7th International Conference on Tethers in Space

Finite Element Model-Based Computational Control and State Estimation for Flexible Space Tether Systems

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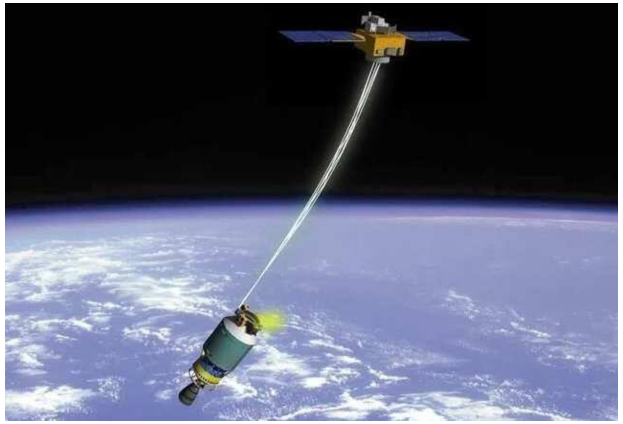
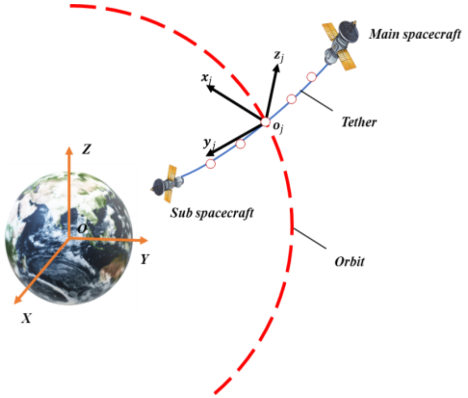
Space Tether System (STS)



- Large-scale
- Continuous deformation
- Complex dynamic characteristics

FE-Model (Computational Mechanics)



- x consists of unmeasurable virtual states
- high-dimensional

Analytical Control

- Closed-form control laws
- Offline computation
- Model-based/non-model based

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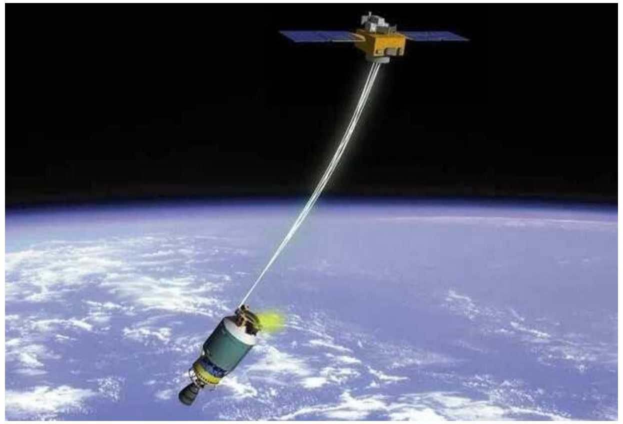
Introduction

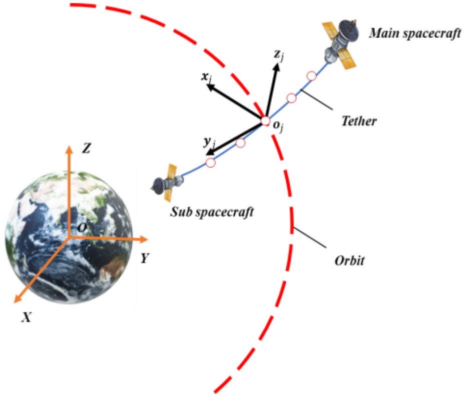
Space Tether System (STS)



- Large-scale
- Continuous deformation
- Complex dynamic characteristics

Framework

FE-Model (Computational Mechanics)



- x consists of thousands of virtual states
- high-dimensional
- not sensible

Analytical Control

- Closed-form control laws
- Offline computation
- Model-based/non-model based

Computational Control

- Numerical algorithm.
- Online computation
- Model-based

Methodology- FE model

The dynamic equation of the element is derived based on the principle of virtual work.

$$\delta U_k - \delta T_k - \delta W_{g,k} - \delta W_{d,k} - \delta W_{c,k} = 0 \quad (1)$$

Work by atmosphere drag

Elastic potential energy

Kinetic energy

Work by gravity force

Work by control force

The motion of equation:

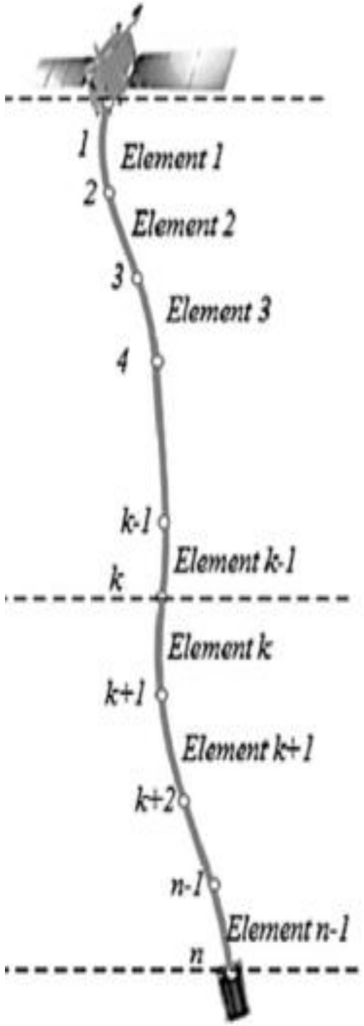
$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{C}\dot{\boldsymbol{X}} + \boldsymbol{K}\boldsymbol{X} = \boldsymbol{F}^{d} + \boldsymbol{F}^{g} + \boldsymbol{B}\boldsymbol{u} \quad (2)$$

State-space equation:

$$\dot{\tilde{\boldsymbol{X}}} = \tilde{\boldsymbol{A}}_d \tilde{\boldsymbol{X}} + \tilde{\boldsymbol{B}}_d \boldsymbol{u}_d + \boldsymbol{F}^{ext} \quad (3)$$

$$\tilde{\boldsymbol{A}}_d = \begin{bmatrix} \boldsymbol{0} & \mathbf{I}_1 \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix} \quad \tilde{\boldsymbol{B}}_d = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}\boldsymbol{B} \end{bmatrix} \quad \boldsymbol{u}_d = \boldsymbol{u} \quad \text{and} \quad \boldsymbol{F}^{ext} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{F}^{d} + \boldsymbol{F}^{g} \end{bmatrix}$$

Discrete model

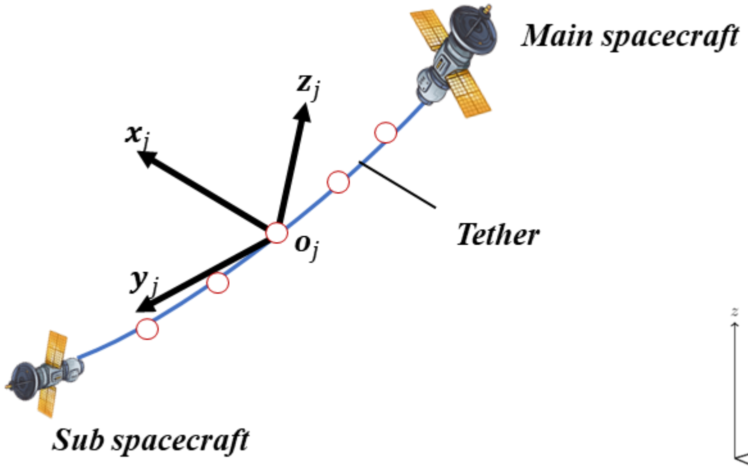


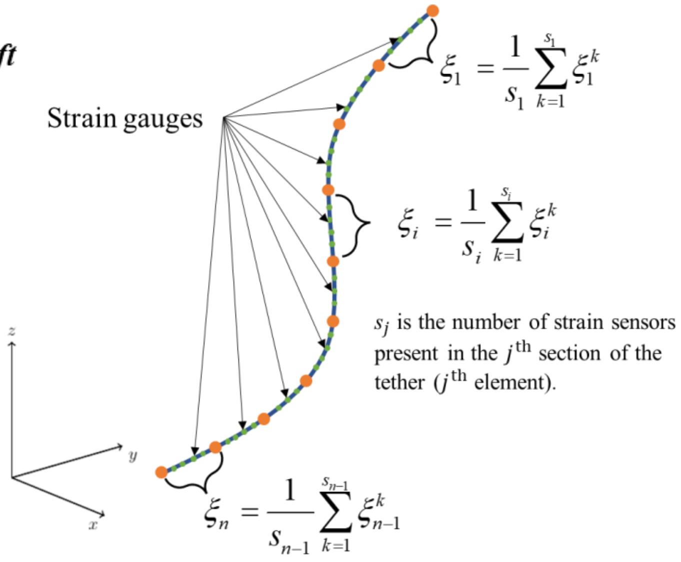
Methodology- State Estimator

Space Tether System

- *NPFEM* → **High Fidelity Dynamic Model**
- *FOSS* → **Strains Along Tether**
- *P&V sensor* → **Observations at Tether Ends**

iNPFEM

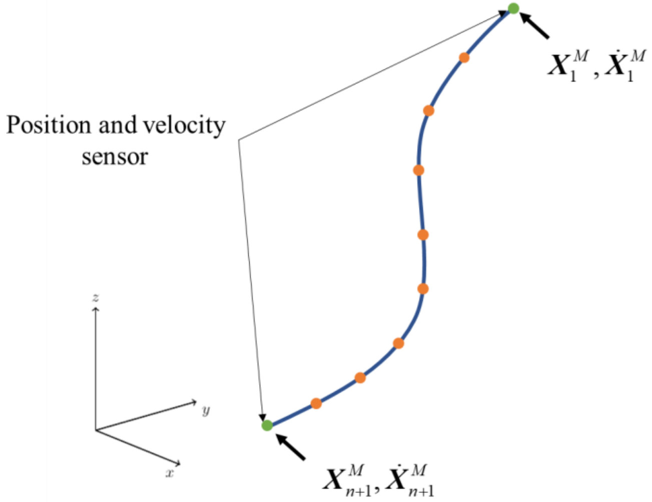




$$\xi_1 = \frac{1}{s_1}\sum_{k=1}^{s_1}\xi_1^k$$

$$\xi_i = \frac{1}{s_i}\sum_{k=1}^{s_i}\xi_i^k$$

$$\xi_n = \frac{1}{s_{n-1}}\sum_{k=1}^{s_{n-1}}\xi_{n-1}^k$$



$X_1^M, \dot{X}_1^M$

$X_{n+1}^M, \dot{X}_{n+1}^M$

$$\boldsymbol{M\ddot{X}} + \boldsymbol{C\dot{X}} + \boldsymbol{KX} = \boldsymbol{F}^d + \boldsymbol{F}^g + \boldsymbol{Bu}$$

$$\Phi_i(\tilde{X}_i) = \left(\xi_i^d - \xi_i\right)^2$$

Measurement (ξ_i^d) Model (ξ_i)

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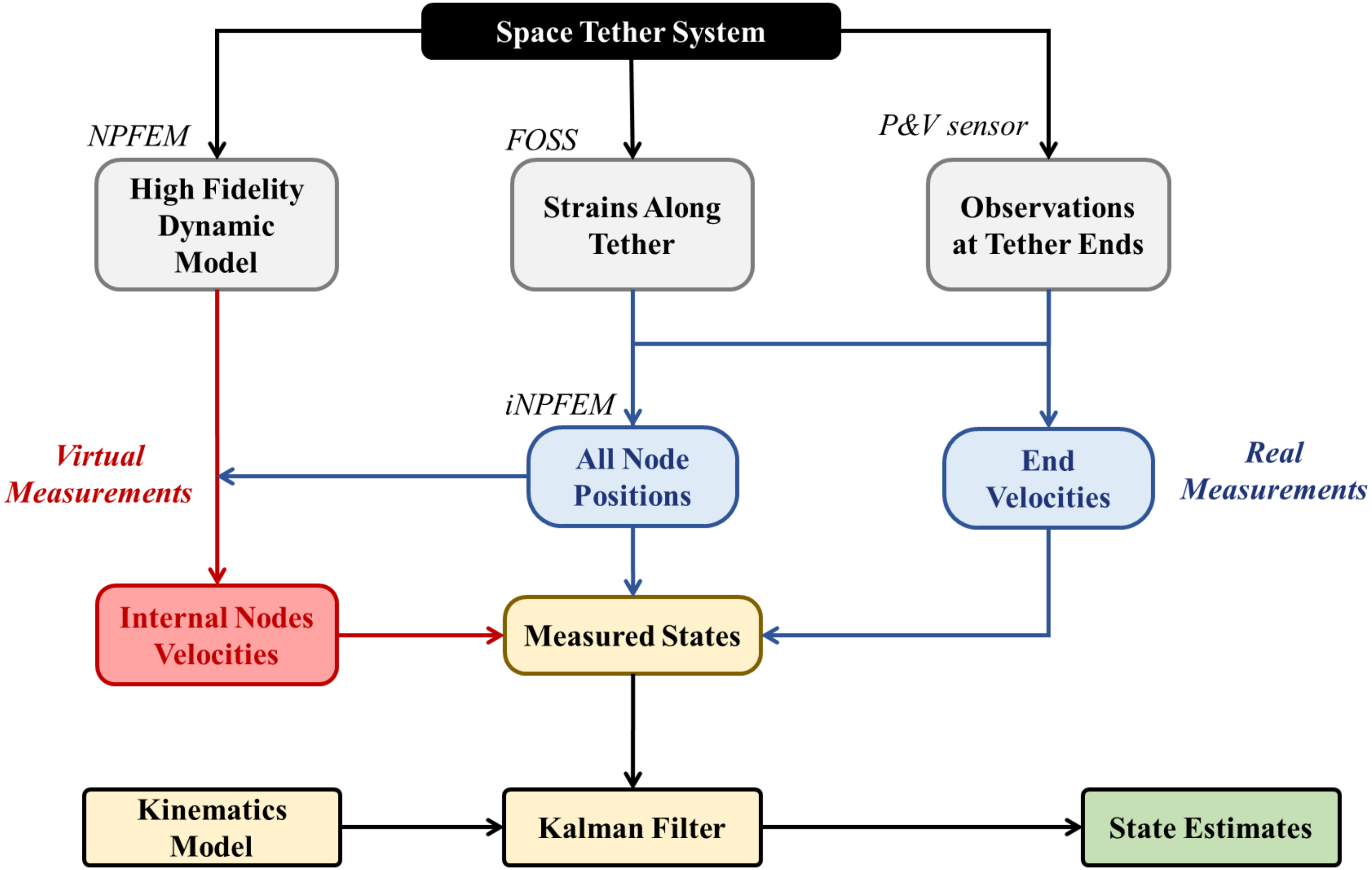
Methodology- State Estimator



Methodology- Computational optimal control

The optimal problem is defined as:

$$\arg\min_{\tilde{\boldsymbol{X}},\boldsymbol{u}_d} J = \frac{1}{2}\left(\tilde{\boldsymbol{X}}_d^{in}(T)-\tilde{\boldsymbol{X}}^{in}(T)\right)^{\mathrm{T}} \boldsymbol{R}_s \left(\tilde{\boldsymbol{X}}_d^{in}(T)-\tilde{\boldsymbol{X}}^{in}(T)\right)+\int_{t_0}^{T}\frac{1}{2}\left(\tilde{\boldsymbol{X}}_d-\tilde{\boldsymbol{X}}\right)^{\mathrm{T}}\boldsymbol{R}\left(\tilde{\boldsymbol{X}}_d-\tilde{\boldsymbol{X}}\right)+\frac{1}{2}\boldsymbol{u}_d^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{u}_d\,dt \qquad (4)$$

The state of the system:

$$\tilde{\boldsymbol{X}}(T)=\begin{bmatrix}\tilde{\boldsymbol{X}}^{in}(T)\\ \tilde{\boldsymbol{X}}^{end}(T)\end{bmatrix}$$

- $\tilde{\boldsymbol{X}}^{in}(T)$ → Internal Nodes
- $\tilde{\boldsymbol{X}}^{end}(T)$ → Spacecraft → $\boldsymbol{u}_d$

Step cost function:

$$L\left(\tilde{\boldsymbol{X}},\bar{\boldsymbol{u}}\right)=\frac{1}{2}\left(\tilde{\boldsymbol{X}}_d-\tilde{\boldsymbol{X}}\right)^{\mathrm{T}}\boldsymbol{R}\left(\tilde{\boldsymbol{X}}_d-\tilde{\boldsymbol{X}}\right)+\frac{1}{2}\bar{\boldsymbol{u}}^{\mathrm{T}}Q\bar{\boldsymbol{u}} \qquad (5)$$

Terminal state at ends: $\tilde{\boldsymbol{X}}^{end}(T)=\tilde{\boldsymbol{X}}_T^{end}$ The target position of the spacecraft

Terminal cost function: $\phi\left(\tilde{\boldsymbol{X}}^{in}(T),T\right)=\frac{1}{2}\left(\tilde{\boldsymbol{X}}_d^{in}(T)-\tilde{\boldsymbol{X}}^{in}(T)\right)^{\mathrm{T}}\boldsymbol{R}_s\left(\tilde{\boldsymbol{X}}_d^{in}(T)-\tilde{\boldsymbol{X}}^{in}(T)\right)$ Vibration reduction

Methodology- Computational optimal control

Define the Hamiltonian function: $$H(\bar{\boldsymbol{x}},\bar{\boldsymbol{u}})=\frac{1}{2}\left(\tilde{\boldsymbol{X}}_d-\tilde{\boldsymbol{X}}\right)^{\mathrm{T}}\boldsymbol{R}\left(\tilde{\boldsymbol{X}}_d-\tilde{\boldsymbol{X}}\right)+\frac{1}{2}\bar{\boldsymbol{u}}^{\mathrm{T}}Q\bar{\boldsymbol{u}}+\boldsymbol{\lambda}_s^{\mathrm{T}}\left(\tilde{\boldsymbol{A}}_d\bar{\boldsymbol{x}}+\tilde{\boldsymbol{B}}_d\boldsymbol{u}+\boldsymbol{F}^{ext}\right) \quad (4)$$

From the stationarity condition: $$\frac{\partial H}{\partial \boldsymbol{u}}=\boldsymbol{Q}\boldsymbol{u}+\boldsymbol{B}^{\mathrm{T}}\boldsymbol{\lambda}_s=0 \Rightarrow \boldsymbol{u}=-\boldsymbol{Q}^{-1}\tilde{\boldsymbol{B}}_d^T\boldsymbol{\lambda}_s \quad (5)$$

State equation: $$\frac{\partial H}{\partial \boldsymbol{\lambda}_s}=\tilde{\boldsymbol{A}}_d\tilde{\boldsymbol{X}}+\tilde{\boldsymbol{B}}_d\boldsymbol{u}+\boldsymbol{F}^{ext}=\tilde{\boldsymbol{A}}_d\tilde{\boldsymbol{X}}-\tilde{\boldsymbol{B}}_d\boldsymbol{Q}^{-1}\tilde{\boldsymbol{B}}_d^{\mathrm{T}}\boldsymbol{\lambda}_s=\dot{\tilde{\boldsymbol{X}}} \quad (6)$$

Costate equation: $$\frac{\partial H}{\partial \tilde{\boldsymbol{X}}}=-\boldsymbol{R}\left(\tilde{\boldsymbol{X}}_d-\tilde{\boldsymbol{X}}\right)+\left(\tilde{\boldsymbol{A}}_d^{\mathrm{T}}\boldsymbol{\lambda}_s+\tilde{\boldsymbol{X}}^{\mathrm{T}}\left(\frac{\partial \tilde{\boldsymbol{A}}_d}{\partial \tilde{\boldsymbol{X}}}\right)^{\mathrm{T}}\boldsymbol{\lambda}_s\right)=-\dot{\boldsymbol{\lambda}}_s \quad (7)$$

Boundary Condition:

$$\tilde{\boldsymbol{X}}^{end}(0)=\tilde{\boldsymbol{X}}_0^{end};\ \tilde{\boldsymbol{X}}^{end}(T)=\tilde{\boldsymbol{X}}_T^{end}$$

$$\boldsymbol{R}\left(\tilde{\boldsymbol{X}}_d^{in}(T)-\tilde{\boldsymbol{X}}^{in}(T)\right)-\boldsymbol{\lambda}^{in}(T)=0$$

$$\boldsymbol{\lambda}_s(T)=\begin{bmatrix}\boldsymbol{\lambda}_{end}(T)\\ \boldsymbol{\lambda}_{in}(T)\end{bmatrix}=0$$

Methodology- Computational optimal control

Two-point boundary value problem
TPBVP

$$\begin{bmatrix} \dot{\tilde{\boldsymbol{X}}} \\ \dot{\boldsymbol{\lambda}}_s \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{A}}_d & \tilde{\boldsymbol{B}}_d^{\mathrm{T}} \boldsymbol{Q}^{-1} \tilde{\boldsymbol{B}}_d \\ \boldsymbol{R} & -\left(\dfrac{\partial \tilde{\boldsymbol{A}}_d}{\partial \tilde{\boldsymbol{X}}} \right)^{\mathrm{T}} \tilde{\boldsymbol{X}} - \tilde{\boldsymbol{A}}_d \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{X}} \\ \boldsymbol{\lambda}_s \end{bmatrix} + \begin{bmatrix} 0 \\ -\boldsymbol{R}\tilde{\boldsymbol{X}}_d \end{bmatrix}$$

$$\tilde{\boldsymbol{X}}^{end}(0) = \tilde{\boldsymbol{X}}_0^{end}; \tilde{\boldsymbol{X}}^{end}(T) = \tilde{\boldsymbol{X}}_T^{end}$$

$$\boldsymbol{R}\left(\tilde{\boldsymbol{X}}_d^{in}(T) - \tilde{\boldsymbol{X}}^{in}(T) \right) - \boldsymbol{\lambda}^{in}(T) = 0 \qquad (8)$$

Explicit Expression:

$$\left(\frac{\partial \tilde{\boldsymbol{A}}_d}{\partial \tilde{\boldsymbol{X}}} \right)_i = \frac{\partial \boldsymbol{A}}{\partial X_i} = \begin{bmatrix} \boldsymbol{0} & 0 \\ -\boldsymbol{M}^{-1} \dfrac{\partial \boldsymbol{K}}{\partial X_i} & -\boldsymbol{M}^{-1} \dfrac{\partial \boldsymbol{C}}{\partial X_i} \end{bmatrix}$$

Element ⇒

$$\boldsymbol{K}_{ji}^{x'} = \frac{\partial \boldsymbol{K}_j}{\partial X_i} = 2EA_0L_0 \left(\frac{\partial \boldsymbol{B}_j}{\partial X_i} \right)^{\mathrm{T}} \boldsymbol{B}_j$$

$$\boldsymbol{C}_{ji}^{x'} = \frac{\partial \boldsymbol{C}}{\partial X_i} = 2\alpha EA_0L_0 \left(\frac{\partial \boldsymbol{B}_j}{\partial X_i} \right)^{\mathrm{T}} \boldsymbol{B}_j$$

Methodology- Closed-loop Control

Closed-loop control

$$\tilde{\boldsymbol{X}}(t_0)=\tilde{\boldsymbol{X}}_0,\ \tilde{\boldsymbol{X}}_{end}(T)=\tilde{\boldsymbol{X}}_T^{end}$$

$$\boldsymbol{R}_s\left(\tilde{\boldsymbol{X}}_d^{in}(T)-\tilde{\boldsymbol{X}}^{in}(T)\right)-\boldsymbol{\lambda}^{in}(T)=0$$

TPBVP

Symplectic adaptive algorithm

$$\begin{bmatrix}\dot{\tilde{\boldsymbol{X}}}\\ \dot{\boldsymbol{\lambda}}_s\end{bmatrix}=\begin{bmatrix}\tilde{\boldsymbol{A}}_d & \tilde{\boldsymbol{B}}_d^{\mathrm{T}}\boldsymbol{Q}^{-1}\tilde{\boldsymbol{B}}_d\\ \boldsymbol{R} & -\left(\dfrac{\partial\tilde{\boldsymbol{A}}_d}{\partial\tilde{\boldsymbol{X}}}\right)^{\mathrm{T}}\tilde{\boldsymbol{X}}-\tilde{\boldsymbol{A}}_d\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{X}}\\ \boldsymbol{\lambda}_s\end{bmatrix}+\begin{bmatrix}0\\ -\boldsymbol{R}\tilde{\boldsymbol{X}}_d\end{bmatrix}$$

Control Input

$$\bar{\boldsymbol{u}}_d=-\boldsymbol{Q}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{\lambda}_s$$

Space tether system

$$\dot{\tilde{\boldsymbol{X}}}=\tilde{\boldsymbol{A}}_d\tilde{\boldsymbol{X}}+\tilde{\boldsymbol{B}}_d\boldsymbol{u}_d+\boldsymbol{F}^{ext}$$

Current state

$$\tilde{\boldsymbol{X}}_0=\tilde{\boldsymbol{X}}(t_0+\Delta t)$$

$$t_0=t_0+\Delta t$$

$$T=T+\Delta t$$

Feedback

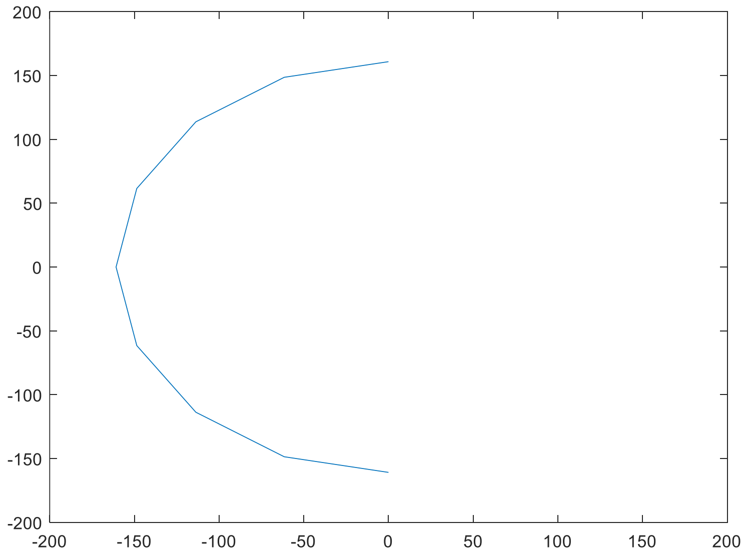
Receded Horizon Control

Simulation

Physical properties of the STS

Parameter	Value
Mass of main/sub-spacecraft (kg)	5
Length of tether (m)	500
Density (kg/m3)	2700
Transverse Area (m2)	2×10^{-7}
Young's Module (MPa)	720
Damping ratio	0

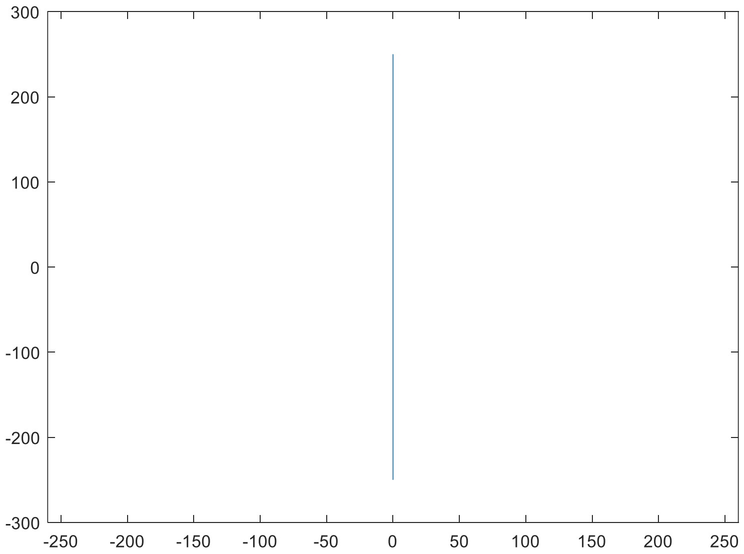
FE Model (8 Elements)



Objective

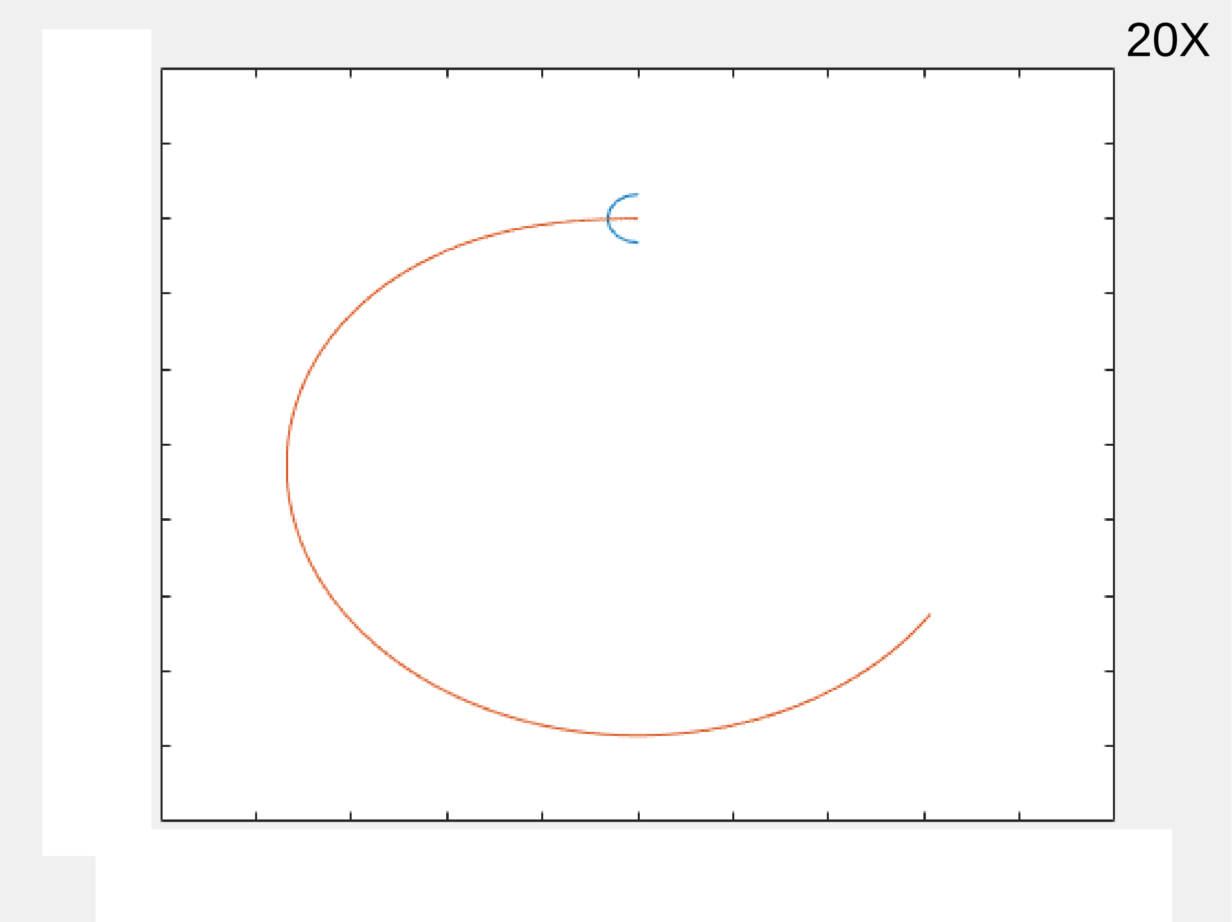
TSS on a fixed orbit

Half circle to Straight line

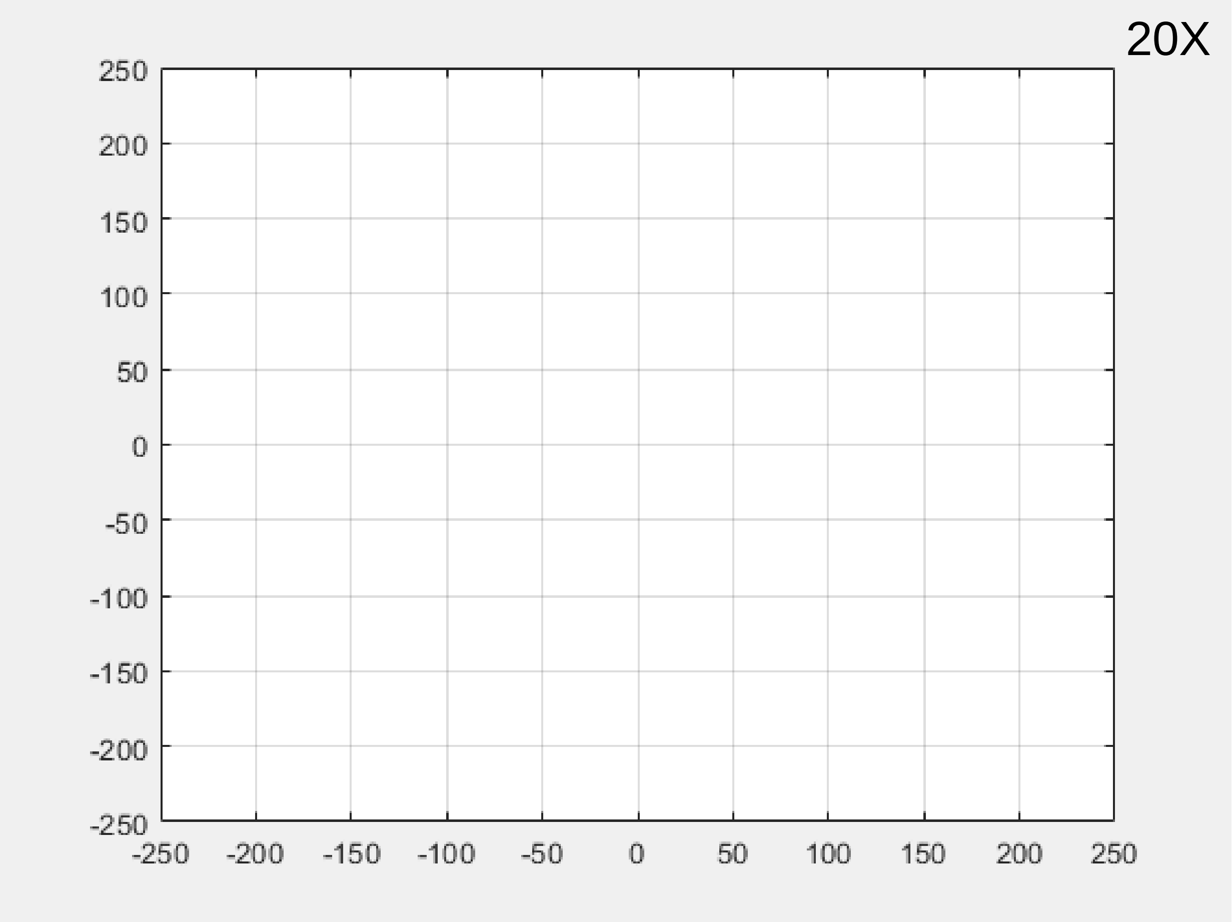


Simulation - Result

The motion on orbit



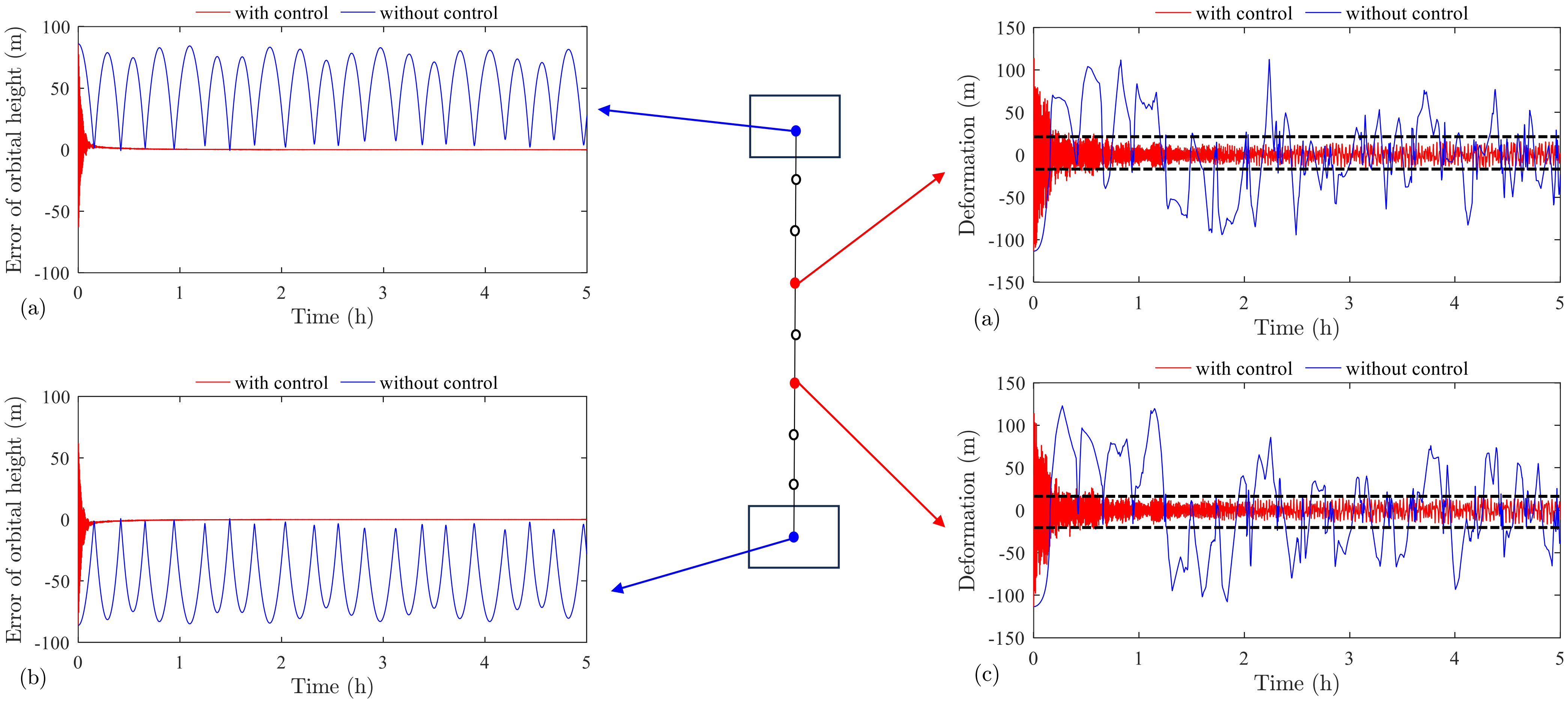
Vibration



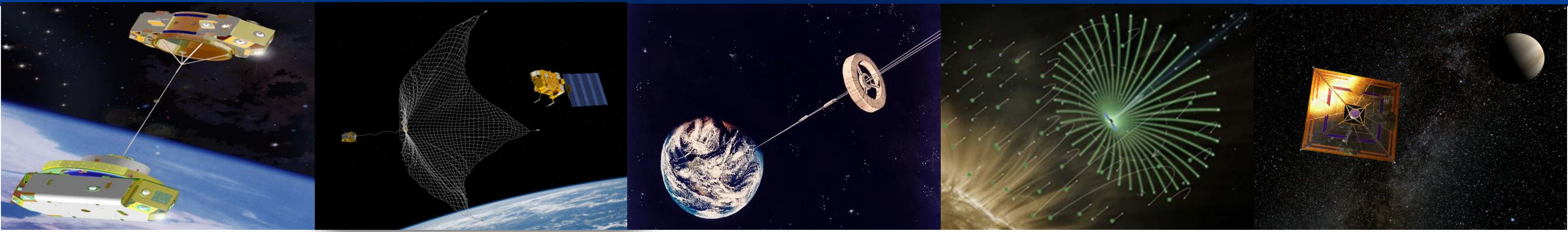
Simulation - Result

TSS on a fixed orbit

Reduce the vibration



Conclusion



1. Established a high-fidelity dynamic model of the space tether system.

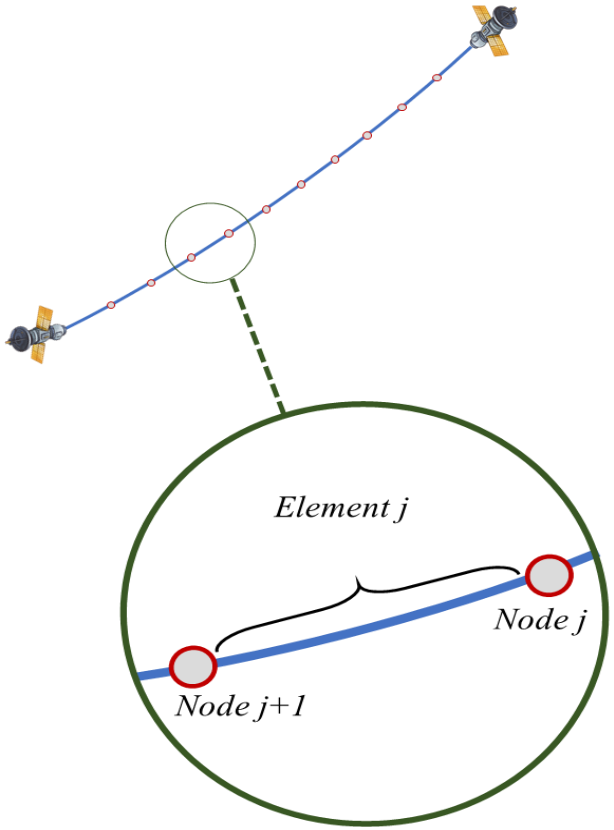
2. Developed an FE-estimator to estimate the elastic state of the space tether system.

3. Proposed a numerical optimal control framework based on variational principle and finite element method.

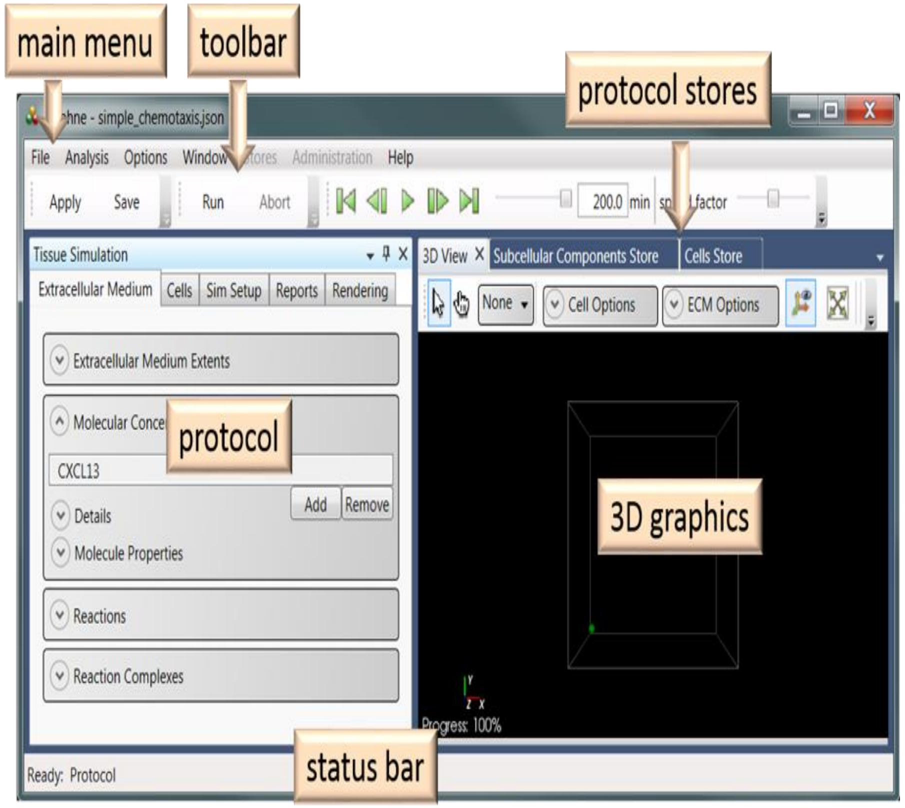
4. Synthesized the modeling & control design into one framework on the space tether system.

Future work

- Design the controller in each element based on the element model and then assemble the control input based on FE theory.



- Build a model-control black box and design a GUI interface.



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Thank you for your attention!

Speaker: Qi Zhang

Date: June 9, 2024