The three-dimensional maneuver control of spinning tether system under a new Lagrangian model

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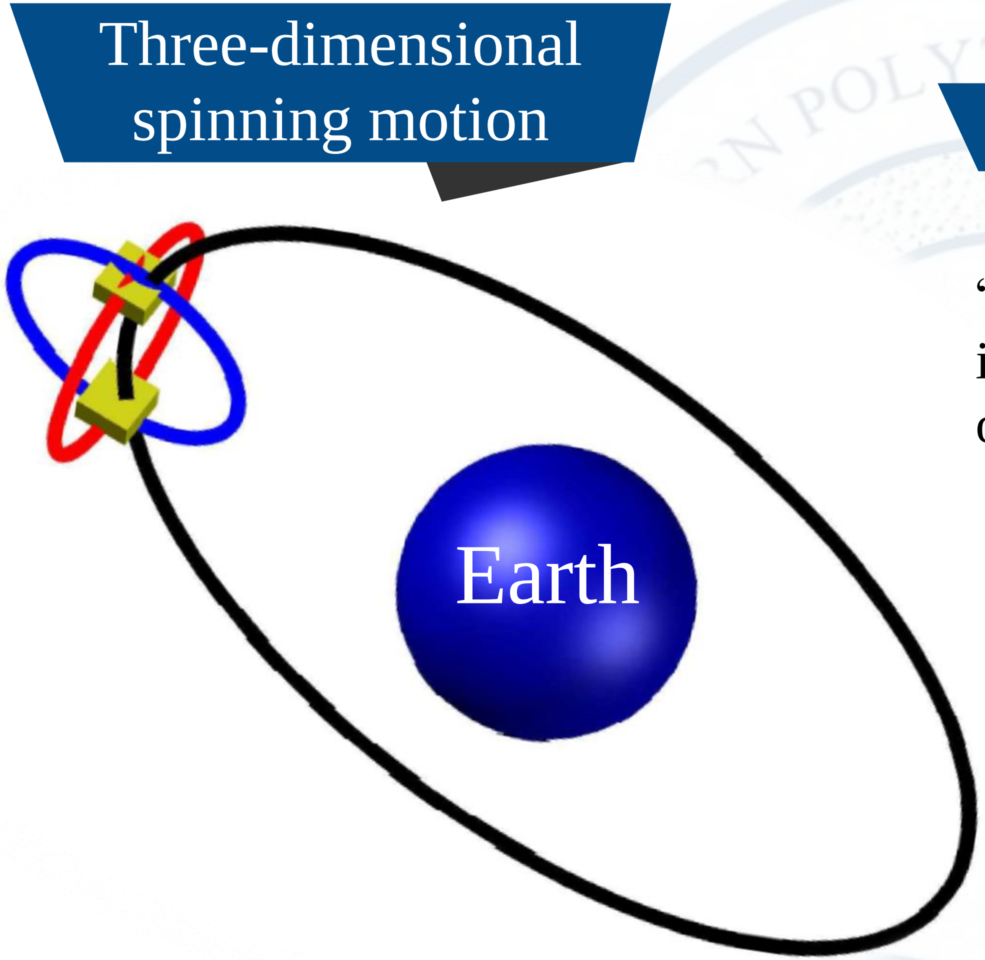
Section 1

Introduction

1 Requirements



Payload tossing

"Additional velocity increment outside the orbital plane"

Artificial gravity

"Variability of artificial gravity environment"

Observation platform

"Spin in different orientations"

Obstacle avoidance, etc.

"Change the direction of the spinning motion to"

The conventional Lagrangian models have singularities in the case of three-dimensional maneuvering.

$$\ddot{\theta} - 2\dot{\beta}\,\mathrm{tg}\,\beta\left(\dot{\theta} + \omega_o\right) + 1.5\omega_o^2 \sin 2\theta = \frac{Q_\theta}{\left(m_e L^2 \cos^2\beta\right)}$$

$$\ddot{\beta} + \left[0.5\left(\dot{\theta} + \omega_0\right)^2 + 1.5v^{-1}\omega_0^2\cos^2\theta\right]\sin 2\beta = \frac{Q_\beta}{m_e L^2}$$

The commonly used lumped suffer from the coupling problem when calculating in-plane and out-of-plane angles from Cartesian coordinate.

$$m_k \frac{d^2\mathbf{R}_k}{dt^2} = \mathbf{G}_k + \mathbf{D}'_k + \mathbf{T}'_k + \mathbf{F}_k,\ k = 1, 2, \ldots n$$

$$L = |\Delta R|, \sin\theta = \frac{\Delta y}{\sqrt{\Delta x^2 + \Delta y^2}}, \sin\beta = \frac{\Delta z}{|\Delta R|}$$

Due to limited thrusts provided by electrical engines, the saturated control problems exist.

$$\tau_\gamma\left(\tau_{c\gamma}\right) = \begin{cases} \bar{\tau}_{c\gamma}\,\mathrm{sign}\left(\tau_{c\gamma}\right) & \text{if } \left|\tau_{c\gamma}\right| > \bar{\tau}_{c\gamma} \\ \tau_{c\gamma} & \text{if } \left|\tau_{c\gamma}\right| \le \bar{\tau}_{c\gamma} \end{cases}$$

Section 2

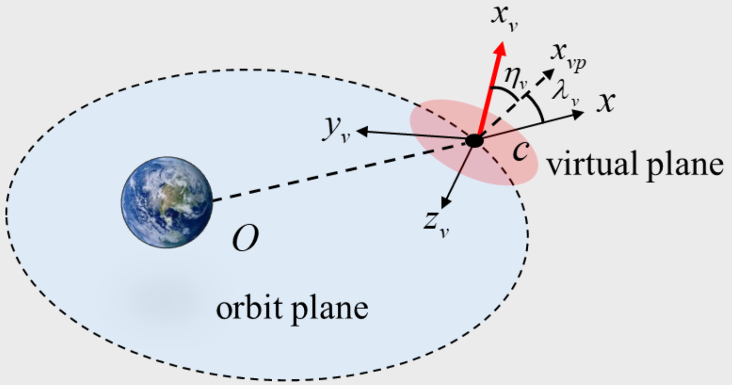
Dynamic model of STS

1 The virtual coordinate system

2 A new singularity-free model

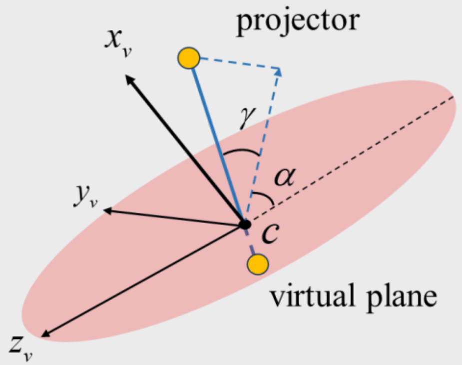
1 The virtual coordinate system

The virtual coordinate system



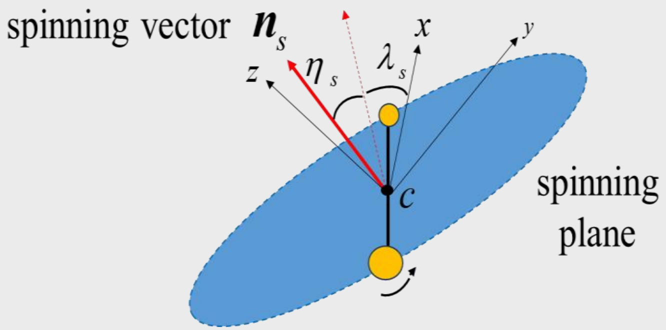
$$cxyz \xrightarrow{L_{cs}=L_\eta L_\lambda} cx_v y_v z_v$$

The virtual tether coordinate system



$$cx_v y_v z_v \xrightarrow{L_{\alpha\gamma}=L_\gamma L_\alpha} cx_{vt} y_{vt} z_{vt}$$

Spinning plane



$$e_v = L_{cs}^T \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} e_{vx} & e_{vy} & e_{vz} \end{bmatrix}^T$$

$$\eta_v = -\arcsin\left(e_{vz}\right)$$

$$\lambda_v = \begin{cases} \dfrac{\pi}{2} & , \ |e_{vz}| = 1 \\ \arcsin\left(\dfrac{e_{vy}}{\sqrt{1-e_{vy}^2}}\right) & , \text{ otherwise} \end{cases}$$

$$\Delta P^{cx_v y_v z_v} = P_1^{cx_v y_v z_v} - P_2^{cx_v y_v z_v} = \begin{bmatrix} p_x & p_y & p_z \end{bmatrix}^T$$

$$\alpha = \arcsin\left(\frac{p_y}{\sqrt{p_y{}^2 + p_z{}^2}}\right)$$

$$\gamma = -\arcsin\left(\frac{p_z}{|\Delta P|}\right)$$

$$L = |\Delta P^{cx_v y_v z_v}|$$

$$n_s = L_{cs}^T \frac{\omega_{vt}}{|\omega_{vt}|} = \begin{bmatrix} n_{sx} & n_{sy} & n_{sz} \end{bmatrix}^T, \omega_{vt} = \begin{bmatrix} \dot{\alpha} \\ 0 \\ 0 \end{bmatrix} + L_\alpha^T \begin{bmatrix} 0 \\ \dot{\gamma} \\ 0 \end{bmatrix}$$

$$\eta_s = -\arcsin\left(n_s\right)$$

$$\lambda_s = \begin{cases} \dfrac{\pi}{2} & , \ |n_{sz}| = 1 \\ \arcsin\left(\dfrac{n_{sy}}{\sqrt{1-n_{sy}^2}}\right) & , \text{ otherwise} \end{cases}$$

2 A new singularity-free model

Assumptions

Assumption 1. STS are considered as rigid rod ,neglecting the flexibility, torsion, elasticity, and other characteristics of the tether.

Assumption 2. The connection between end-satellites are rigid, end-satellites are point masses and the tether is a massless rigid rod.

Assumption 3. STS is only subject to the gravitational force of the Earth and thrust.

Assumption 4. The virtual coordinate system is defined. Angles λ_v, η_v and all their derivatives are known.

Lagrangian equations

$$\frac{d}{dt}\left(\frac{T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}=Q_i$$

The generalized coordinates

$$\boldsymbol{q}=\begin{bmatrix}\alpha & \gamma & L\end{bmatrix}^T$$

The thrust in the virtual tether coordinate system $cx_{vt}y_{vt}z_{vt}$

$$\boldsymbol{\tau}_1=\begin{bmatrix}-\tau_\gamma & \tau_\alpha & \tau_L\end{bmatrix}^T$$

$$\boldsymbol{\tau}_2=-\boldsymbol{\tau}_1$$

2 A new singularity-free model

The position of end-satellites in the virtual coordinate system

$$\boldsymbol{P}_1^{cx_v y_v z_v} = \begin{bmatrix} L_1 \sin\gamma & -L_1 \cos\gamma \sin\alpha & L_1 \cos\gamma \cos\alpha \end{bmatrix}^T$$

$$\boldsymbol{P}_2^{cx_v y_v z_v} = \begin{bmatrix} -L_2 \sin\gamma & L_2 \cos\gamma \sin\alpha & -L_2 \cos\gamma \cos\alpha \end{bmatrix}^T$$

The position of the end-satellites in the orbital coordinate system

$$\boldsymbol{P}_i^{cxyz} = \boldsymbol{L}_{cs}^T \boldsymbol{P}_i^{cx_v y_v z_v}$$

$$\boldsymbol{P}_i^{OX_0 Y_0 Z_0} = \boldsymbol{L}_\vartheta^T \left(\boldsymbol{P}_i^{cxyz} + \boldsymbol{R}_c \right)$$

The total kinetic energy of STS

$$\begin{aligned} T &= \frac{1}{2} m_1 \left(\dot{\boldsymbol{P}}_1^{OX_0 Y_0 Z_0} \right)^T \dot{\boldsymbol{P}}_1^{OX_0 Y_0 Z_0} + \frac{1}{2} m_2 \left(\dot{\boldsymbol{P}}_2^{OX_0 Y_0 Z_0} \right)^T \dot{\boldsymbol{P}}_2^{OX_0 Y_0 Z_0} \\ &= \frac{1}{2} ML^2 \left(\dot{\eta}_v \cos\alpha + \dot{\gamma} + \cos\eta_v \sin\alpha \left(\dot{\vartheta} + \dot{\lambda}_v \right) \right)^2 + \frac{1}{2} M\dot{L}^2 + \frac{1}{2} m \left(\dot{r}_c^2 + \dot{\vartheta}^2 r_c^2 \right) + \\ &\quad \frac{1}{2} ML^2 \left(\dot{\eta}_v \sin\alpha \sin\gamma + \dot{\alpha} \cos\gamma + \left(-\cos\gamma \sin\eta_v - \cos\eta_v \cos\alpha \sin\gamma \right) \left(\dot{\vartheta} + \dot{\lambda}_v \right) \right)^2 \end{aligned}$$

2 A new singularity-free model

The generalized forces

$$Q_\alpha = \tau_\alpha L\cos\gamma - \frac{\partial E_p}{\partial \alpha}, Q_\gamma = \tau_\gamma L - \frac{\partial E_p}{\partial \gamma}, Q_L = 2\tau_L - \frac{\partial E_p}{\partial L} - F_T$$

The gravitational potential energy of STS

$$E_p = -\frac{\mu m}{r_c} - \frac{\mu M L^2}{2r_c^3}\left[3\left(\cos\eta_v\cos\lambda_v\sin\gamma + \cos\gamma\left(\cos\alpha\cos\lambda_v\sin\eta_v + \sin\alpha\sin\lambda_v\right)\right)^2 - 1\right]$$

Substituting them into Lagrangian equation,the new Lagrangian model is derived as follows.

$$\ddot{L} = -M\left(F_T - 2\tau_L\right) + f_L$$

$$\ddot{\gamma} = \frac{\tau_\gamma}{ML} + f_\gamma$$

$$\ddot{\alpha} = \frac{\tau_\alpha}{ML\cos\gamma} + f_\alpha$$

f_γ, f_L, f_α are shown as formulas (19) in my paper. Although model (19) still has singularities different from conventional models, these singularities can be avoided by reasonably defining a virtual coordinate system.

Section 3

Reference Maneuvering Trajectories

1 *Maneuvering scheme for the spinning plane*

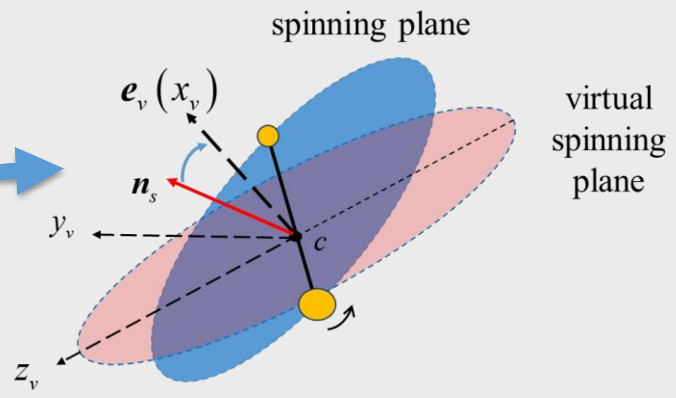
2 *Control objective and open-loop control*

1 Maneuvering scheme for the spinning plane

Control scheme of the spinning plane

$$\boldsymbol{n}_s \rightarrow \boldsymbol{e}_v$$

$$\lambda_s \rightarrow \lambda_v, \eta_s \rightarrow \eta_v$$



Smooth transition scheme

$$\lambda_v = \left(\lambda_f - \lambda_0\right)\frac{1}{1+\exp\left(-\kappa_\lambda (t-t_f)\right)} + \lambda_0$$

$$\eta_v = \left(\eta_f - \eta_0\right)\frac{1}{1+\exp\left(-\kappa_\eta (t-t_f)\right)} + \eta_0$$

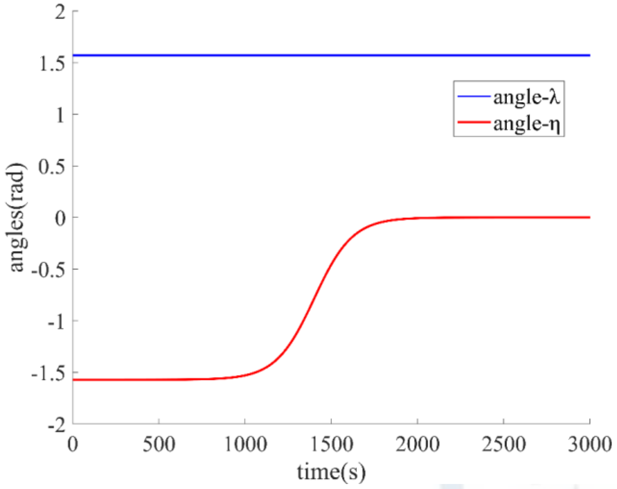
Typically

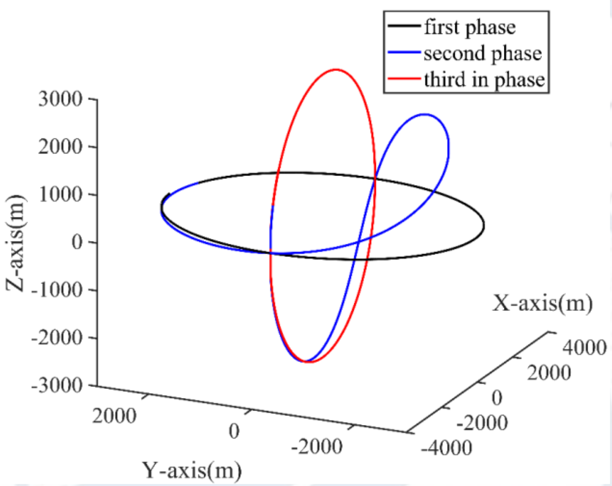
$$\lambda = \frac{\pi}{2}$$

$$\eta = \left(0-\left(-\frac{\pi}{2}\right)\right)\frac{1}{1+\exp\left(-0.009(t-1400)\right)} - \frac{\pi}{2}$$

1 Maneuvering scheme for the spinning plane

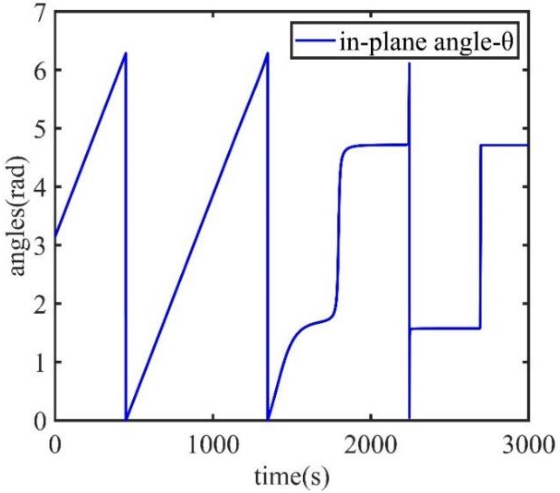
Angles scheme and positions scheme is

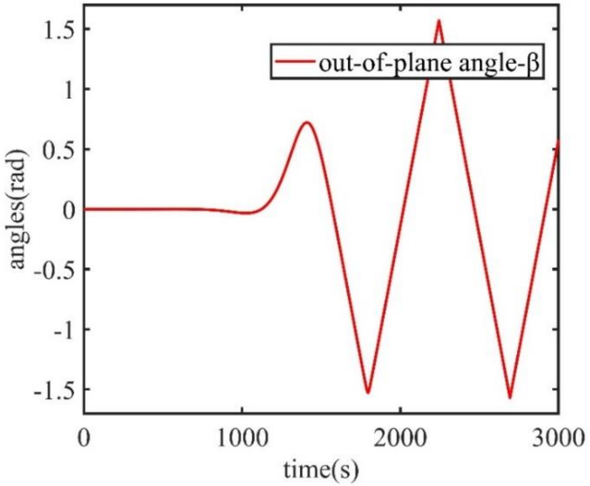




No coupling and singularities problems occur during motion out of the orbital plane..

Correspondingly, the in-plane and out-of-plane angles scheme is





Coupling and singularities problems occur during motion out of the orbital plane.

The control objective for the new model is as follows.

$$\boldsymbol{n}_s \to \boldsymbol{e}_v \qquad\qquad \gamma_d = 0, \alpha_d = \dot{\psi} t$$

$$\lambda_s \to \lambda_v, \eta_s \to \eta_v \qquad\qquad \dot{\gamma}_d = 0, \dot{\alpha}_d = \dot{\psi}$$

This paper assume that the system is always spinning and the initial state is spinning in the plane cxy of $cxyz$

$$\begin{cases} \left[\dot{\gamma}(0) \quad \dot{\alpha}(0)\right] = \left[\dot{\psi} \quad 0\right]^T \\ \left[\ddot{\gamma}(0) \quad \ddot{\alpha}(0)\right] = \left[0 \quad 0\right]^T \\ \dot{\psi} = 0.007 \end{cases}$$

Open-loop objective is $\ddot{\gamma}_d = 0, \ddot{\alpha}_d = 0$. Thus, the open-loop controller solved are

$$\tau_\gamma = -MLf_\gamma, \tau_\alpha = -ML\cos\gamma\, f_\alpha$$

Section 4

Closed-loop controller design

1 System description

Control system

$$\ddot{\gamma} = \frac{\tau_\gamma}{ML} + f_\gamma + \chi_\gamma$$

$$\ddot{\alpha} = \frac{\tau_\alpha}{ML\cos\gamma} + f_\alpha + \chi_\alpha$$

$$|\chi_i| \le \iota_i$$

Disturbance χ_i are caused by the elongation etc. of the flexible tether

Input saturation

$$\tau_\gamma(\tau_{c\gamma}) = \begin{cases} \bar{\tau}_{c\gamma}\,\mathrm{sign}(\tau_{c\gamma}) & \text{if} \quad |\tau_{c\gamma}| > \bar{\tau}_{c\gamma} \\ \tau_{c\gamma} & \text{if} \quad |\tau_{c\gamma}| \le \bar{\tau}_{c\gamma} \end{cases}$$

$$\tau_\alpha \le 0.5N, \tau_\gamma \le 2.5N$$

$$\delta_\gamma = \tau_\gamma - \tau_{c\gamma}$$

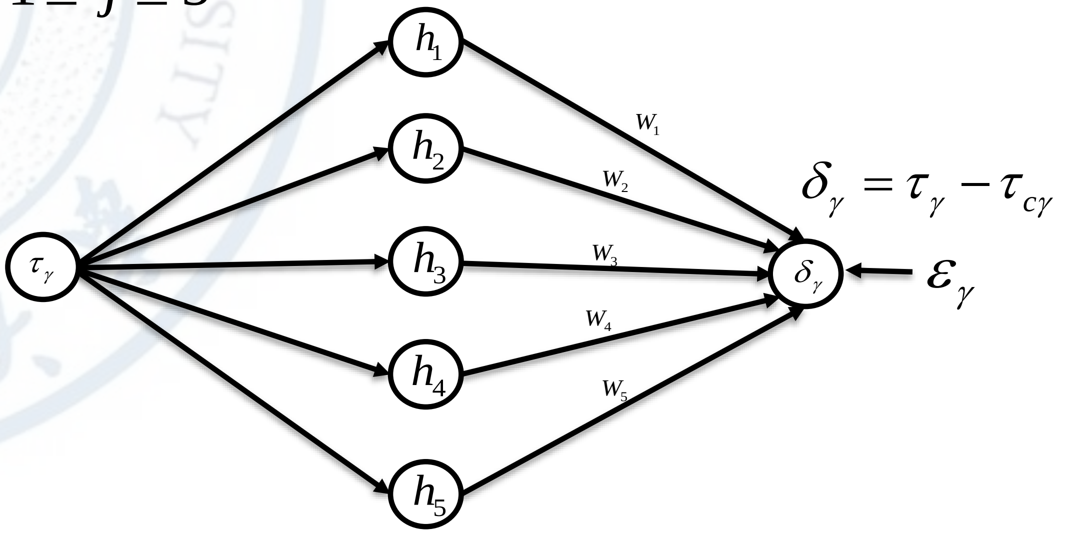
RBF network

$$h_j = \exp\left(\frac{\left\|x_\gamma - \varsigma_j\right\|^2}{2b_\gamma^2}\right)$$

$$\delta_\gamma = \boldsymbol{W}_\gamma^{*T}\boldsymbol{h}_\gamma(x_\gamma) + \varepsilon_\gamma$$

$$1 \le j \le 5$$

An ideal RBF network of 3 layers and 5 neurons in this section is used to approximate parameter.



The sliding-mode functions.

$$s_i = c_i e_i + \dot{e}_i, i = \gamma, \alpha$$

The sliding-mode saturated controller with RBF network compensation is as follows.

$$\tau_\gamma = ML\left(-c_\gamma \dot{e}_\gamma - f_\gamma - \kappa_\gamma \operatorname{sgn}\left(s_\gamma\right)\right) - \hat{\delta}_\gamma$$

$$\tau_\alpha = ML\cos\gamma\left(-c_\alpha \dot{e}_\alpha - f_\alpha - \kappa_\alpha \operatorname{sgn}\left(s_\alpha\right)\right)$$

The output of the RBF network

$$\hat{\delta}_\gamma = \hat{\boldsymbol{W}}_\gamma^T \boldsymbol{h}_\gamma$$

Take adaptive laws are as follows.

$$\dot{\hat{\boldsymbol{W}}}_\gamma = \frac{1}{\Xi_\gamma} s_\gamma \frac{1}{ML} \boldsymbol{h}_\gamma, \Xi_\gamma > 0$$

3 Closed-loop controller

The difference between the ideal RBF network and the RBF network of controller is follows.

$$\tilde{\boldsymbol{W}}_\gamma = \hat{\boldsymbol{W}}_\gamma - \boldsymbol{W}_\gamma^*$$

$$\delta_\gamma - \hat{\delta}_\gamma = \boldsymbol{W}_\gamma^{*T}\boldsymbol{h}_\gamma + \varepsilon_\gamma - \hat{\boldsymbol{W}}_\gamma^T\boldsymbol{h}_\gamma = \left(\boldsymbol{W}_\gamma^{*T} - \hat{\boldsymbol{W}}_\gamma^T\right)\boldsymbol{h}_\gamma + \varepsilon_\gamma = -\tilde{\boldsymbol{W}}_\gamma^T\boldsymbol{h}_\gamma + \varepsilon_\gamma$$

Lyapunov functions

$$V_\gamma = \frac{1}{2}s_\gamma^2 + \frac{1}{2}\Xi_\gamma \tilde{W}_\gamma \tilde{W}_\gamma^T$$

$$V_\alpha = \frac{1}{2}s_\alpha^2$$

If the design parameters hold $\kappa_\gamma \geq \iota_\gamma + \frac{1}{ML}\varepsilon_{\max}$, $\kappa_\alpha \geq \iota_\alpha$, the system with closed-loop controller is asymptotically stable.

$$\begin{aligned}\dot{V}_\gamma &= -\kappa_\gamma|s_\gamma| + \left(\chi_\gamma + \frac{1}{ML}\varepsilon_\gamma\right)s_\gamma \\ &\leq -\left(\kappa_\gamma - \left(\iota_\gamma + \frac{1}{ML}\varepsilon_{\max}\right)\right)|s_\gamma| \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\dot{V}_\alpha &= -\kappa_\alpha|s_\alpha| + \chi_\alpha s_\alpha \\ &\leq -(\kappa_\alpha - \iota_\alpha)|s_\alpha| \\ &\leq 0\end{aligned}$$

Section 3

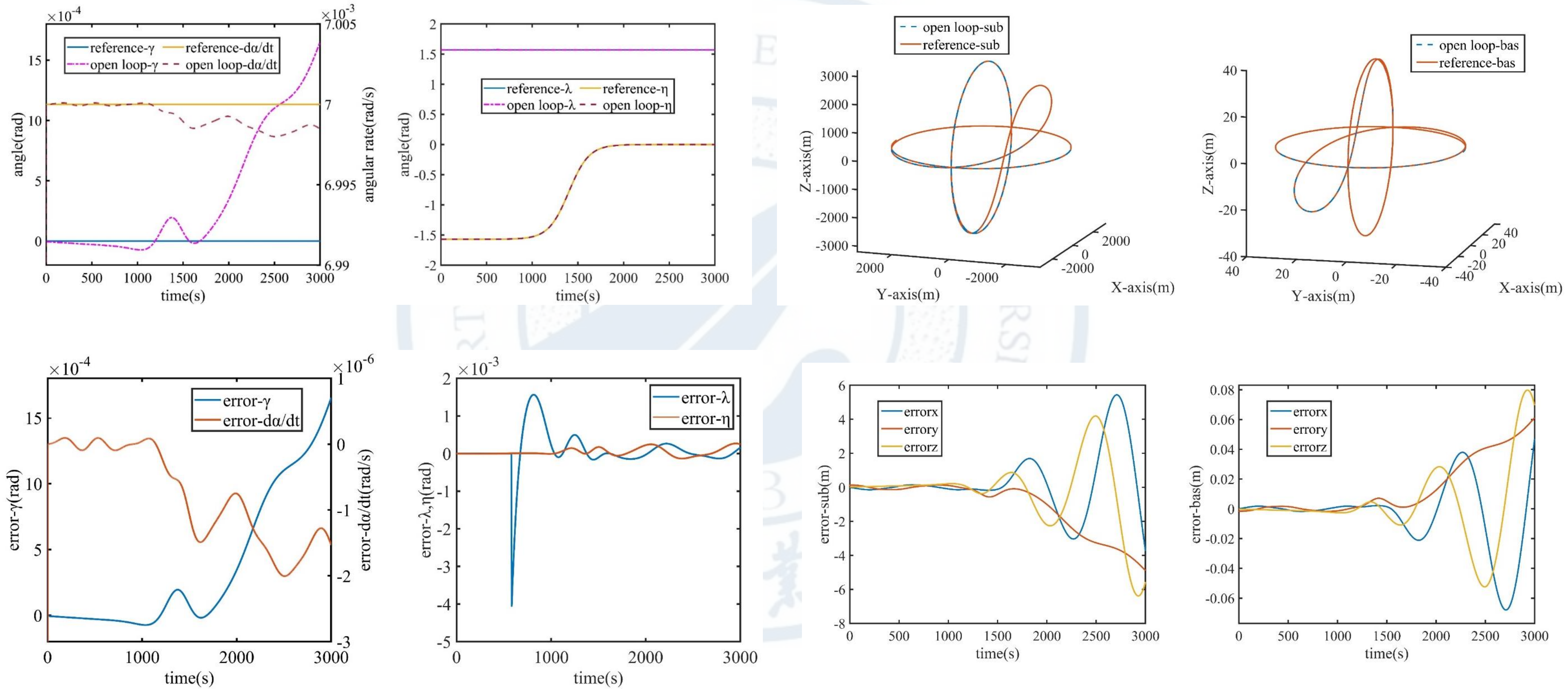
Simulation results

1 ***System motion under open-loop Controller***

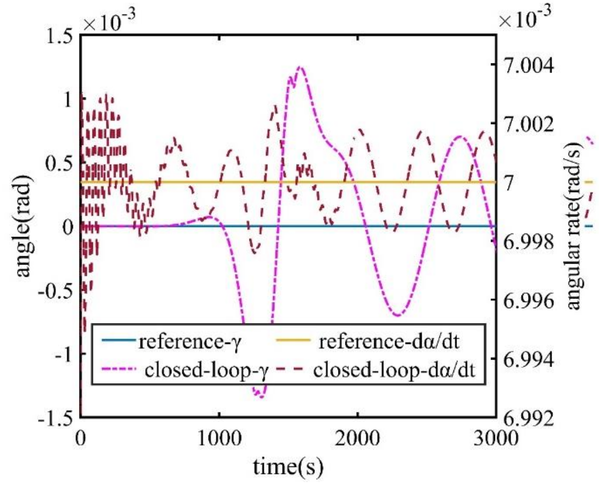
2 ***System motion under closed-loop controller***

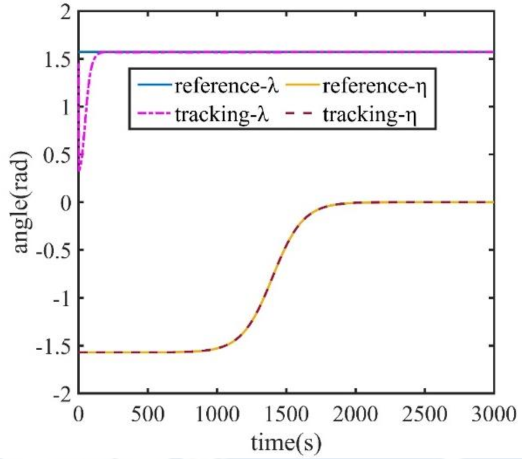
3 ***Thrust and tension during numerical process***

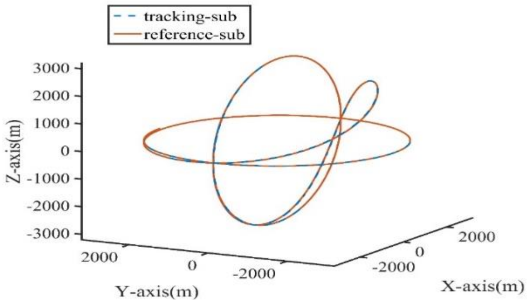
1 System motion under open-loop Controller

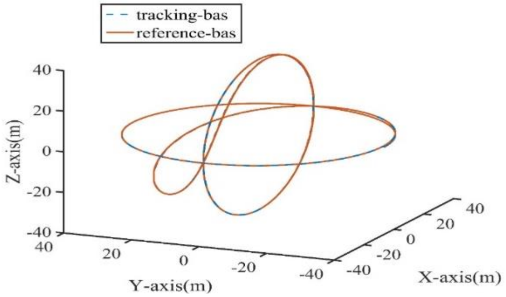


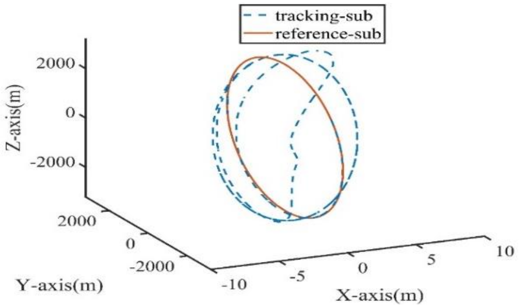
System motion under closed-loop controller

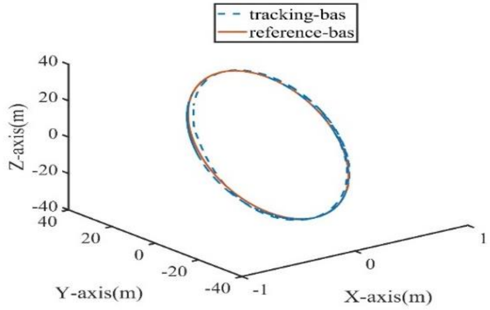


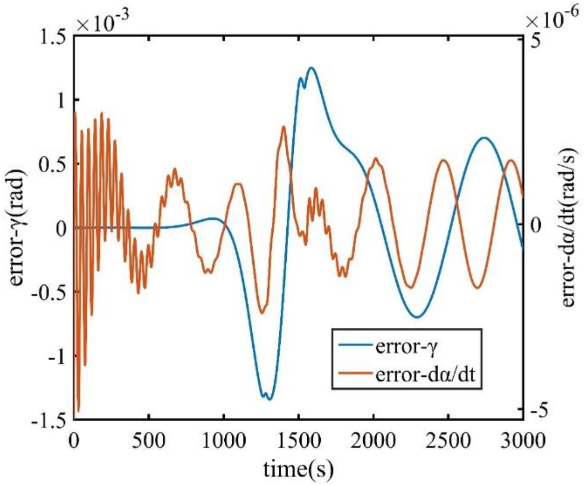


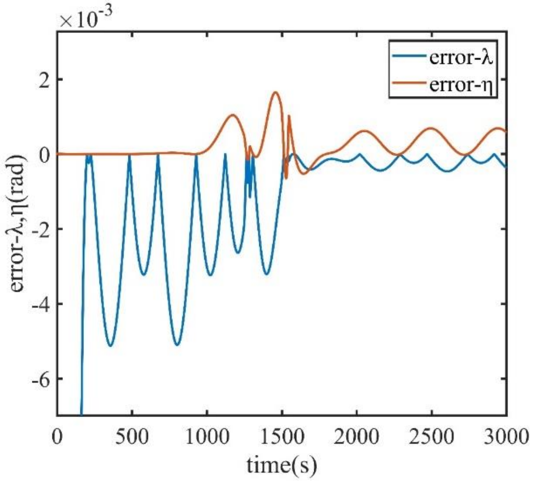


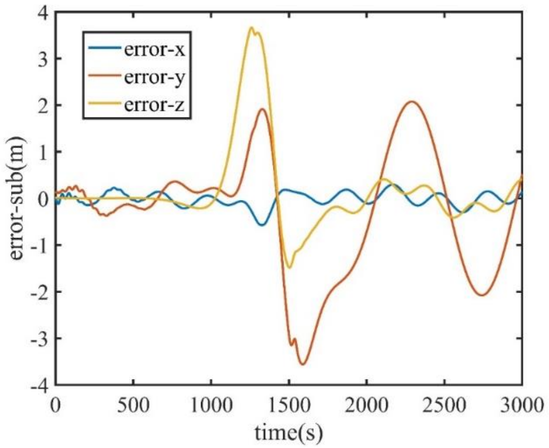


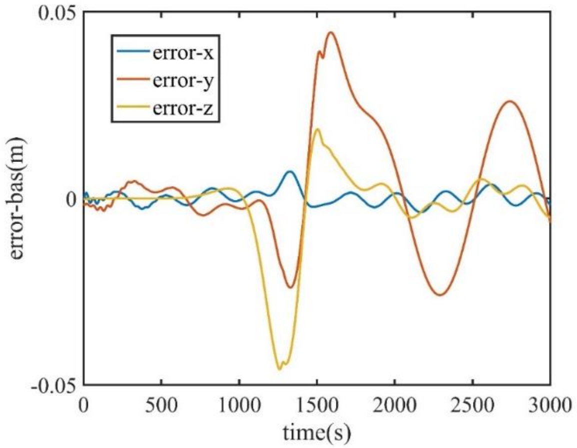






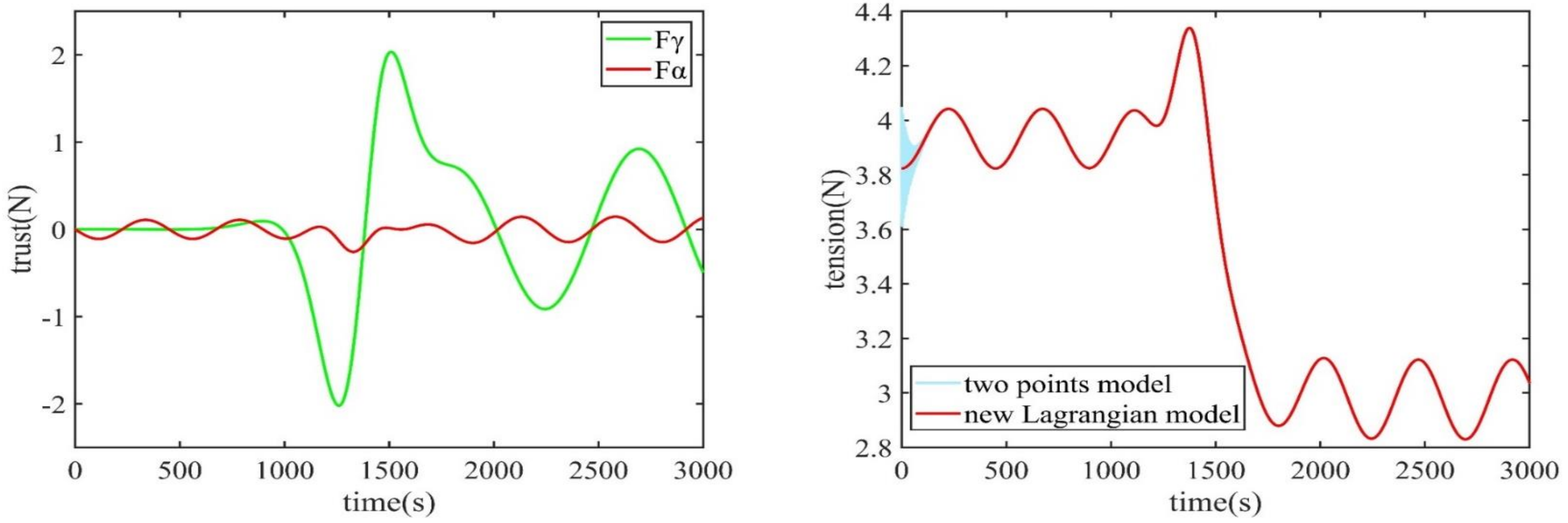




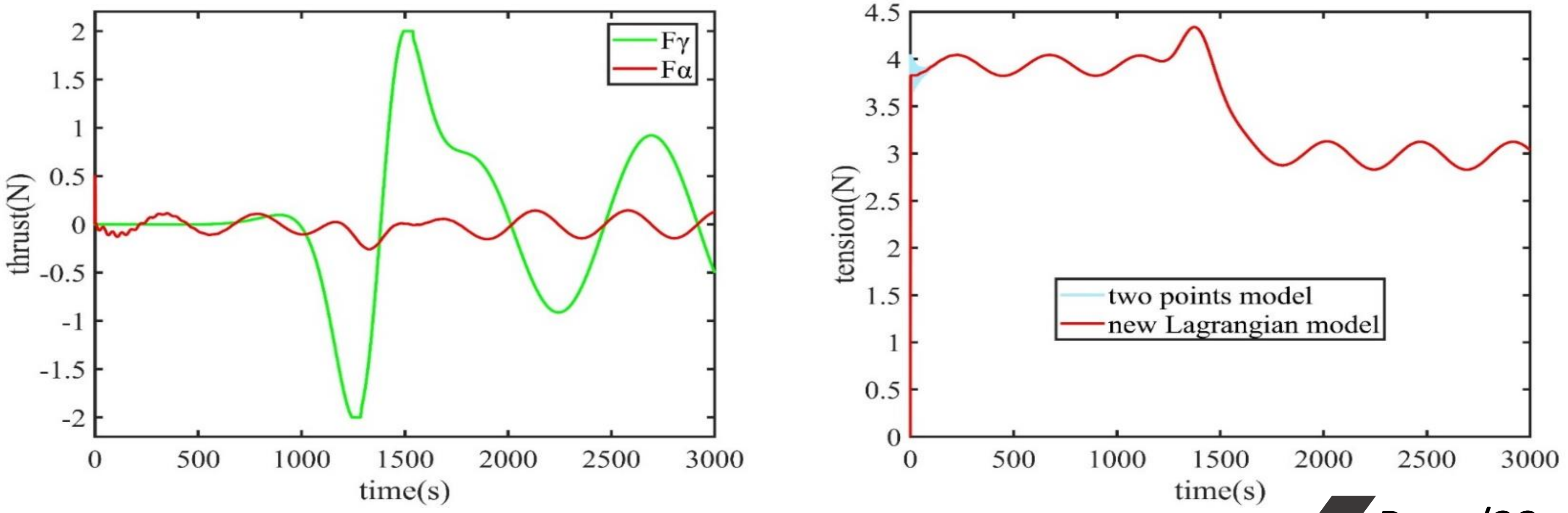


3 The thrusts in open-loop control and closed-loop control

Open-loop control



Closed-loop control



Section 6

Conclusions

Conclusions

Conclusions

This paper investigates the dynamics and control in three-dimensional motion with input saturation.

To solve the singularities and coupling problems of conventional models, this paper proposed a novel coordinate system and A new singularity-free model, which avoids singularities and coupling problems in the case of three-dimensional maneuvering.

Subsequently, to complete the three-dimensional maneuver of STS between spinning planes, a smooth maneuvering scheme and control objectives are presented. Accordingly, an open-loop controller is given.

The new Lagrangian model accurately describes the spinning motion of STS without singularities and coupling problems. To hold and maneuver spinning motion with limited thrust, it's necessary to design a closed-loop anti-saturation controller.

As a result, the proposed Lagrangian model does not exhibit singularities and coupling problems during STS three-dimensional maneuvering processes, and the designed closed-loop controller can successfully control STS maneuvering processes. The thrust can rapidly return to a non-saturated state with the assistance of anti-saturation of closed-loop controller.

Thanks for Listening

Question & Answer