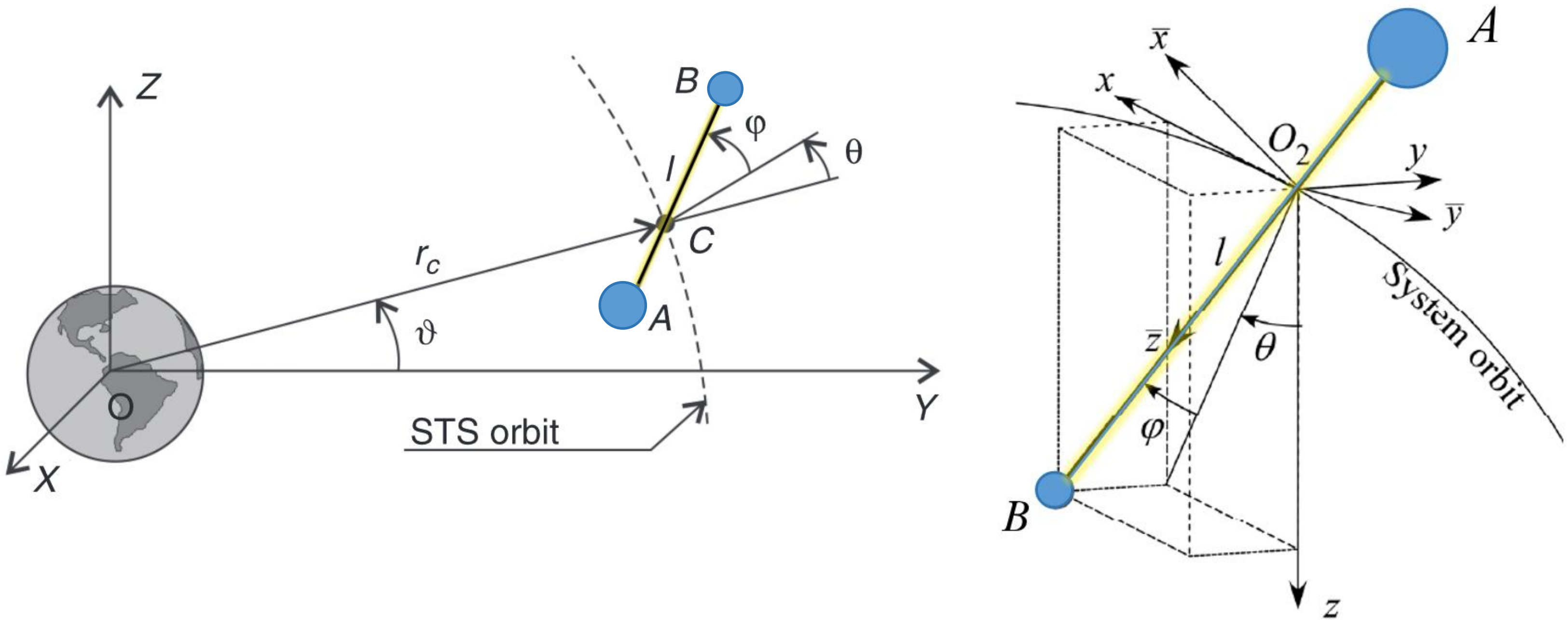


Fig 4. Scheme of space tether system.

Motivation

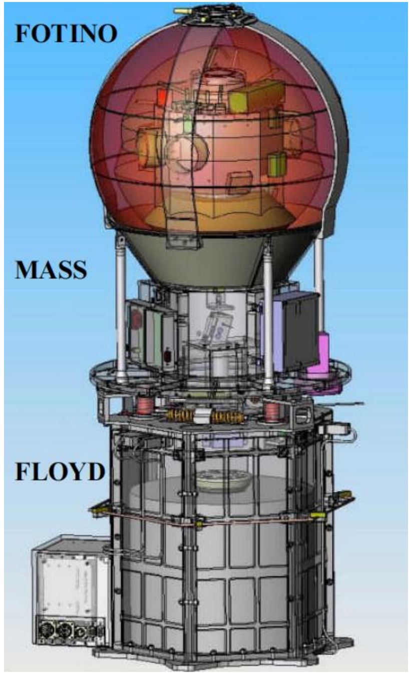


Fig 5. YES2 contains FLOYD , MASS and Fotino, the spherical re-entry capsule.

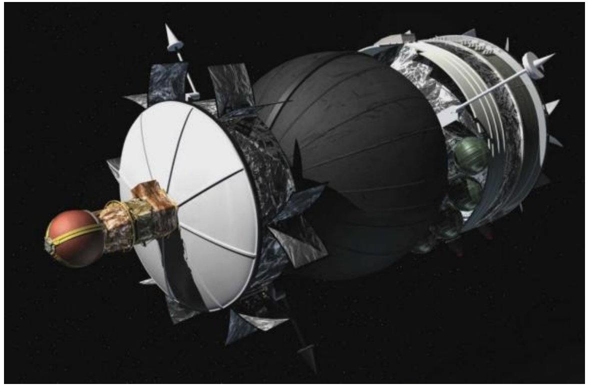


Fig 6. YES2 on Foton.

Problem

Velocity filter overestimated the speed in the first minute after ejection, leading the controller to command additional braking, which lead to a decrease of velocity.

[1] Kruijff, M., Summary of Data Analysis of the YES2 Tethered SpaceMail Experiment, Delft University of Technology, 2008.
[2] I.V. Belokonov, M.V. Bondar, I.A. Kudryavtsev, Problems of Navigational Support of a Tether System Deployment by an Example of the YES2 Experiment Aboard Foton M3, 1 (2010).

Motivation

the Kalman filter algorithm

[3] T.S. Denney, M.E. Greene, **On state estimation for an orbiting single tether system**, IEEE Trans. Aerosp. Electron. Syst. 27 (1991) 689–695. https://doi.org/10.1109/7.85043.

[4] P. Williams, Electrodynamic Tethers Under Forced-Current Variations Part II: **Flexible-Tether Estimation** and Control, J. Spacecr. Rockets 47 (2010) 320–333.

[5] G. Li, Z.H. Zhu, **Estimation of flexible space tether state based on end measurement by finite element Kalman filter state estimator**, Adv. Space Res. 67 (2021) 3282–3293. https://doi.org/10.1016/j.asr.2021.01.057.

[6] P. Tortora, L. Somenzi, L. Iess, R. Licata, Small Mission Design for Testing In-Orbit an Electrodynamic Tether Deorbiting System, J. Spacecr. Rockets 43 (2006) 883–892. https://doi.org/10.2514/1.15359.

Estimate the **unmeasure states** of the tether system

In [6], Tortora et al. designed an extended Kalman filter to estimate the in-plane and out-of-plane libration angles and **angular velocities** for Space Tether attitude control.

A significant computational source is required to process a set of sampling points (sigma points).

Methodology - Immersion and Invarance （I&I） Technique

[7] Ø.N. Stamnes, O.M. Aamo, G.-O. Kaasa, A constructive speed observer design for general Euler–Lagrange systems, Automatica 47 (2011) 2233–2238. https://doi.org/10.1016/j.automatica.2011.08.006.

[8] A. Astolfi, D. Karagiannis, R. Ortega, Nonlinear and Adaptive Control with Applications, Springer London, London, 2008. https://doi.org/10.1007/978-1-84800-066-7.

[9] A. Astolfi, R. Ortega, A. Venkatraman, A Globally Exponentially Convergent Immersion and Invariance Speed Observer for N Degrees of Freedom Mechanical Systems, Proc. 48h IEEE Conf. Decis. Control CDC Held Jointly 2009 28th Chin. Control Conf. (2009) 6508–6513. https://doi.org/DOI: 10.1109/CDC.2009.5399984.

I&I adaptive Control

Nonlinear Observer Design

- Electrical System
- Machanical System
- Electromechanical System

the in-plane pitch angle θ and out-of-plane roll angle ϕ

[10] H. Wen, Z.H. Zhu, D. Jin, H. Hu, Exponentially Convergent Velocity Observer for an Electrodynamic Tether in an Elliptical Orbit, J. Guid. Control Dyn. 39 (2016) 1113–1118. https://doi.org/10.2514/1.G001532.

Methodology - Immersion and Invarance （I&I） Technique

Manifold : $\mathcal{M}=\left\{x\in\mathbf{R}^n \middle| s(x)=0\right\}=\left\{x\in\mathbf{R}^n \middle| x=\pi(\xi),\xi\in\mathbf{R}^p\right\}$

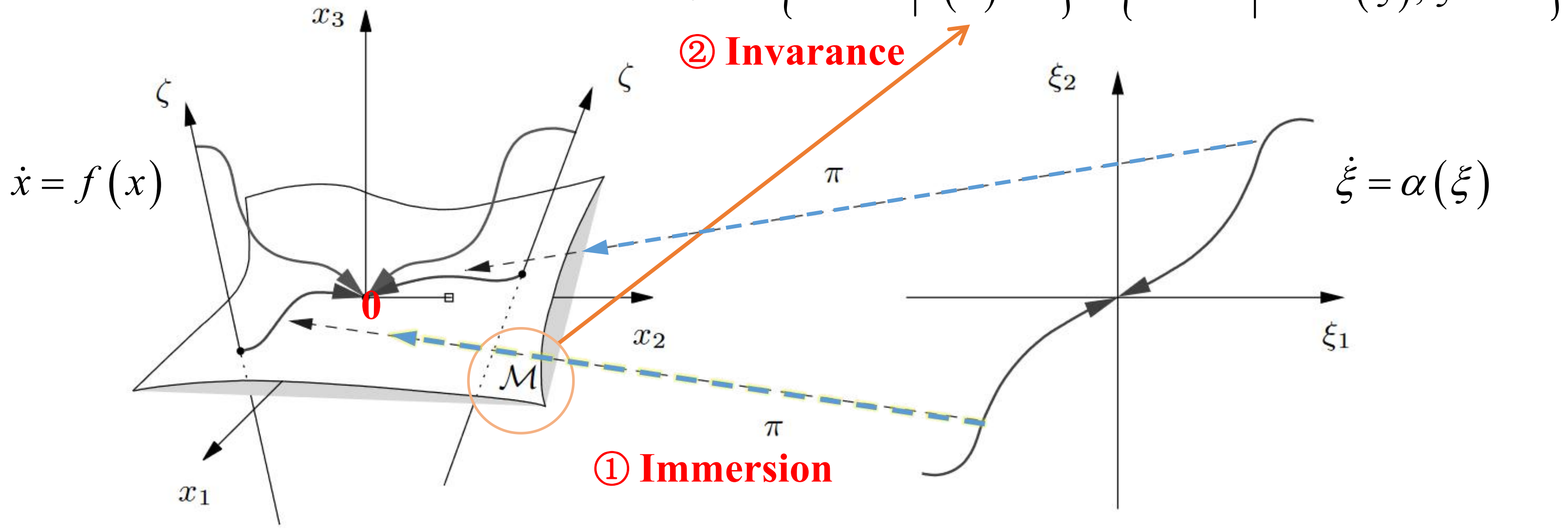


Fig. 5 Graphical illustration of the mapping between the trajectories of the system to be controlled and the target system.

[8] Astolfi, D. Karagiannis, R. Ortega, Nonlinear and Adaptive Control with Applications, Springer London, London, 2008. https://doi.org/10.1007/978-1-84800-066-7.

Contents

System Description

The dynamic equations of the space tether system during the deployment

$$
\begin{aligned}
& l'' - l\left[\left(\theta' + \Omega\right)^2 + \Omega^2\left(3\cos^2\theta - 1\right)\right] = -\frac{\bar{T}}{m_0} \\
& l^2\theta'' + 2ll'\left(\Omega + \theta'\right) + \frac{3}{2}l^2\Omega^2 \sin 2\theta = 0
\end{aligned}
\qquad (1)
$$

Define the following nondimensional variables:

$$
\lambda = l/l_n,\ \tilde{T} = \bar{T}/\left(m_0 l_n \Omega^2\right),\ \tau = \Omega t,\ \dot{s} = ds/dt \qquad (2)
$$

The nondimensional equations of tethered system (1) can be presented as following

$$
\begin{aligned}
& \ddot{\lambda} - \lambda\left[\left(\dot{\theta} + 1\right)^2 - 1 + 3\cos^2\theta\right] = -\tilde{T} \\
& \lambda^2\ddot{\theta} + 2\lambda\dot{\lambda}\left(1 + \dot{\theta}\right) + 3\lambda^2 \sin\theta\cos\theta = 0
\end{aligned}
\qquad (3)
$$

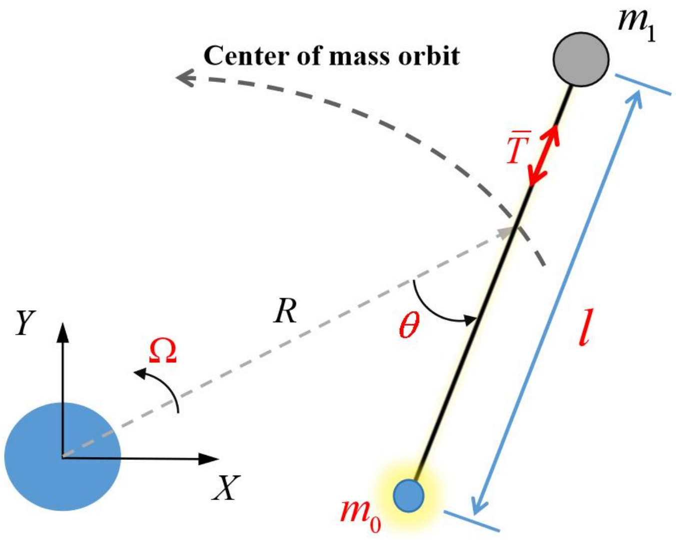
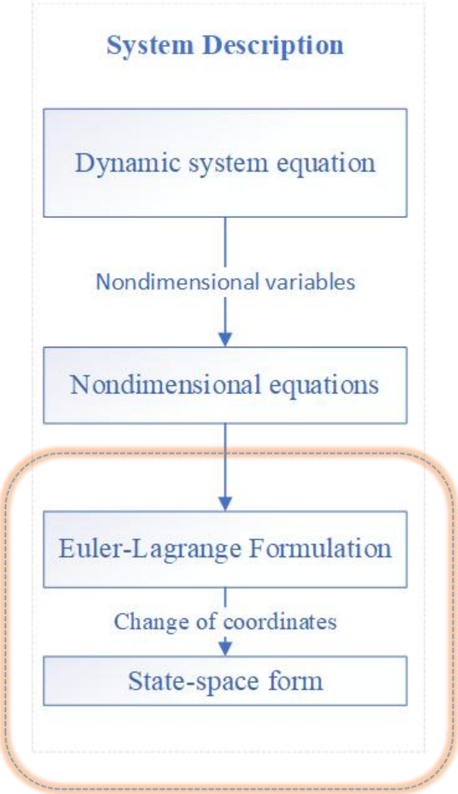


Fig. 6 Schematic diagram of STS

System Description



Furthermore, the system can be described in Euler-Lagrange by

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=u \tag{4}$$

where $q=[\lambda \quad \theta]^{\top}, u=[-\tau \quad 0]^{\top} \quad M(q)=M^{\top}(q)>0$

Then, a **factorization** of the generalized inertia matrix is used as

$$M(q)=T^{\top}(q)T(q), \quad T(q)=\begin{bmatrix} 1 & \\ & \lambda \end{bmatrix} \tag{5}$$

And the change of coordinates $(\dot{q} \quad q)^{\top} \mapsto (x \quad y)^{\top}$

where $x=T(q)\dot{q}, y=q$

Define the mapping and as $L(q)=T^{-1}(q), F(q,u)=T^{-\top}(q)(u-G(q))$ (6)

Therefore, the system (3) allows to be converted to a state-space form

$$\dot{y} = L(y)x \tag{7}$$

$$\dot{x} = S(x,y)x + F(y,u) \tag{8}$$

System Description - Velocity Observer Design

Main Result

Proposition 1. An exponentially convergent speed observer is designed for the STS (2) as follows, and are the observer states.

$$\dot{\eta} = S_1(\eta+\beta,y)\cdot(\eta+\beta)+S_2\cdot(\eta+\beta)+F(y,u)-\frac{\partial\beta}{\partial y}L(y)\cdot(\eta+\beta)-\frac{\partial\beta}{\partial\hat{y}}\dot{\hat{y}}-\frac{\partial\beta}{\partial\hat{x}}\dot{\hat{x}}$$
$$-r^2\left[\frac{\partial\beta}{\partial y}L(y)\right]^{\top}e_x-r^2L^{\mathrm{T}}(y)e_y \tag{12}$$

And the observer state $\hat{y},\hat{x}\in\mathbb{R}^2$ obtained from the filter

$$\dot{\hat{y}} = L(y)(\eta+\beta)-\varphi_1 e_y \tag{13}$$

$$\dot{\hat{x}} = S_1(\eta+\beta,y)\cdot(\eta+\beta)+S_2\cdot(\eta+\beta)+F(y,u)-\varphi_2 e_x-r^2\left[\frac{\partial\beta}{\partial y}L(y)\right]^{\top}e_x-r^2L^{\top}(y)e_y \tag{14}$$

where $\varphi_1:\mathbb{R}^2\times\mathbb{R}_+\to\mathbb{R}_+$ and $\varphi_2:\mathbb{R}^2\times\mathbb{R}_+\to\mathbb{R}_+$ are gain functions, the mapping $\beta\in\mathbb{R}^{2\times2\times2}\to\mathbb{R}^2$ is an intermediate variable, which is defined below, and e_x,e_y are the error of the system state .

System Description

Observer Design

Define a manifold for system

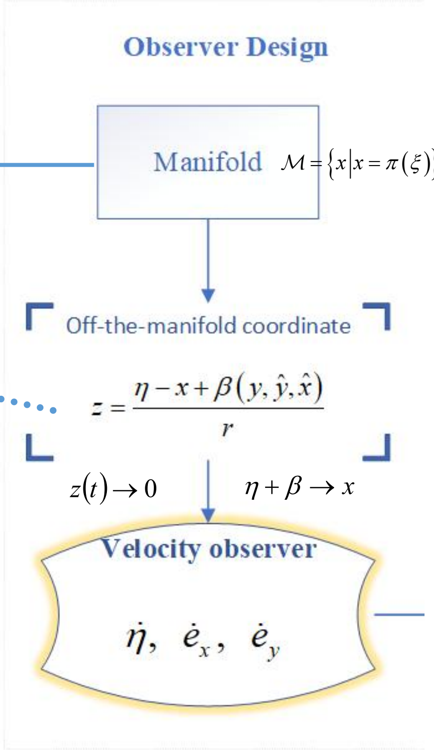
$$\pi := \left\{ \left(y, x, \eta, \hat{y}, \hat{x} \right) : \eta + \beta\left(y, \hat{y}, \hat{x} \right) - x = 0 \right\} \subset \mathbb{R}^2 \tag{15}$$

Then give the off-the-manifold coordinate

$$z = \frac{\eta - x + \beta\left(y, \hat{y}, \hat{x} \right)}{r} \tag{16}$$

Differentiate the aforementioned equation, yields

$$\begin{aligned} \dot{z} &= \frac{\left[\dot{\eta} - \dot{x} + \dot{\beta}\left(y, \hat{y}, \hat{x} \right) \right] r - \dot{r}\left[\eta - x + \beta\left(y, \hat{y}, \hat{x} \right) \right]}{r^2} \\ &= \frac{{\color{red}\dot{\eta}} - S_1\left(x, y \right) x - S_2 x - F\left(y, u \right) + \nabla_y \beta \cdot L\left(y \right) \cdot x + \nabla_{\hat{y}} \beta \dot{\hat{y}} + \nabla_{\hat{x}} \beta \dot{\hat{x}}}{r} \\ &\quad - \frac{\dot{r}}{r} z \end{aligned} \tag{17}$$



System Description - Velocity Observer Design

Assigning

$$\dot{\eta} = F(y,u) - \frac{\partial \beta}{\partial y} L(y)\cdot(\eta+\beta) - \frac{\partial \beta}{\partial \hat{y}}\dot{\hat{y}} - \frac{\partial \beta}{\partial \hat{x}}\dot{\hat{x}} + S_2\cdot(\eta+\beta)$$
$$+ S_1(\eta+\beta, y)\cdot(\eta+\beta) - r^2\left[\frac{\partial \beta}{\partial y} L(y)\right]^{\top} e_x - r^2 L^{\top}(y) e_y \qquad (18)$$

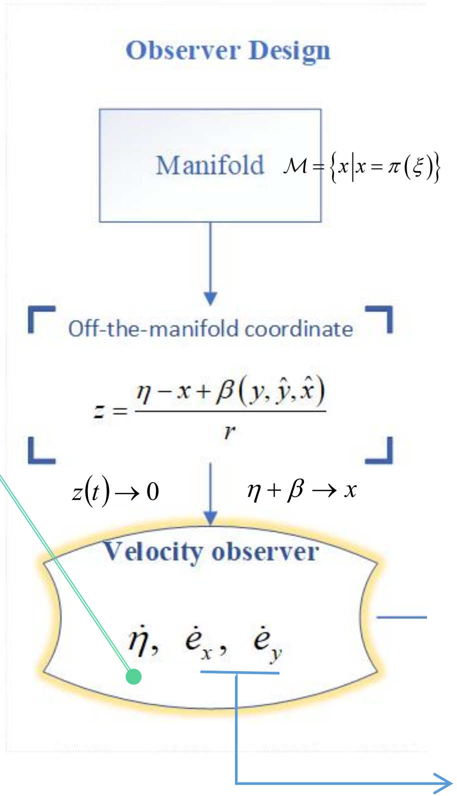
Meanwhile, defining the errors of the states x and y

$$e_y = \hat{y} - y \qquad (19)$$

$$e_x = \hat{x} - \eta - \beta \qquad (20)$$

Replacing equation (18) in (17) , and using the ***lemma 1***, yields

$$\dot{z} = \frac{\dot{\eta} - S_1(x,y)\cdot x - S_2\cdot x - F(y,u) + \nabla_y \beta L(y)\cdot x + \nabla_{\hat{y}}\beta\dot{\hat{y}} + \nabla_{\hat{x}}\beta\dot{\hat{x}}}{r} - \frac{\dot{r}}{r}z$$
$$= S_1(x,y)\cdot z + \bar{S}_1(\eta+\beta,y)\cdot z + S_2\cdot z - \frac{\partial \beta}{\partial y}L(y)\cdot z - r\left[\frac{\partial \beta}{\partial y}L(y)\right]^{\top} e_x$$
$$- rL^{\top}(y) e_y - \frac{\dot{r}}{r}z \qquad (21)$$



System Description

Differentiate equations (19) and (20), respectively,

$$e_y = \hat{y} - y \qquad (19)$$
$$e_x = \hat{x} - \eta - \beta \qquad (20)$$

$$\begin{aligned}\dot{e}_y &= \dot{\hat{y}} - \dot{y} = L(y)\cdot(\eta+\beta) - \varphi_1 e_y - L(y)x \\ &= L(y)\cdot(\eta+\beta) - L(y)\cdot(\eta+\beta-rz) - \varphi_1 e_y \\ &= L(y)\cdot rz - \varphi_1 e_y\end{aligned}$$

$$\dot{\hat{y}} = L(y)(\eta+\beta) - \varphi_1 e_y$$
$$x = \eta + \beta - rz \qquad (22)$$

meanwhile, substituting $\dot{\hat{x}}$ (14) and $\dot{\eta}$ (18) in differential (20), yields

$$\dot{e}_x = \dot{\hat{x}} - \dot{\eta} - \dot{\beta} = \frac{\partial\beta}{\partial y} L(y)\cdot rz - \varphi_2 e_x \qquad (23)$$

The error system can be expressed in general Hamiltonian frame, with

$$H(z, e_y, e_x) = \frac{1}{2} z^\top z + \frac{1}{2} e_y^\top e_y + \frac{1}{2} e_x^\top e_x$$

$$\begin{bmatrix} \dot{z} \\ \dot{e}_y \\ \dot{e}_x \end{bmatrix} = \begin{bmatrix} S_1(x,y) & -rL^\top(y) & -r\left(\nabla_y \beta L(y)\right)^\top \\ L(y)r & 0 & 0 \\ \nabla_y \beta L(y) r & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial z} \\ \frac{\partial H}{\partial e_y} \\ \frac{\partial H}{\partial e_x} \end{bmatrix} - \begin{bmatrix} -\overline{S}_1(\eta+\beta, y) - S_2 + \nabla_y \beta L(y) + \frac{\dot{r}}{r} & 0 & 0 \\ 0 & \varphi_1 & 0 \\ 0 & 0 & \varphi_2 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial z} \\ \frac{\partial H}{\partial e_y} \\ \frac{\partial H}{\partial e_x} \end{bmatrix} \qquad (24)$$

System Description

Define the partial differential equation (**PDE**)

$$\frac{\partial \beta}{\partial y} = \nabla_y \beta = \bar{S}_1(\hat{x},\hat{y}) L^{\dagger}(\hat{y}) + k_1 L^{\dagger}(\hat{y}) \tag{25}$$

$$\beta = \bar{S}_1(\hat{x},\hat{y}) L^{\dagger}(\hat{y}) \cdot y + k_1 L^{\dagger}(\hat{y}) \cdot y \tag{26}$$

let the error mapping $\Delta = \Delta_x(y,\hat{x},e_x) + \Delta_y(y,\hat{x},e_y)$

$$= \bar{S}_1(\eta+\beta, y) - \bar{S}_1(\hat{x},\hat{y}) + \left[\bar{S}_1(\hat{x},\hat{y}) + k_1 I\right]\left[L^{\dagger}(y) - L^{\dagger}(\hat{y})\right] L(y) + S_2 \tag{27}$$

which are satisfied with $\Delta_x(y,\hat{x},0) = 0,\ \Delta_y(y,\hat{x},0) = 0$

the Hamiltonian function (24) can be transformed into

$$\begin{bmatrix} \dot{z} \\ \dot{e}_y \\ \dot{e}_x \end{bmatrix} = \begin{bmatrix} S_1(x,y) & -rL^{\top}(y) & -r\left(\nabla_y \beta L(y)\right)^{\top} \\ L(y) r & 0 & 0 \\ \nabla_y \beta L(y) r & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial z} \\ \frac{\partial H}{\partial e_y} \\ \frac{\partial H}{\partial e_x} \end{bmatrix} - \begin{bmatrix} k_1 I - \Delta + \frac{\dot{r}}{r} & 0 & 0 \\ 0 & \varphi_1 & 0 \\ 0 & 0 & \varphi_2 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial z} \\ \frac{\partial H}{\partial e_y} \\ \frac{\partial H}{\partial e_x} \end{bmatrix} \tag{28}$$

System Description - Velocity Observer Design

Stability

Proposition 2. Assume the scaling factor

$$\dot{r} = -\frac{k_1}{4}(r-1) + \frac{r}{2k_1}\|\Delta\|^2, \quad r(0) \ge 1 \tag{29}$$

and notice the factor r satisfied $\{r \in \mathbb{R} \mid r \ge 1\}$ and $k_1 > 0$

Proof:

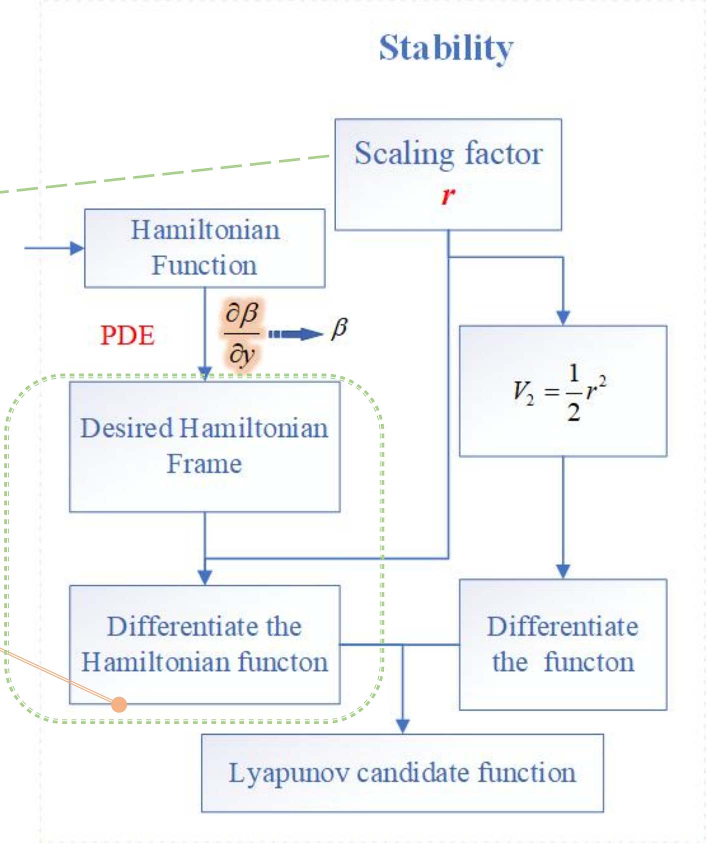
Step 1. **(proof of stability of the general Hamiltonian function)**

$$\dot{H}(z, e_y, e_x) = -z^\top \left(k_1 I - \Delta + \frac{\dot{r}}{r}\right) z - e_y^\top \varphi_1 e_y - e_x^\top \varphi_2 e_x \tag{30}$$

satisfies the property of $k_1 I - \Delta + \dot{r}/r \ge 0$

get $\dot{H}(z, e_y, e_x) \le 0$ (31)

when $\{z, e_y, e_x \in \mathbb{R}^n \mid \dot{H}(z, e_y, e_x) = 0\} = \{z, e_y, e_x \mid z = 0, e_y = 0, e_x = 0\}$



System Description - Velocity Observer Design

Select the positive-definite function $\boxed{V_1 = \frac{1}{2} z^\top z}$ and its derivative is

$$\dot{V}_1 = -z^\top \left(k_1 I - \Delta + \frac{\dot{r}}{r} \right) z = -k_1 z^\top z + z^\top \Delta z - \frac{\dot{r}}{r} |z|^2 \tag{32}$$

According the Young's inequality, yields $z^\top \Delta z \le \frac{k_1}{2} |z|^2 + \frac{1}{2k_1} \|\Delta\|^2 |z|^2$

Therefore

$$\begin{aligned}
\dot{V}_1 &= -z^\top \left(k_1 I - \Delta + \frac{\dot{r}}{r} \right) z = -k_1 z^\top z + z^\top \Delta z - \frac{\dot{r}}{r} |z|^2 \\
&\le -k_1 |z|^2 + \left(\frac{k_1}{2} |z|^2 + \frac{1}{2k_1} \|\Delta\|^2 |z|^2 \right) - \frac{\dot{r}}{r} |z|^2 = -\frac{k_1}{2} |z|^2 + \frac{1}{2k_1} \|\Delta\|^2 |z|^2 - \frac{\dot{r}}{r} |z|^2 \\
&\le -\frac{k_1}{2} |z|^2 + \frac{k_1 (r-1)}{4r} |z|^2
\end{aligned}$$

$(r-1)/r \le 1$

$$\boxed{\dot{V}_1 \le -\frac{k_1}{4} |z|^2} \tag{33}$$

$$\dot{H}(z, e_y, e_x) = -z^\top \left(k_1 I - \Delta + \frac{\dot{r}}{r} \right) z - e_y^\top \varphi_1 e_y - e_x^\top \varphi_2 e_x = V_1 - e_y^\top \varphi_1 e_y - e_x^\top \varphi_2 e_x \le 0$$

System Description - Velocity Observer Design

***Step 2.* (proof of stability of the extended system)** Select the proper Lyapunov candidate function for the extended system such that

$$V\left(z,e_y,e_x,r\right)=H\left(z,e_y,e_x\right)+V_2=\frac{1}{2}z^{\top}z+\frac{1}{2}e_y^{\top}e_y+\frac{1}{2}e_x^{\top}e_x+\frac{1}{2}r^2 \qquad \boxed{V_2=\frac{1}{2}r^2} \qquad (34)$$

Then, the derivative of the function

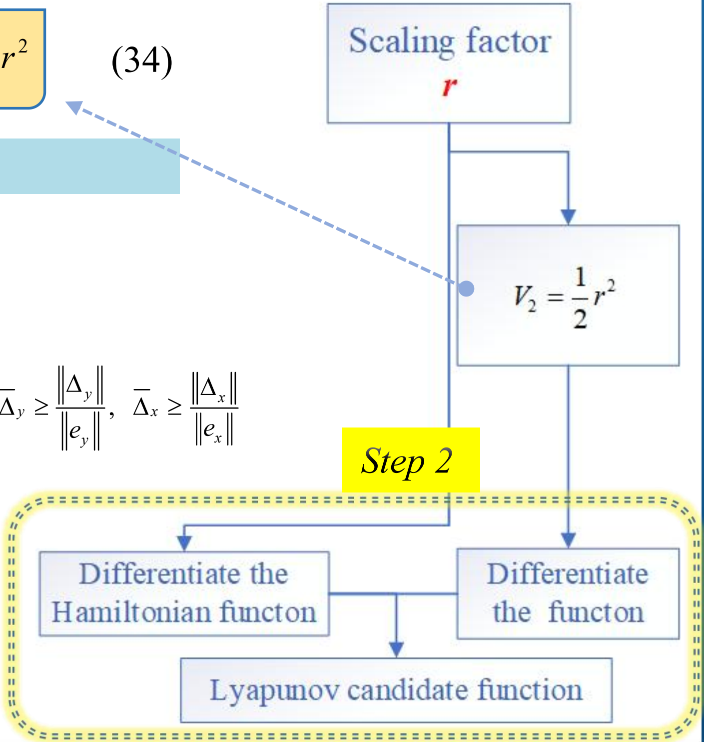
$$\begin{aligned}\dot{V}\left(z,e_y,e_x,r\right) &\le -\frac{k_1}{4}|z|^2-e_y^{\top}\varphi_1 e_y-e_x^{\top}\varphi_2 e_x+r\dot{r}\\ &=-\frac{k_1}{4}|z|^2-e_y^{\top}\varphi_1 e_y-e_x^{\top}\varphi_2 e_x-\frac{k_1}{4}\left(r^2-r\right)+\frac{r^2}{2k_1}\|\Delta\|^2\\ &\le -\frac{k_1}{4}|z|^2-\varphi_1 e_y^2-\varphi_2 e_p^2-\frac{k_1}{4}\left(r^2-r\right)+\frac{r^2}{k_1}\|\Delta_y\|^2+\frac{r^2}{k_1}\|\Delta_x\|^2\\ &\le -k_1|z|^2-\bar{\varphi}_1 e_y^2-\bar{\varphi}_2 e_x^2-\frac{k_1}{4}\boxed{\left(r^2-r\right)}\end{aligned}$$

$$\bar{\Delta}_y \ge \frac{\|\Delta_y\|}{\|e_y\|},\quad \bar{\Delta}_x \ge \frac{\|\Delta_x\|}{\|e_x\|}$$

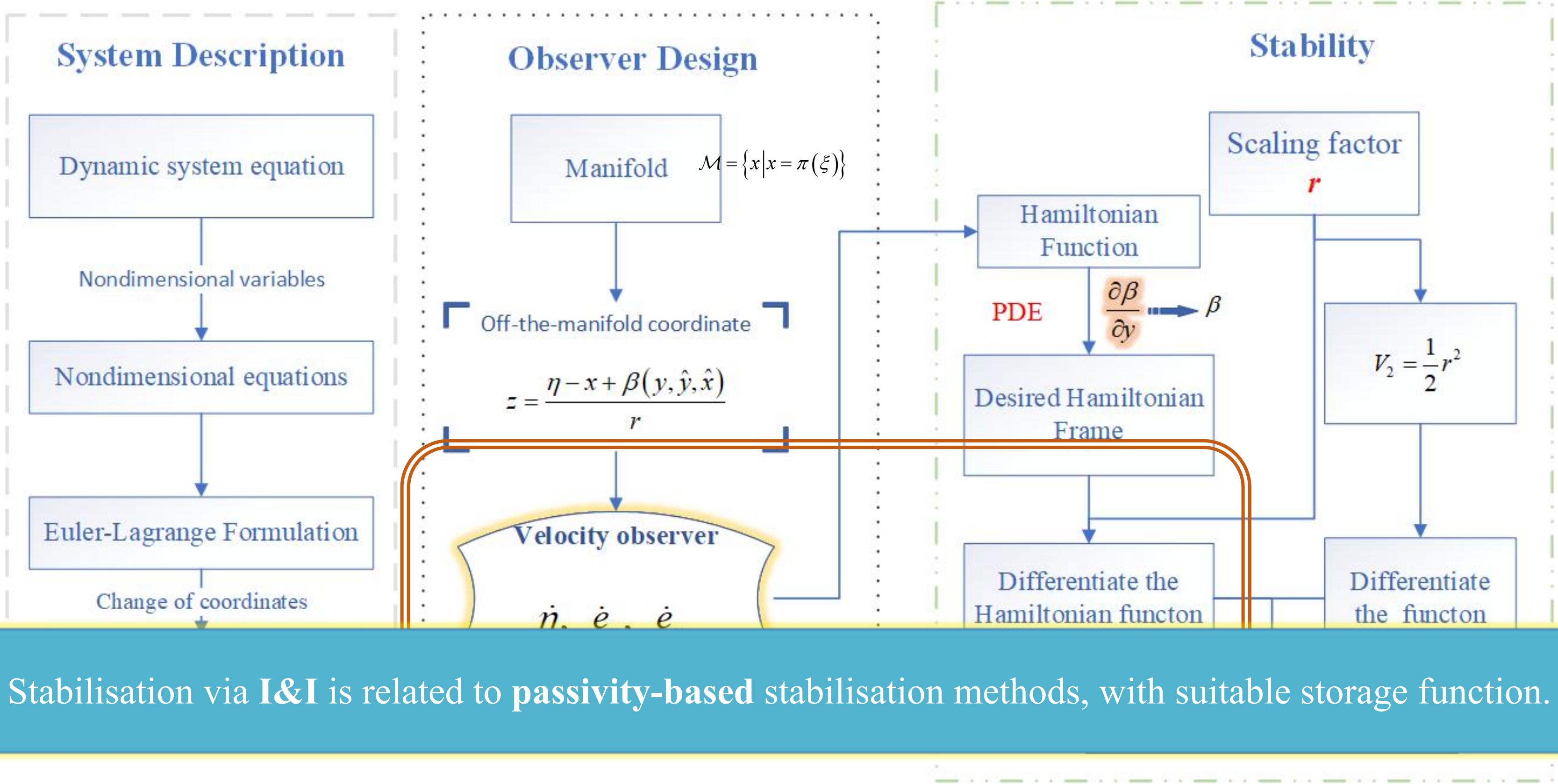
$$\boxed{r^2-r\ge 0, r(0)\ge 1}$$

Therefore $\boxed{\dot{V}\left(z,e_y,e_x,r\right)\le 0}$

Stability



System Description



Stabilisation via **I&I** is related to **passivity-based** stabilisation methods, with suitable storage function.

Contents

Simulation Results

Proposition 1.

$$\dot{\eta} = S_1(\eta+\beta,y)\cdot(\eta+\beta)+S_2\cdot(\eta+\beta)+F(y,u)-\frac{\partial\beta}{\partial y}L(y)\cdot(\eta+\beta)$$
$$-\frac{\partial\beta}{\partial\hat{y}}\dot{\hat{y}}-\frac{\partial\beta}{\partial\hat{x}}\dot{\hat{x}}-r^2\left[\frac{\partial\beta}{\partial y}L(y)\right]^{\top}e_x-r^2L^{\mathrm{T}}(y)e_y$$

$$\dot{\hat{y}} = L(y)(\eta+\beta)-\varphi_1 e_y$$

$$\dot{\hat{x}} = S_1(\eta+\beta,y)\cdot(\eta+\beta)+S_2\cdot(\eta+\beta)+F(y,u)-\varphi_2 e_x-r^2\left[\frac{\partial\beta}{\partial y}L(y)\right]^{\top}e_x-r^2L^{\top}(y)e_y$$

Input	Simulation parameters and initial conditions	
$u=\left[-\tau \quad 0\right]^{\top} =\left[-3y_1\cos^2 y_2 \quad 0\right]^{\top}$	$k_1=5, k_1=10$	$\overline{\varphi}_1=\overline{\varphi}_2=0.175$
	$y_1(0)=0.01, y_2(0)=0$	$x_1(0)=0.5, x_2(0)=0$
	$\hat{q}_1(0)=0.05, \hat{q}_2(0)=0$	$\hat{x}_1(0)=0.75, \hat{x}_2(0)=0$
	$\eta_1(0)=0.74, \eta_2(0)=0$	$r(0)=1.5$

Simulation Results

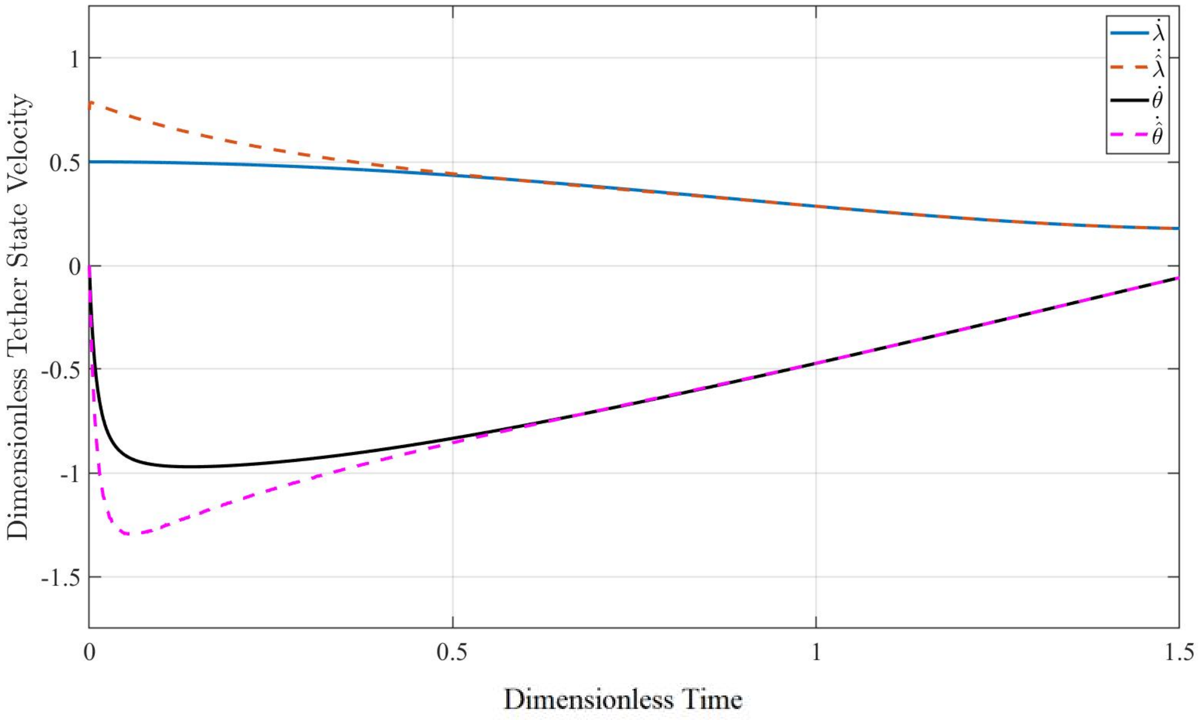


Fig. 7 Dimensionless time histories of comparison between **real velocities** $\left(\dot{\lambda},\dot{\theta}\right)$ and **estimated velocities** $\left(\dot{\hat{\lambda}},\dot{\hat{\theta}}\right)$.

Simulation Results

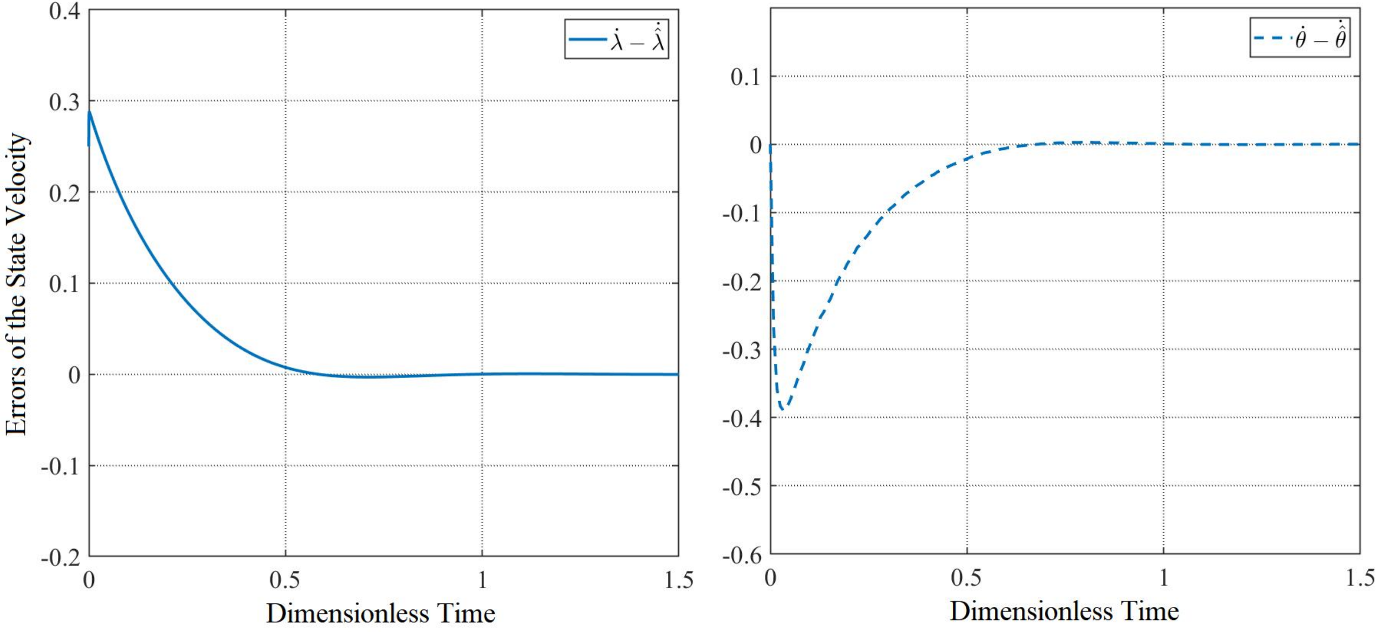


Fig. 2 Dimensionless time histories of the error between the real velocities and the estimated velocities

Simulation Results

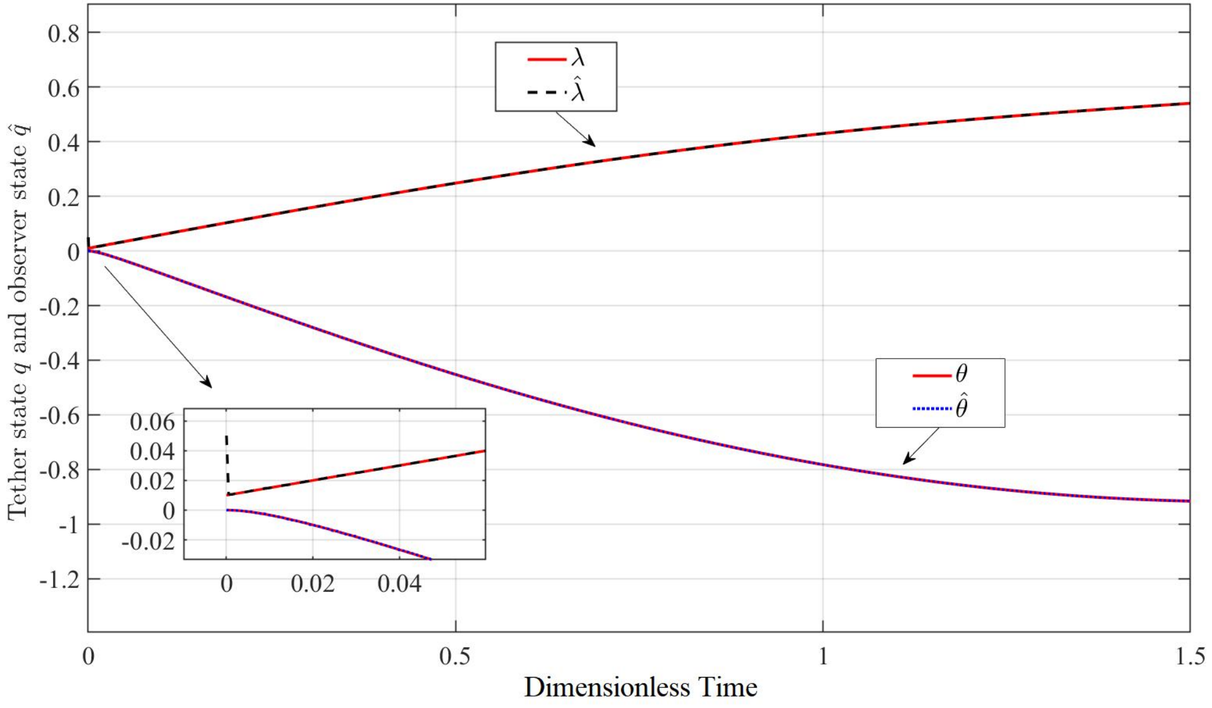


Fig.8 Dimensionless time histories of the comparison between $y = q = (\lambda \quad \theta)^{\top}$ and $\hat{y} = \hat{q} = (\hat{\lambda} \quad \hat{\theta})^{\top}$.

Contents

Conclusions

- A nonlinear velocity observer to estimate the in-plane deployment velocities of the space tether system has been presented in this paper.
- A crucial PDE
- The stability analysis of the proposed velocity observer can be demonstrated based on the Hamiltonian principle

Future Work

- Add perturbation to the the measured signals.

Thank you for listening！