西安電子科技大學
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A Comprehensive Investigation of Electric Solar Wind Sail Coning Motion

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CONTENT

1 Introduction

Background

Electric solar wind sail (E-sail) is a novel space propulsion technology that harnesses energy by repelling the protons in solar wind. The most prominent E-sail design features a central spacecraft connected to long, thin tethers (main tethers) with remote units at the tips of each tether. The remote units are connected by non-conductive tethers (auxiliary tethers) to prevent the main tethers from winding together. The positively charged main tethers create a static electric field that repels protons, generating thrust for the E-sail.

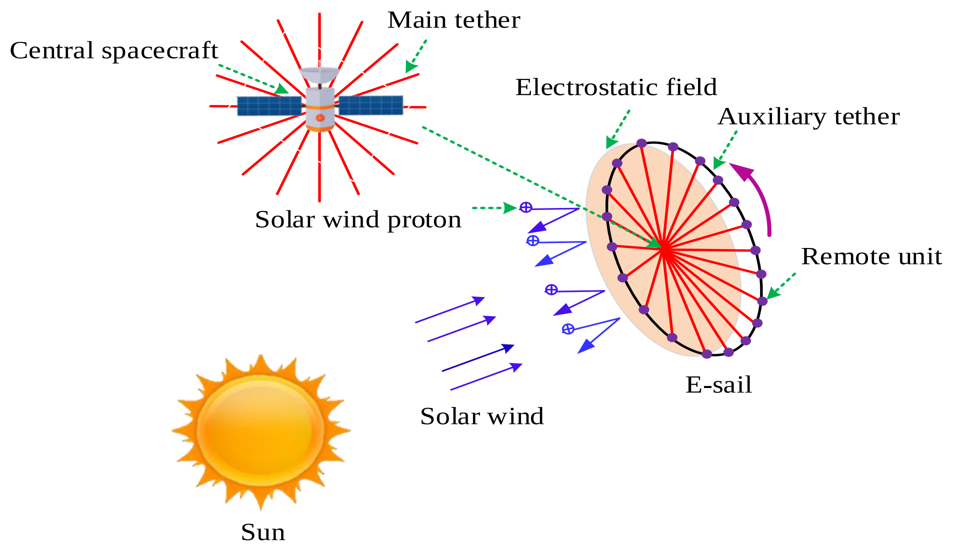
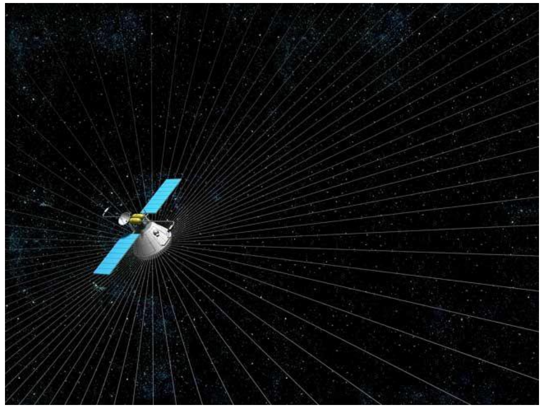
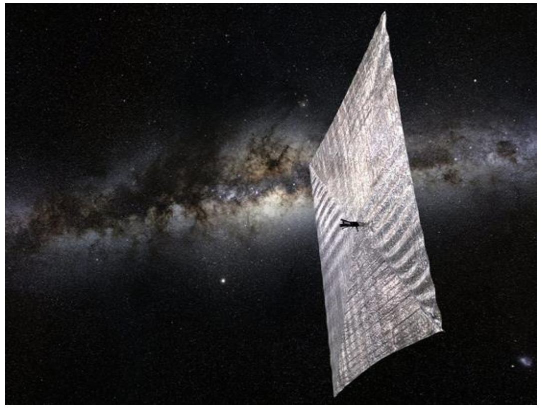


Fig. 1. Configuration and operational principle of E-sail.

Advantage

		Payload fractions	Degradation of propulsive force over distances	Attitude control
E-sail		Higher	Slower	Easier-to-tune
Solar sail		Lower	Faster	Harder-to-tune

Coning motion

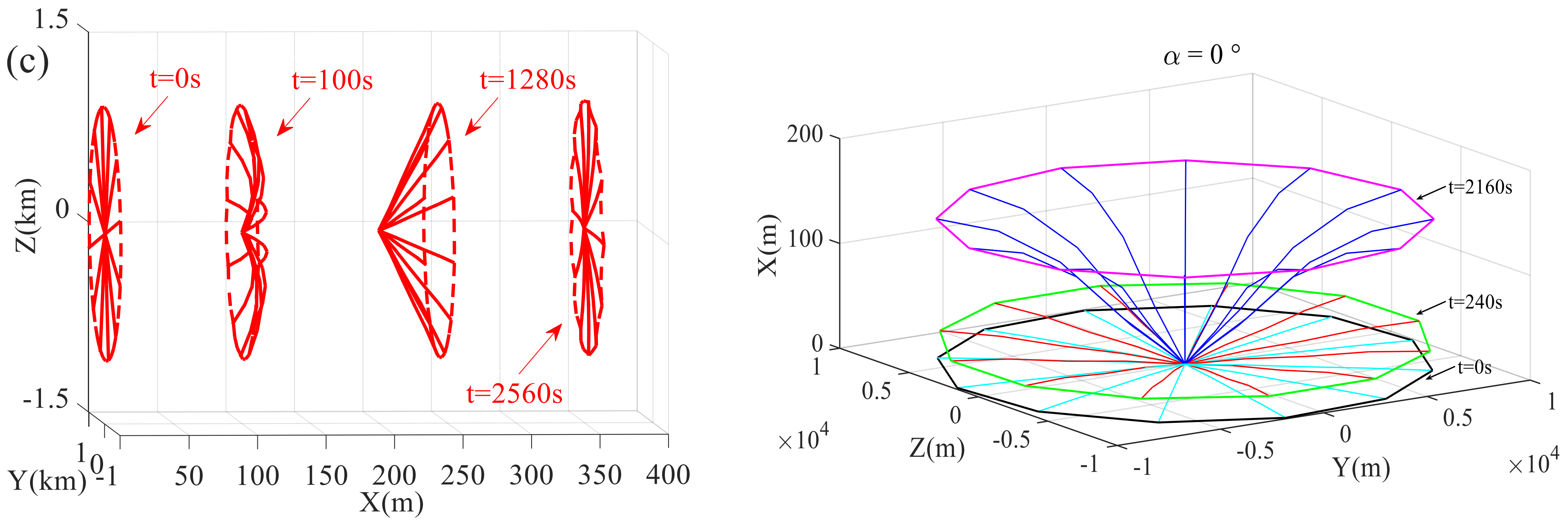


Fig. 2. Schematic diagram of periodic coning motion.

Probable perils

The E-sail's prolonged lifespan and thin tether diameter pose severe probable perils if the continuous coning motion is not properly controlled.

Peril 1

Accumulative fatigue from repeated stress cycles in tethers heighten the risk of tether breakage and compromising the E-sail's functionality.

Peril 2

The coning motion adds complexity to controlling the E-sail's attitude and trajectory.

Peril 3

Chronic tether bending degrades electric conductivity, reducing propulsive force and causing trajectory deviations

2 Mechanism of periodic coning motion

Mechanism

Consider the i_{th} main tether as a sample as shown in Fig. 3(a) in the $O_oX_oY_oZ_o$ coordinate system, the propulsive force will generate a torque τ_t that pushes the tether rotating out of the nominal spin plane with respect to the central spacecraft, and the centrifugal forces due to the inertia of the i_{th} main tether and remote unit will generate a restoring torque τ_c to push the main tether rotating back to the nominal spin plane.

$$\begin{cases} \tau_t > \tau_c \longrightarrow \dot{\beta}\uparrow \\ \tau_t = \tau_c \longrightarrow \dot{\beta}_{\max} \\ \tau_t < \tau_c \longrightarrow \dot{\beta}\downarrow \end{cases}$$

Where β and $\dot{\beta}$ are the coning angle and its velocity of the E-sail

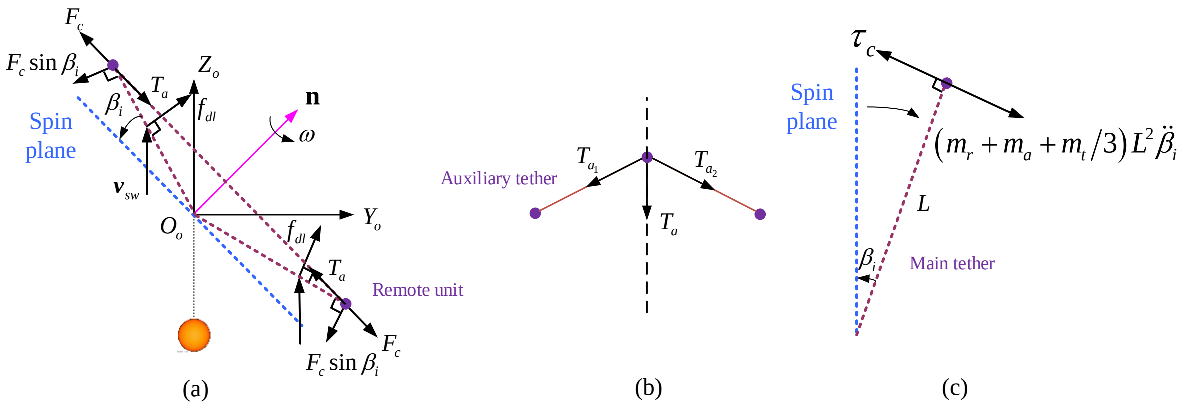


Fig. 3. Free-body diagram of an E-sail.

Period

From the free-body diagram, the equation of the coning motion is derived from the moment balance about the central spacecraft, as shown in Fig. 3(c), that is,

$$(m_r + m_a + m_t/3)L^2\ddot{\beta}_i = -\tau_c$$

For a small β_i, $\cos\beta_i \approx 1$ and $\sin\beta_i \approx \beta_i$. Therefore, the Equation is simplified as

$$\ddot{\beta}_i + \left\{\omega^2 - T_a/\left[(m_r + m_a + m_t/3)L\right]\right\}\beta_i = 0$$

Thus, the angular frequency of the coning motion is obtained as,

$$\hat{\omega} = \sqrt{\omega^2 - T_a/\left[(m_r + m_a + m_t/3)L\right]}$$

According, the period of the coning motion is,

$$T_p = \frac{2\pi}{\hat{\omega}}$$

3 Equilibrium state of axially symmetric E-sail

Initial elongation of the main tether

Assume that all tethers are straight and taut. Then, the elongation of the main tethers, ΔL_m , after the E-sail is fully deployed can be determined by the free-body diagram of the remote unit as shown in Fig. 4(a), such that,

$$
\begin{cases}
F_c = T \\
F_c = (m_r + m_{au})(L_{m0} + \Delta L_m)\omega^2 \\
\quad + \int_0^{L_{m0}+\Delta L_m} \rho l L_{m0} \omega^2 / (L_{m0} + \Delta L_m) dl \\
\quad = m_{eff} L_{total} \omega^2 \\
T = T_m + 2T_{au} \sin(\pi/\mathbb{N})
\end{cases}
$$

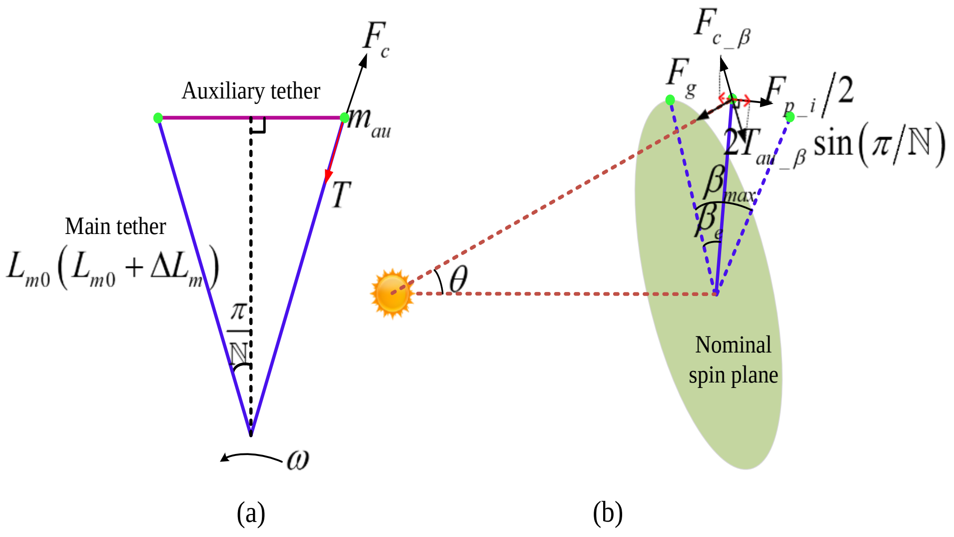


Fig. 4. (a) Geometric relationship among main and auxiliary tethers; (b) Coning motion equilibrium position.

Initial elongation of the main tether

where

$$m_{eff} = \left(m_r + m_{au} + L_{m0}\rho/2\right)$$

T_m and T_{au} are determined by Hooke' s law,

$$\begin{cases} T_m = E_m A_m \Delta L_m / L_{m0} \\ T_{au} = E_{au} A_{au} \Delta L_{au} / L_{au0} \end{cases}$$

$$\begin{cases} L_{au0} = 2L_{m0}\sin\left(\pi/\mathbb{N}\right) \\ \Delta L_{au} = 2\Delta L_m \sin\left(\pi/\mathbb{N}\right) \end{cases}$$

Thus, the initial elongation of the main tether is

$$\Delta L_m = \frac{m_{eff} L_{m0}^2 \omega^2}{E_m A_m + 2E_{au}A_{au}\sin\left(\pi/\mathbb{N}\right) - m_{eff} L_{m0}\omega^2}$$

Coning angle

Recalling that the E-sail’ s coning motion is harmonic, the equilibrium state of E-sail can be obtained through the coning motion’ s equilibrium position where the restoring force is zero, as shown in Fig. 4(b), such that

$$F_{c\_\beta}\sin\beta_e + F_g\cos(\beta_e+\theta) = F_{p\_i}/2 + 2T_{au\_\beta}\sin(\pi/\mathbb{N})\sin\beta_e$$

where the centrifugal force F_{c_β}, gravitational force F_g and tension T_{au_β} can be written as

$$\begin{cases} F_{c\_\beta} = F_c\cos\beta_e \quad T_{au\_\beta} = T_{au}\cos\beta_e \\ F_g = -\mu_\odot m_{eff}/d_{sr}^2 \end{cases}$$

Coning angle

Then, the propulsive force acting on the main tether ($\boldsymbol{F}_{p\_i}$) is derived as

$$\boldsymbol{F}_{p\_i} = \boldsymbol{f}_{dl}\left(L_{m0} + \Delta L_m\right)$$

where

$$\boldsymbol{f}_{dl} = 0.18\max\left(0, V_i - V_w\right)\sqrt{\varepsilon_0 m_p n}\ \boldsymbol{v}_\perp = \sigma \boldsymbol{v}_\perp$$

$$\boldsymbol{v}_{\perp\_i}^b = \nu\left(\boldsymbol{l}_i^b \times \boldsymbol{r}^b\right)\times \boldsymbol{l}_i^b$$

$$\begin{cases} \boldsymbol{l}_i^b = \left[\cos\beta_e\cos\xi_i \quad \cos\beta_e\sin\xi_i \quad \sin\beta_e\right]^T \\ \xi_i = 2\pi\left(i-1\right)/\mathbb{N} \end{cases}$$

$$\boldsymbol{v}_{\perp\_i}^b = \nu\sin\alpha\begin{bmatrix} \cos^2\beta_e\cos^2\xi_i - \cot\alpha\sin\beta_e\cos\beta_e\cos\xi_i - 1 \\ \cos^2\beta_e\sin\xi_i\cos\xi_i - \cot\alpha\sin\beta_e\cos\beta_e\sin\xi_i \\ \cot\alpha\cos^2\beta_e + \sin\beta_e\cos\beta_e\cos\xi_i \end{bmatrix}$$

Coning angle

To solve the coning angle β_e , the first main tether $i=1$ is selected to obtain $\boldsymbol{F}_{p\_1}$ without the loss of generality, which results

$$\boldsymbol{v}_{\perp\_1}^{b} = \nu \begin{bmatrix} -\sin\alpha\sin^2\beta_e - \cos\alpha\sin\beta_e\cos\beta_e \\ 0 \\ \cos\alpha\cos^2\beta_e + \sin\alpha\sin\beta_e\cos\beta_e \end{bmatrix}$$

$$F_{p\_1} = \left|\boldsymbol{F}_{p\_1}\right| = \sigma\nu L_{total}\left(\sin\alpha\sin\beta_e + \cos\alpha\cos\beta_e\right)$$

For a small β_e, $\cos\beta_e \approx 1$ and $\sin\beta_e \approx \beta_e$. Thus, the propulsion force is simplified as

$$F_{p\_1} = \sigma\nu L_{total}\left(\beta_e\sin\alpha + \cos\alpha\right)$$

Coning angle

Then, the coning angle at the equilibrium state can be obtained as

$$\beta_e = \frac{\sigma \nu L_{total} \cos\alpha}{2\left(F_c + F_g - 2T_{au}\sin\left(\pi/\mathbb{N}\right)\right) - \sigma \nu L_{total} \sin\alpha}$$

Furthermore, the maximum coning angle, , can be determined intuitively from the geometry shown in Fig. 4(b), such that,

$$\beta_{max} = 2\beta_e = \frac{\sigma \nu L_{total} \cos\alpha}{\left(F_c + F_g - 2T_{au}\sin\left(\pi/\mathbb{N}\right)\right) - \sigma \nu L_{total} \sin\alpha/2}$$

4 Simulation results and discussion

Mechanism and period

Table 1. Parameters of the parametric investigation.

Label	$m_r\,(kg)$	$\omega_0\,(rad/s)$	E-sail configuration	Initial coning angle (degree)
Case A	0.5	0.004	No auxiliary tether	0
Case B	1.5	0.004	No auxiliary tether	0
Case C	0.5	0.004	Auxiliary tether	0
Case D	1.5	0.004	Auxiliary tether	0
Case E	1.5	0.003	No auxiliary tether	0
Case F	1.5	0.003	Auxiliary tether	0

Mechanism and period

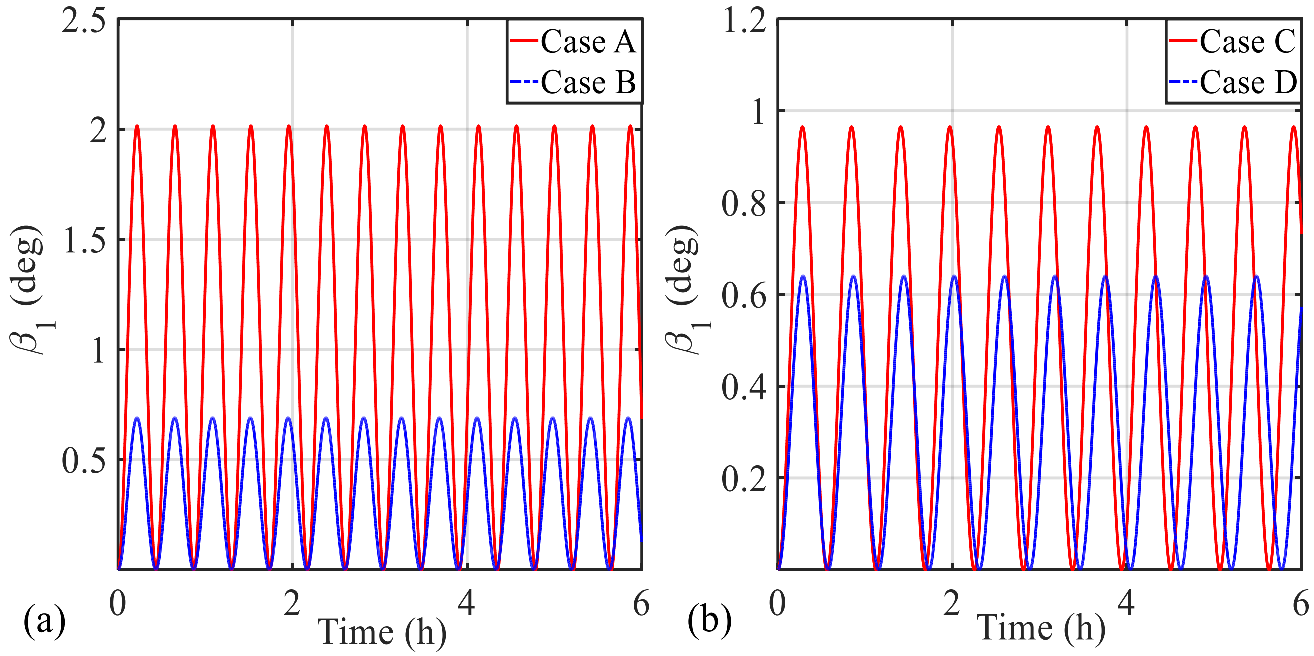


Fig. 5. Time histories of the coning motion of different remote unit masses without (a) and with (b) the auxiliary tether.

Mechanism and period

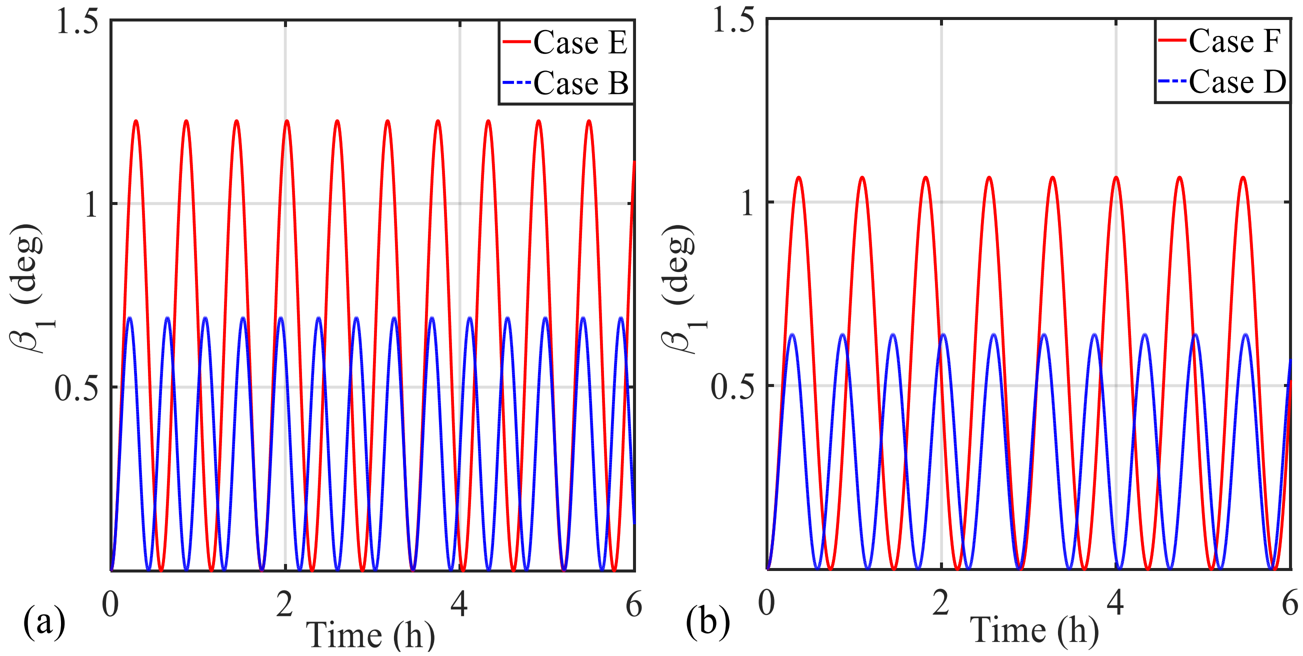


Fig. 6. Time histories of the coning motion of different initial spin rates without (a) and with (b) the auxiliary tether.

Maximum coning angle

Table 2. Physical Properties of tethers.	
Parameters	Values
Remote unit mass (kg)	1.5
Main tether Young’ s modulus (GPa)	70
Auxiliary tether Young’ s modulus (GPa)	2.5
Main tether linear density (kgm^{-1})	1.155×10^{-5}
Auxiliary tether linear density (kgm^{-1})	2.705×10^{-4}
Main tether radius (m)	3.690

Maximum coning angle

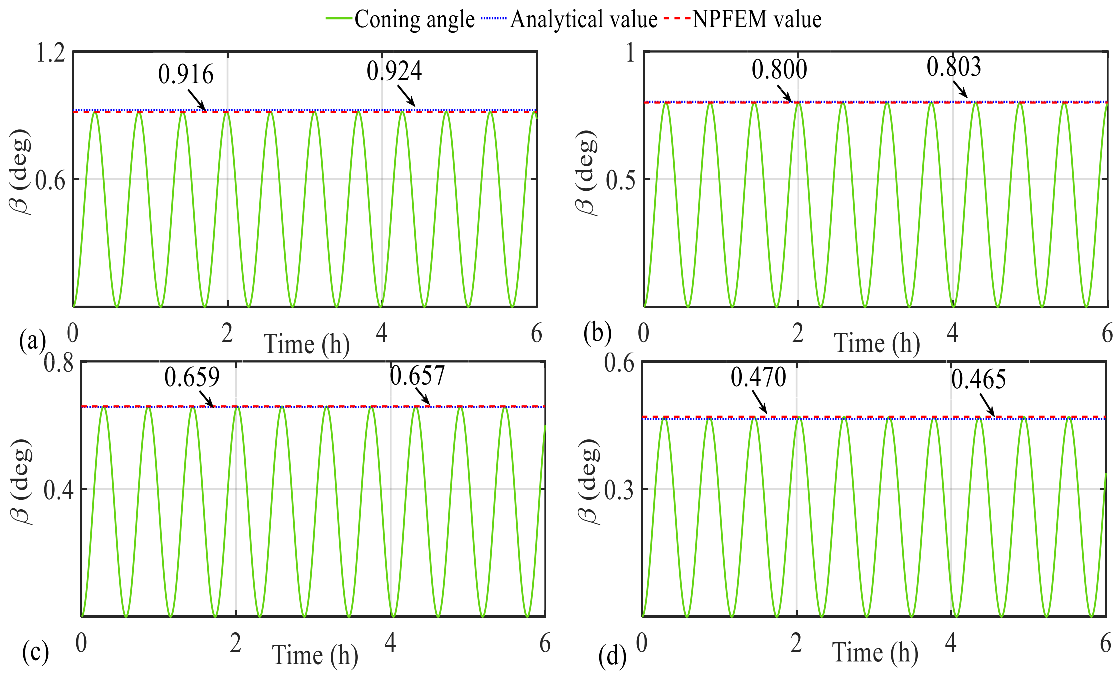


Fig. 7. Time histories of the maximum coning angle with different sail angles (a) $\alpha = 0^\circ$, (b) $\alpha = 30^\circ$, (c) $\alpha = 45^\circ$, and (d) $\alpha = 60^\circ$.

Maximum coning angle

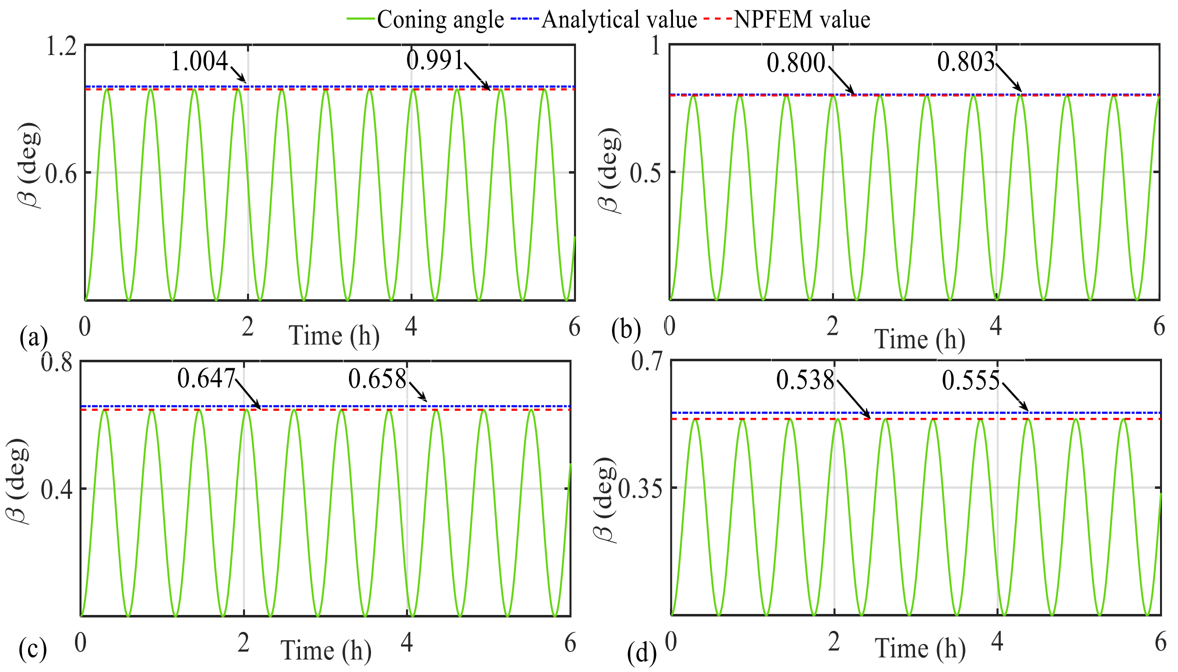


Fig. 8. Time histories of the maximum coning angle with different main tether lengths (a) 5km, (b) 10km, (c) 15km, and (d) 20km.

Coning angle at equilibrium state

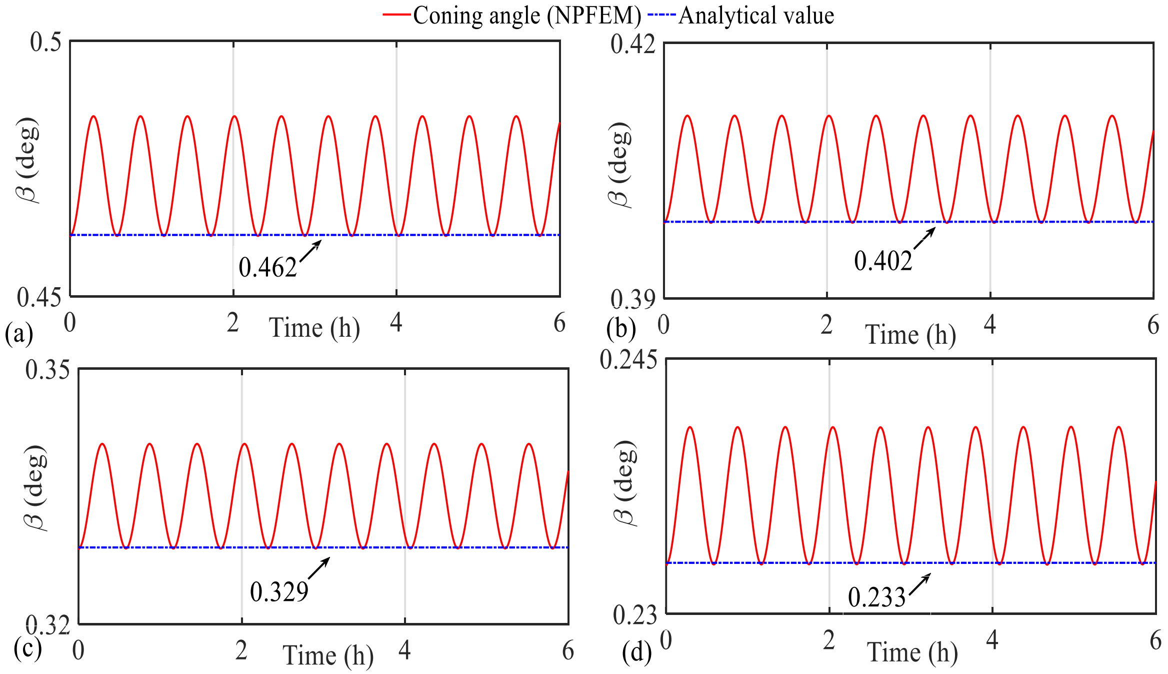


Fig. 9. Time histories of the coning angle under the equilibrium with different sail angles (a) $\alpha = 0^\circ$, (b) $\alpha = 30^\circ$, (c) $\alpha = 45^\circ$, and (d) $\alpha = 60^\circ$.

Coning angle at equilibrium state

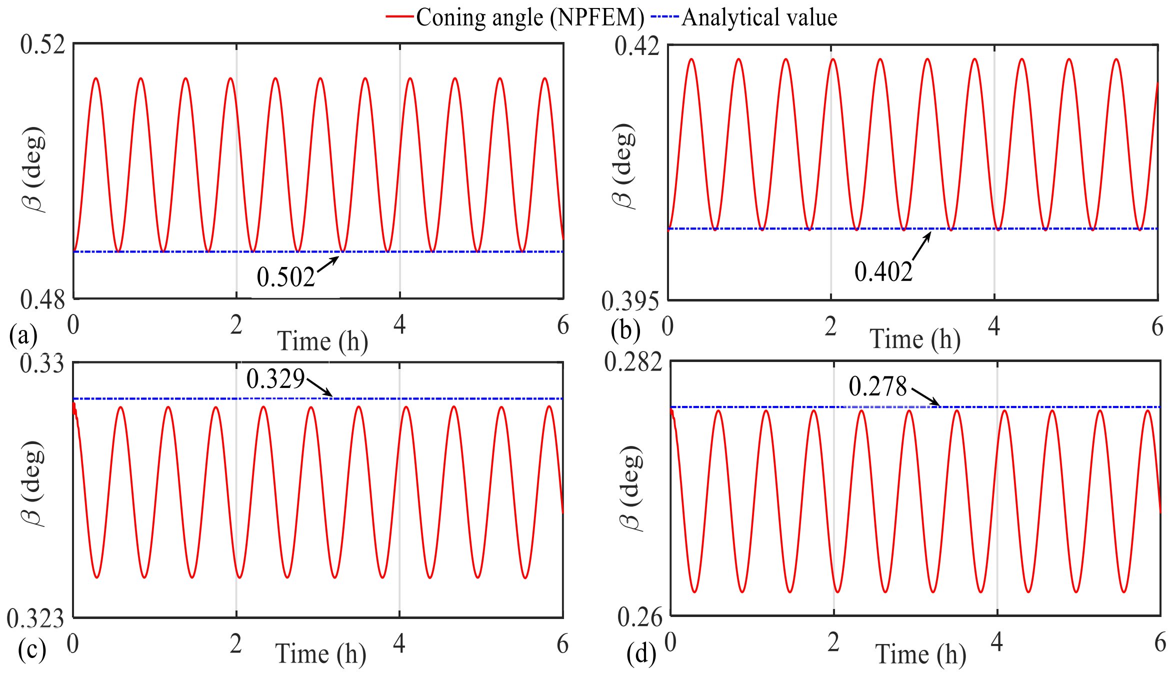


Fig. 10. Time histories of the coning angle under the equilibrium with different main tether lengths (a) 5km, (b) 10km, (c) 15km, and (d) 20km.

5 Conclusions

Conclusions

This work provides a comprehensive study of the periodic coning motion of an axially symmetric E-sail with auxiliary tethers at arbitrary angles.

- This work unveils the underlying mechanism behind the periodic coning motion of the E-sail, and provides the analytic solution for its oscillation frequency.
- This work determines the maximum coning angle and equilibrium state of the E-sail.
- This work examines the E-sail's ability to maintain its equilibrium state from an initial equilibrium configuration.

The research presented in this study lays a solid foundation for the practical application of the E-sail system.

Thank you!