Optimal Control Approach for Stable E-Sail Transitions

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Objectives

- Analyze the feasibility of stable transition for an underactuated E-sail.
- Validate the applicability of a procedure that may be extended to more accurate E-sail models and operating conditions and maneuvers.

Define simplified E-sail model

Define steady-state

Compute reference controls for steady transition

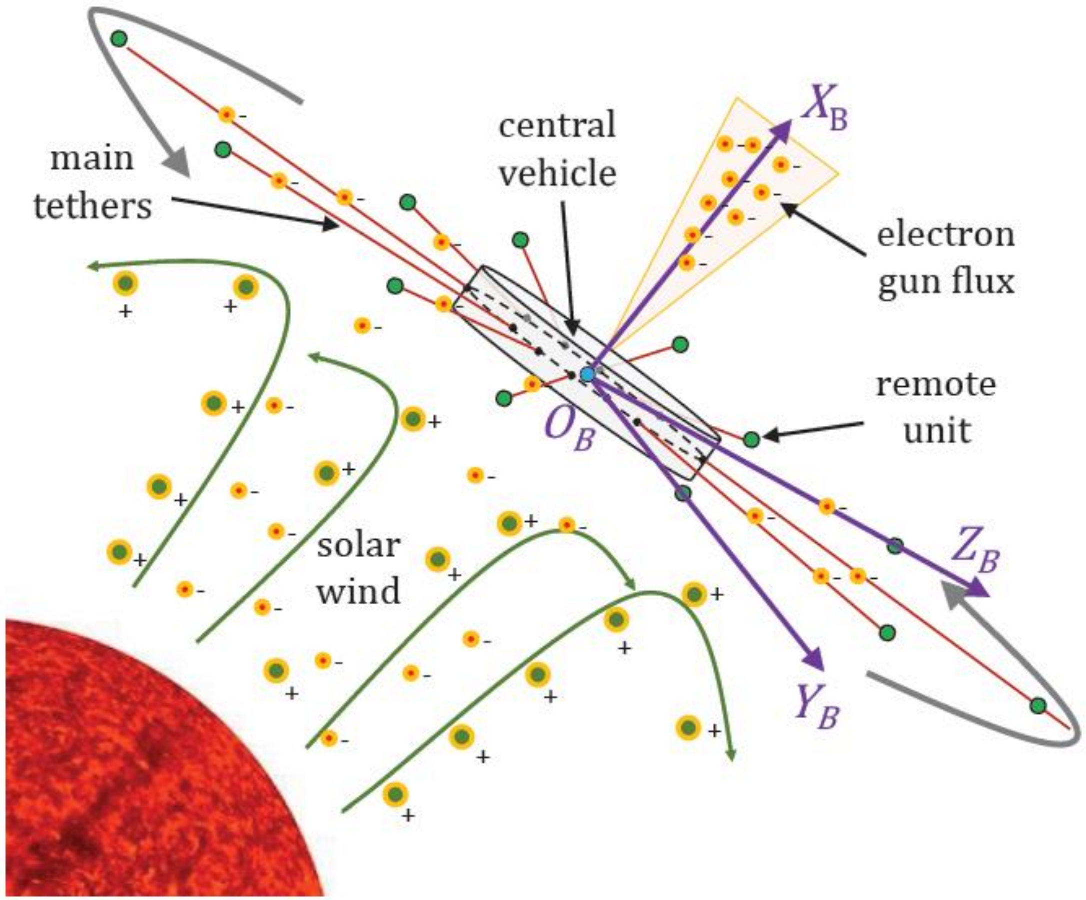
Optimal Control Problem (OCP)

Apply control for tracking

Model Predictive Control (MPC)

Description of E-sail model

- Multi-body perspective:
  - Central vehicle is a rigid cylinder
  - Tethers
    - Straight and rigid
    - No secondary tethers
  - Remote units
    - Punctual masses
- Contributions
  - Inertial forces
  - Coulomb forces
  - Control moment
  - <u>Orbit forces neglected</u>

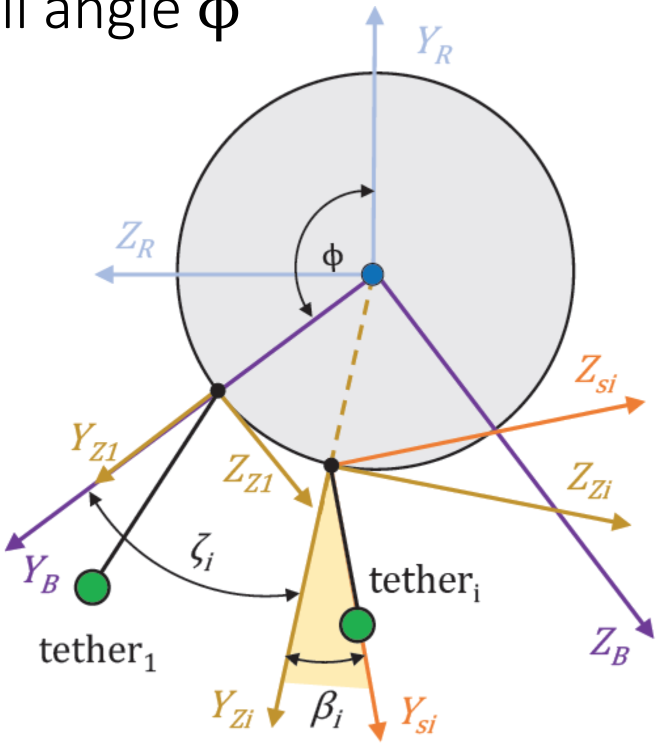


Description of E-sail model

Formulation considering a minimum set of coordinates in ODE form.

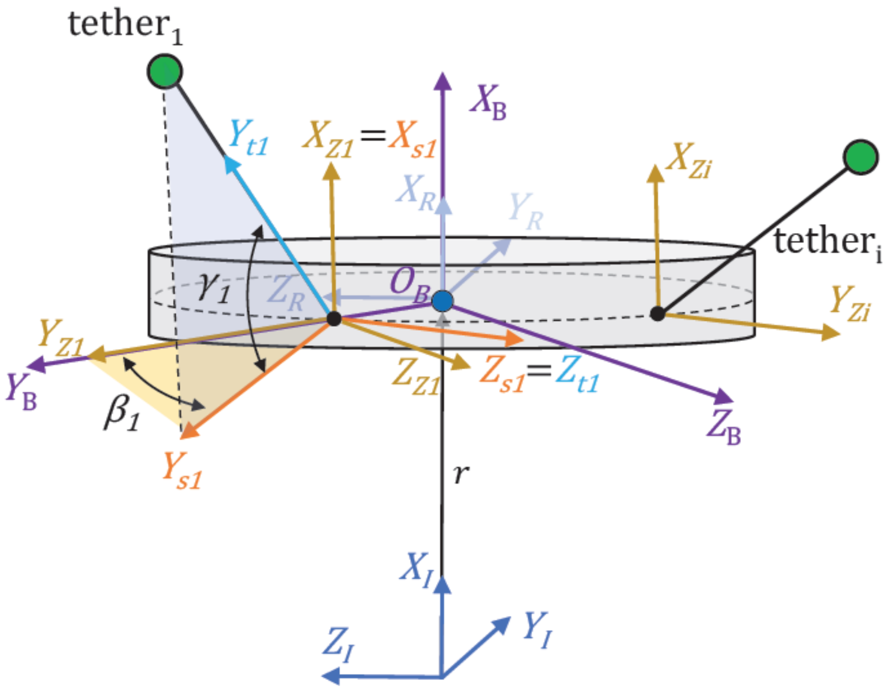
Central vehicle

- Movement along X_I : r
- Roll angle ϕ



Tethers

- Coning angle vector γ
- Lagging angle vector β.



Description of E-sail model

Formulation considering a minimum set of coordinates in ODE form.

- Controls
  - Voltage modulation vector $\overline{\boldsymbol{v}}$
  - Control moment at central vehicle $\overline{\boldsymbol{m}}_c$

- Nonlinear dynamic in states space representation

$$M(\boldsymbol{x})\,\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u})$$

being

$$\boldsymbol{x} = \left[\dot{\overline{\phi}}, \dot{\overline{\boldsymbol{\gamma}}}^T, \dot{\overline{\boldsymbol{\beta}}}^T \phi, \boldsymbol{\gamma}^T, \boldsymbol{\beta}^T\right]^T \qquad \boldsymbol{u} = \left[\overline{\boldsymbol{v}}^T, \overline{m}_c\right]^T$$

Problem Formulation - Steady-state

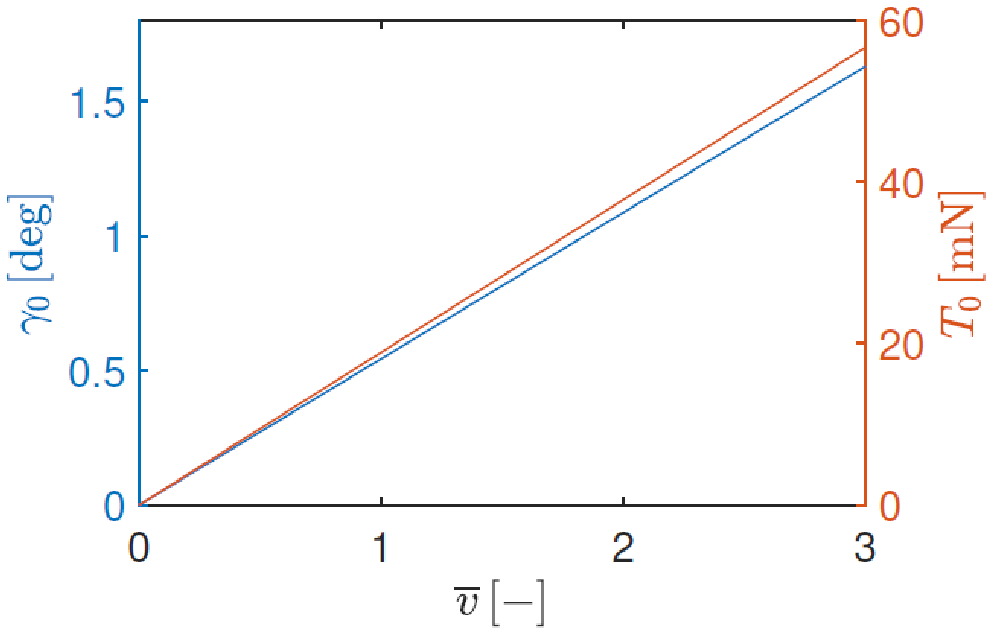
Definition:

- Central spacecraft angular velocity equal to the nominal value $\dot{\overline{\phi}} = \dot{\overline{\phi}}_0$,
- And constant linear acceleration of the E-sail $\ddot{\overline{r}} = \ddot{\overline{r}}_0$,
- Null lagging angle, $\boldsymbol{\beta} = \mathbf{0}$,
- Uniform coning angle, $\boldsymbol{\gamma} = \boldsymbol{\gamma}_0$,
- Null tether angular velocities, $\dot{\overline{\boldsymbol{\gamma}}} = \dot{\overline{\boldsymbol{\beta}}} = \mathbf{0}$,
- Null angular accelerations, $\ddot{\overline{\boldsymbol{\gamma}}} = \ddot{\overline{\boldsymbol{\beta}}} = \mathbf{0}$.

Problem Formulation - Steady-state

$$\left(\bar{J}\,\sin(2\gamma_0) + \bar{m}\,\sin(\gamma_0)\right)\dot{\bar{\phi}}_0^2 + \frac{\left(p\,\bar{m}\,\cos^2(\gamma_0) - 1\right)}{\bar{R}}\bar{f}_{v0}\,\bar{v}_0\,\cos(\gamma_0) = 0$$

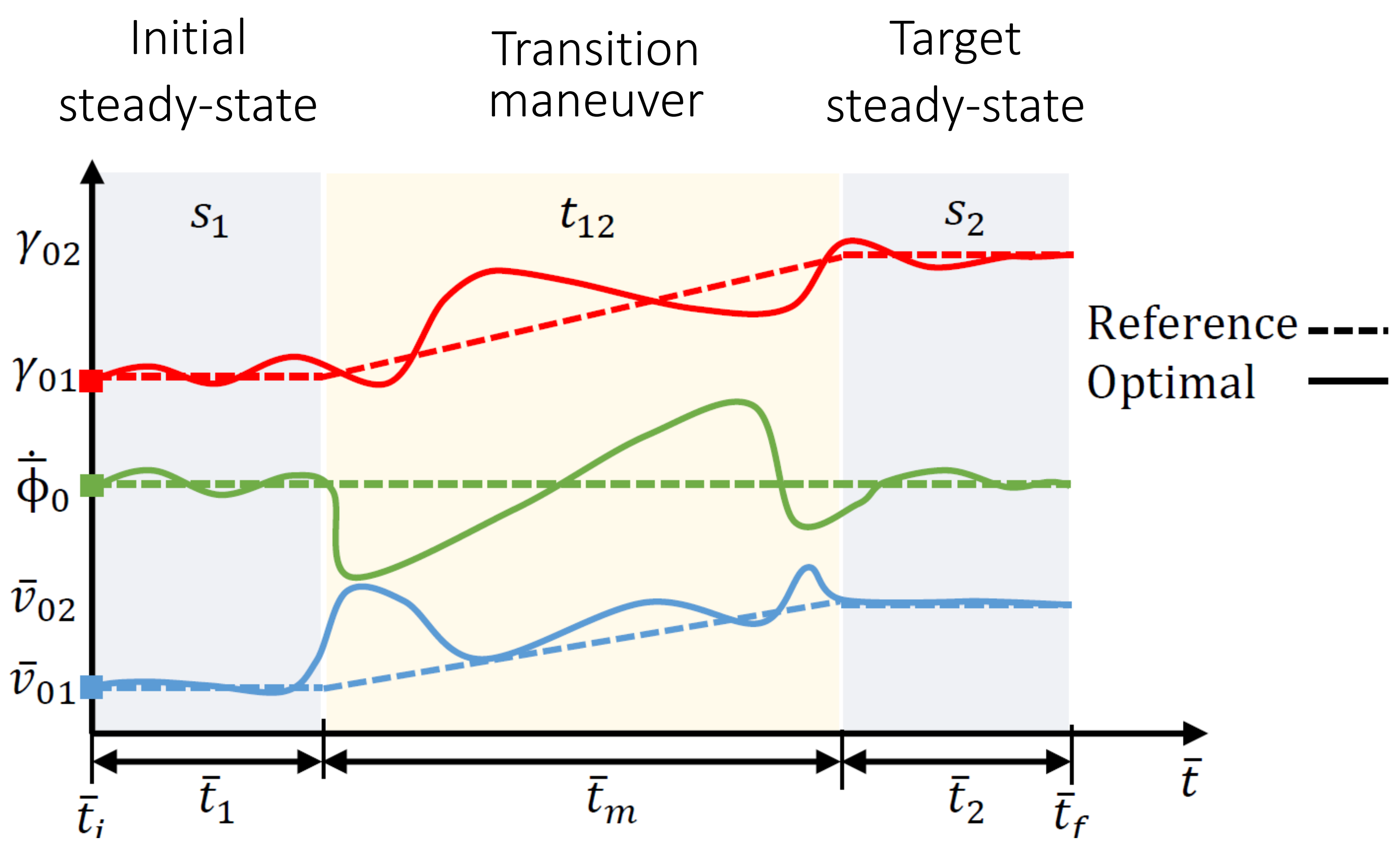
$$\ddot{\bar{r}}_0 - p\,\bar{f}_{v0}\,\bar{v}_0\,\cos^2(\gamma_0) = 0$$



$\bar{J}$: normalized tether inertia
$\bar{m}$: normalized E-sail mass
p: number of tethers
$\bar{R}$: normalized central spacecraft radius
$\bar{f}_{v0}$: normalized reference Coulomb force
$\dot{\bar{\phi}}_0$: normalized nominal angular speed

$\bar{v}_0$: normalized voltage modulation for steady state
γ_0: steady-state coning angle
$\ddot{\bar{r}}_0$: normalized steady-state propulsive acceleration

Problem Formulation - The transition maneuver



Problem Formulation - Optimal planning

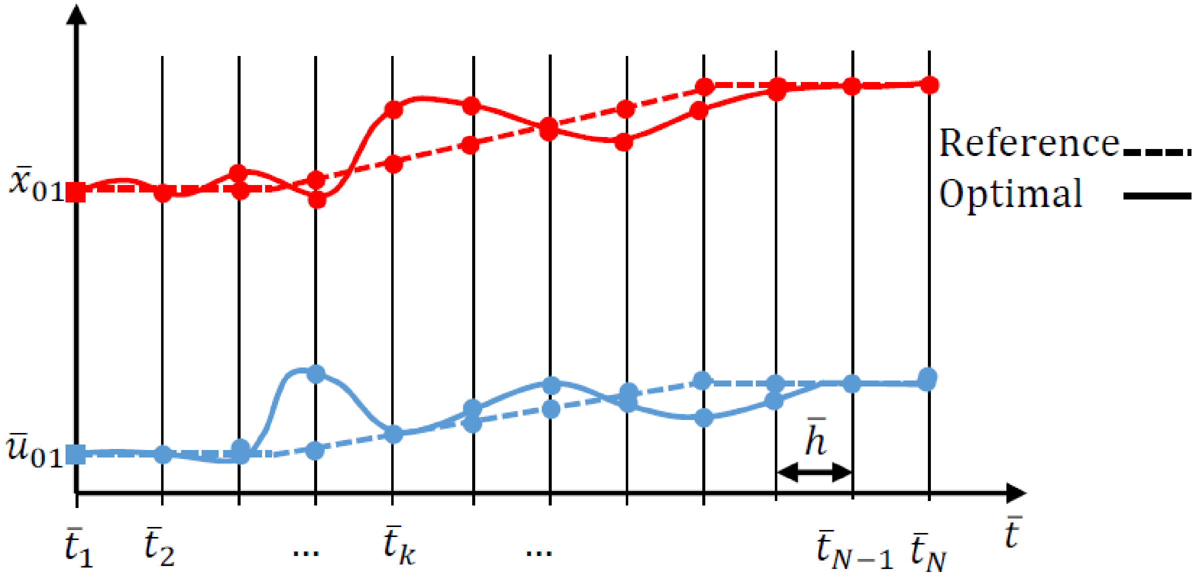
- Inversion of the non-linear E-sail dynamics:
  - Achieve stable transition between steady-states.
  - Considering an underactuated system.

- Solution approach:
  - Formulation from the optimal control problem (OCP) perspective.
  - Solved considering a direct transcription method.
  - Leading to a nonlinear programming problem (NLP).

Problem Formulation - Optimal planning

- Design variables:

$$\overline{\boldsymbol{X}} = (\overline{\boldsymbol{x}}_1; \overline{\boldsymbol{u}}_1; \dots; \overline{\boldsymbol{x}}_k; \overline{\boldsymbol{u}}_k; \dots; \overline{\boldsymbol{x}}_N; \overline{\boldsymbol{u}}_N)$$



- Cost function:

$$J^d = \sum_{k=1}^{N-1} \bar{h} \left(\widehat{\boldsymbol{x}_k^T} Q_k \widehat{\boldsymbol{x}_k} + \widehat{\boldsymbol{u}_k^T} R_k \widehat{\boldsymbol{u}_k} \right) + \widehat{\boldsymbol{x}_N^T} Q_N \widehat{\boldsymbol{x}_N} + \widehat{\boldsymbol{u}_N^T} R_N \widehat{\boldsymbol{u}_N}$$

$\widehat{\boldsymbol{x}_k} / \widehat{\boldsymbol{u}_k}$: normalized state/control vector error respect to reference.

Q_k / R_k : state/control weight matrix.

Q_N / R_N : state/control terminal weight matrix.

Problem Formulation - Optimal planning

- Reference:

$$\bar{\mathbf{x}}_k^r = \begin{cases} \begin{bmatrix} \bar{\phi}_0 & \mathbf{0}^T & \mathbf{0}^T & (k-1)\,\bar{h}\,\bar{\phi}_0 & \gamma_{01}^T & \mathbf{0}^T \end{bmatrix}^T & \text{if } (k-1)\,\bar{h} \le \bar{t}_1 \\ \begin{bmatrix} \bar{\phi}_0 & \mathbf{0}^T & \mathbf{0}^T & (k-1)\,h\,\bar{\phi} & \bar{\gamma}_{01}^T + \frac{(k-1)\,\bar{h}-\bar{t}_1}{\bar{t}_\mathrm{m}} \left(\bar{\gamma}_{02} - \bar{\gamma}_{01}\right)^T & \mathbf{0}^T \end{bmatrix}^T & \text{if } \bar{t}_1 < (k-1)\,\bar{h} \le \bar{t}_1 + \bar{t}_\mathrm{m} \\ \begin{bmatrix} \bar{\phi}_0 & \mathbf{0}^T & \mathbf{0}^T & (k-1)\,h\,\bar{\phi}_0 & \bar{\gamma}_{02}^T & \mathbf{0}^T \end{bmatrix}^T & \text{if } \bar{t}_1 + \bar{t}_m < (k-1)\,\bar{h} \le \bar{t}_1 + \bar{t}_m + \bar{t}_2 \end{cases}$$

$$\bar{\mathbf{u}}_k^r = \begin{cases} \begin{bmatrix} 0 & \bar{\mathbf{v}}_{01}^T & \mathbf{0}^T \end{bmatrix}^T & \text{if } (k-1)\,\bar{h} \le \bar{t}_1 \\ \begin{bmatrix} 0 & \bar{\mathbf{v}}_{01} + \frac{\left((k-1)\,\bar{h}-\bar{t}_1\right)}{\bar{t}_\mathrm{m}} \left(\bar{\mathbf{v}}_{02} - \bar{\mathbf{v}}_{01}\right)^T & \mathbf{0}^T \end{bmatrix}^T & \text{if } \bar{t}_1 < (k-1)\,\bar{h} \le \bar{t}_1 + \bar{t}_\mathrm{m} \\ \begin{bmatrix} \mathbf{0}^T & \bar{\mathbf{v}}_{02} & \mathbf{0}^T \end{bmatrix}^T & \text{if } \bar{t}_1 + \bar{t}_m < (k-1)\,\bar{h} \le \bar{t}_1 + \bar{t}_m + \bar{t}_2 \end{cases}$$

Problem Formulation - Optimal planning

- Constraints equations:
  - Nonlinear associated to integration scheme RK-4.
  - Linear associated to initial boundary condition.

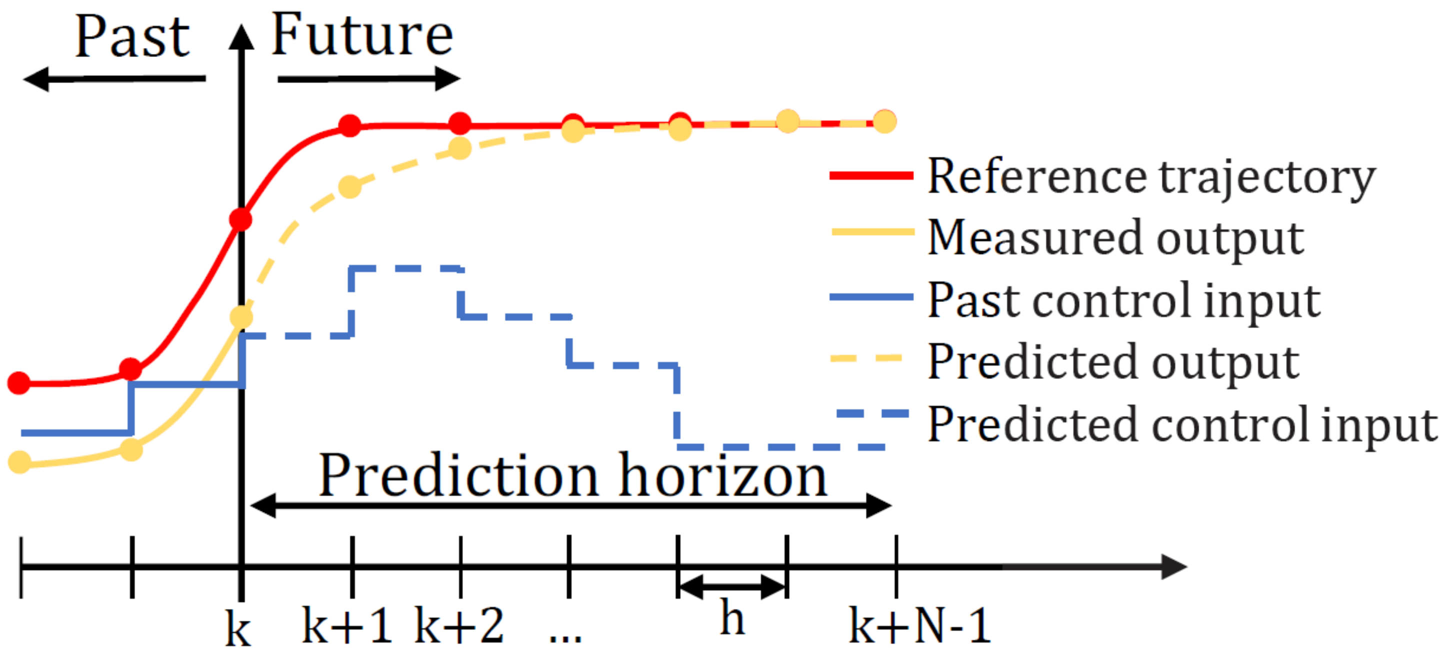
$$\hat{\boldsymbol{c}} = \left[\widehat{\boldsymbol{c}_{i_1}^T} \ \widehat{\boldsymbol{c}_{i_2}^T} \ \dots \ \widehat{\boldsymbol{c}_{i_{N-1}}^T} \ \widehat{\boldsymbol{c}_{b_1}^T}\right]^T$$

- Initial iterant is defined equal to reference states and controls.

- Resolution:
  - Considering matlab *fmincon* function.
  - Optimal planning $\mathbf{X}_p$.
  - Optimal open-loop control law, $\overline{\boldsymbol{u}}_p(\bar{t}_k)$, is extracted at the discretization points for $k = 1, \dots, N$.

Problem Formulation - Tracking

- Application of open-loop optimal control law not ensures tracking the optimal reference.
- Feedback control is needed: Model Predictive Control (MPC).

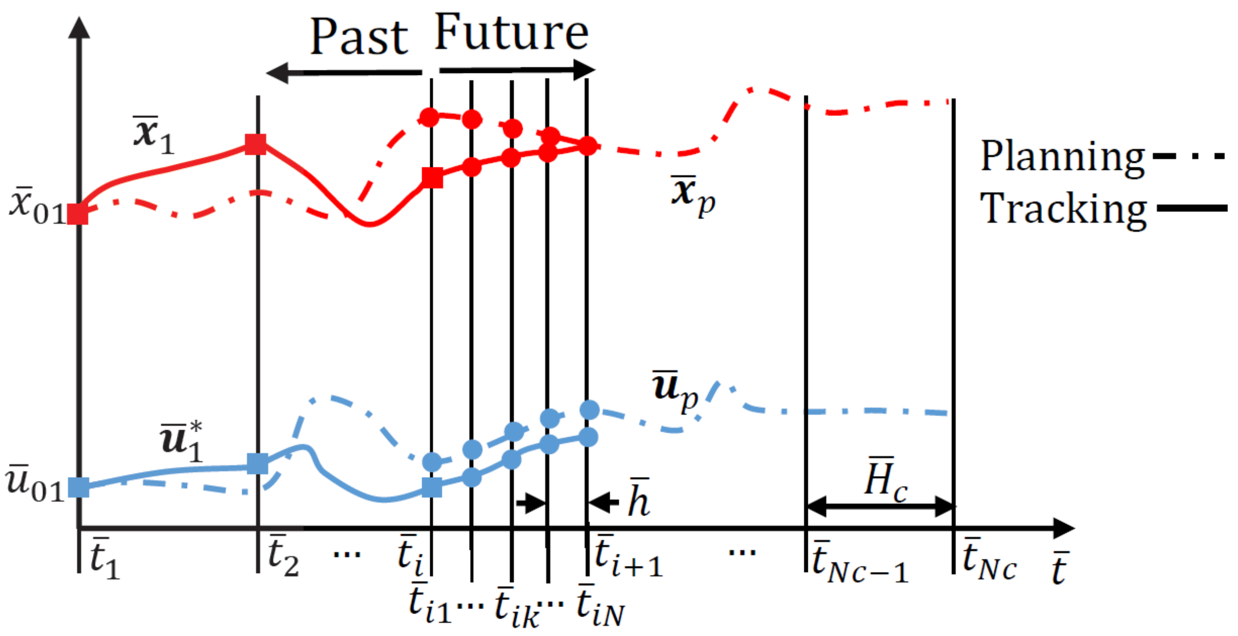


Problem Formulation - Tracking

Based on predicted system evolution solve OCPs for $i = 1, \dots, N_c$

$$\boldsymbol{X}_i = (\overline{\boldsymbol{x}}_{i1}; \overline{\boldsymbol{u}}_{i1}; \dots; \overline{\boldsymbol{x}}_{ik}; \overline{\boldsymbol{u}}_{ik}; \dots; \overline{\boldsymbol{x}}_{iN}; \overline{\boldsymbol{u}}_{iN})$$

- Initial conditions are imposed.
- Initial iterant based on optimal planning.
- Solve i[th]-OCP to find X_i^*.
- Define $\overline{\boldsymbol{u}}_i^*$.



Simulation results

Baseline parameters

Body	Dimension	Value
Main spacecraft	height, h_r [m]	2
	outer radius, R_r [m]	1
	density, ρ_r [kg/m^3]	884
Tethers	number of tethers, p[-]	4
	nominal length, L_0 [km]	10
	section area, A_t [mm^2]	$4.28 \cdot 10^{-3}$
	section inertia, I_t [mm^4]	$1.47 \cdot 10^{-6}$
	density, ρ_t [kg/m^3]	7653
	Young modulus, E_t [GPa]	70
	Voltage, V_0 [kV]	20
Remote unit	mass, m_u [kg]	1.5

Transition definition

- Considering:

$$\bar{\boldsymbol{v}}_{01} = 0$$

$$\bar{t}_1 = \bar{t}_2 = 0.25$$

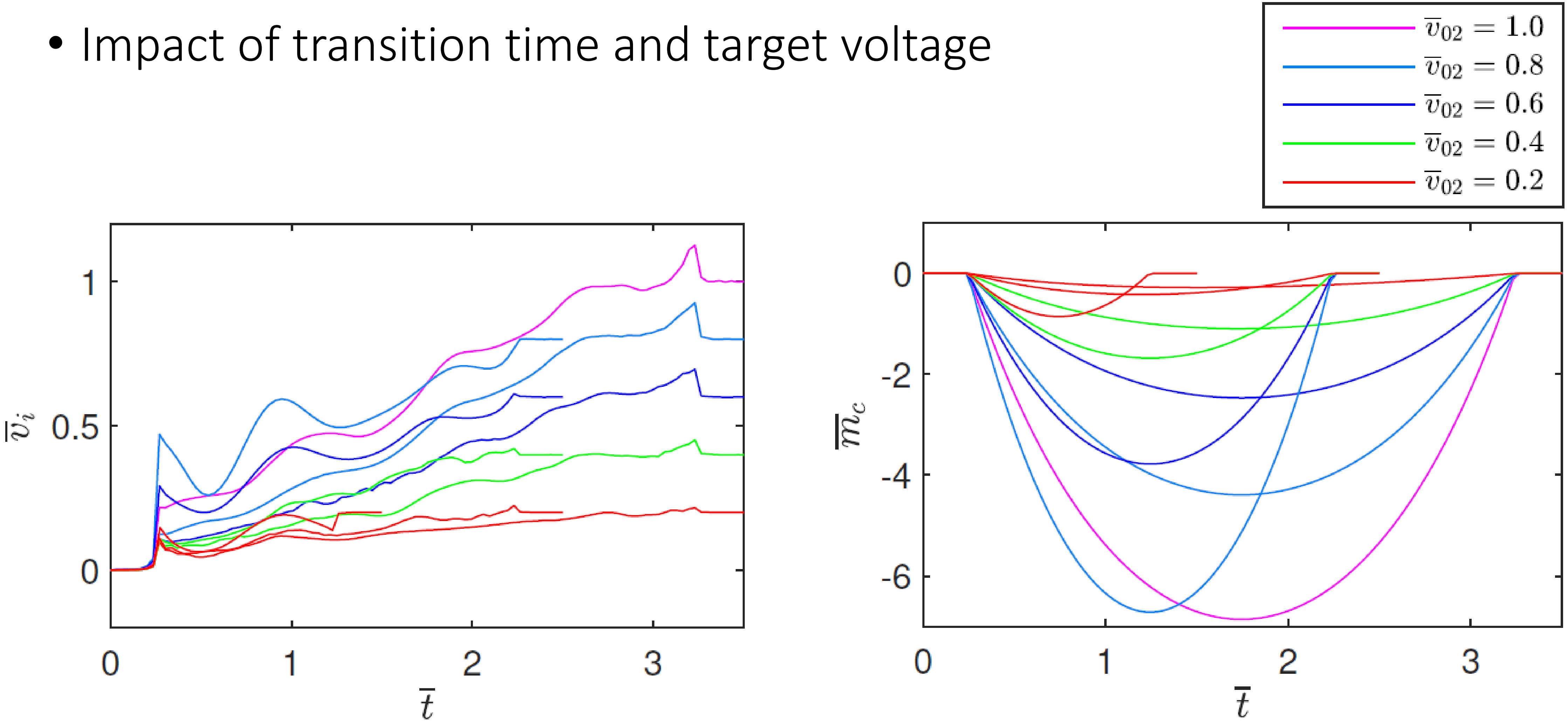
- Evaluate the impact of

$$\bar{t}_m = 1.0, 2.0, 3.0$$

$$\bar{\boldsymbol{v}}_{02} = 0.2, 0.4, 0.6, 0.8, 1.0$$

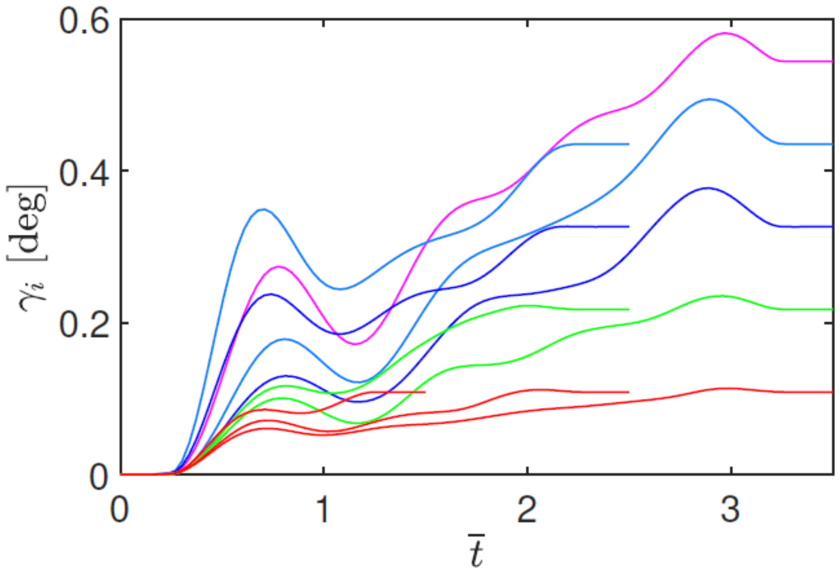
Simulation results - Optimal planning

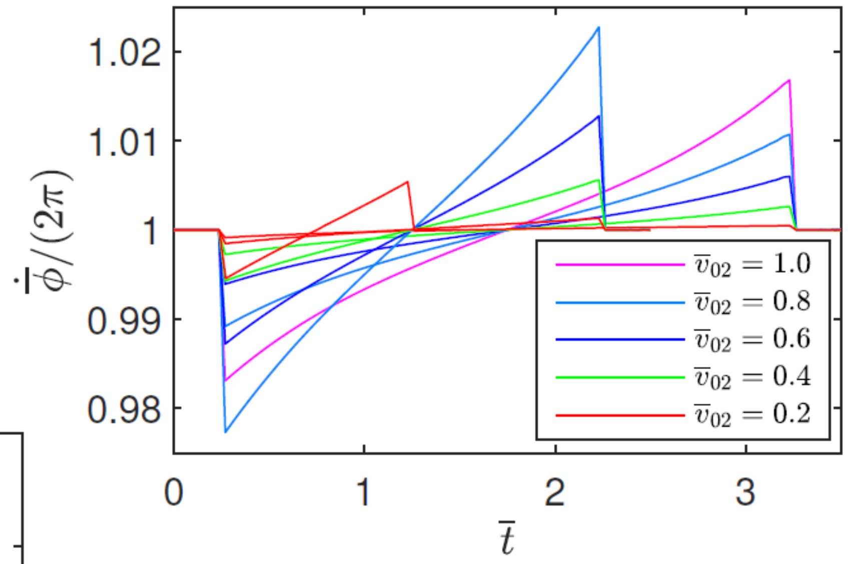
- Impact of transition time and target voltage

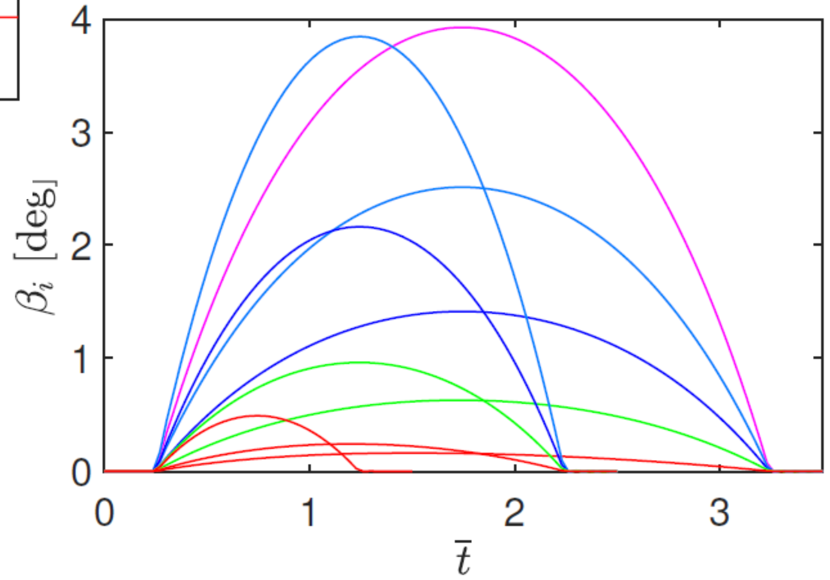


Simulation results - Optimal planning

- Impact of transition time and target voltage

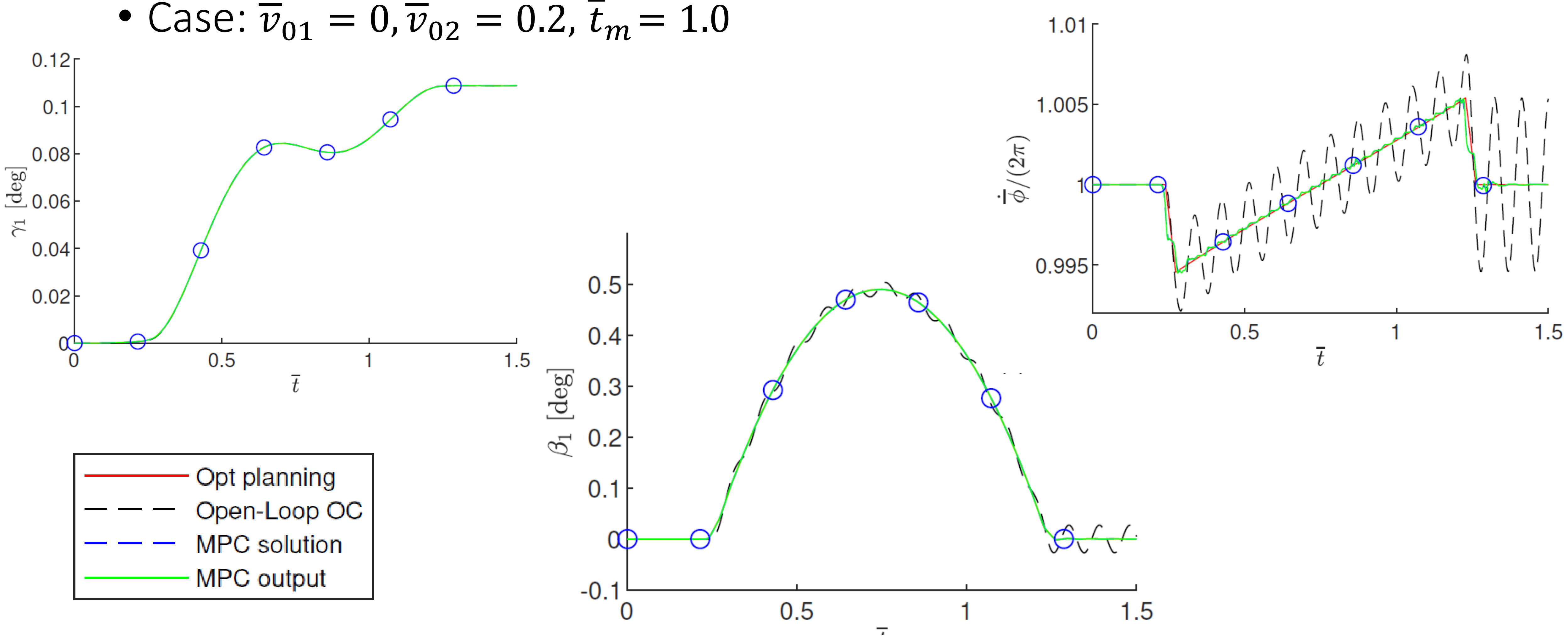






Simulation results - Tracking

- Case: $\overline{v}_{01} = 0, \overline{v}_{02} = 0.2, \overline{t}_m = 1.0$



Simulation results - Tracking

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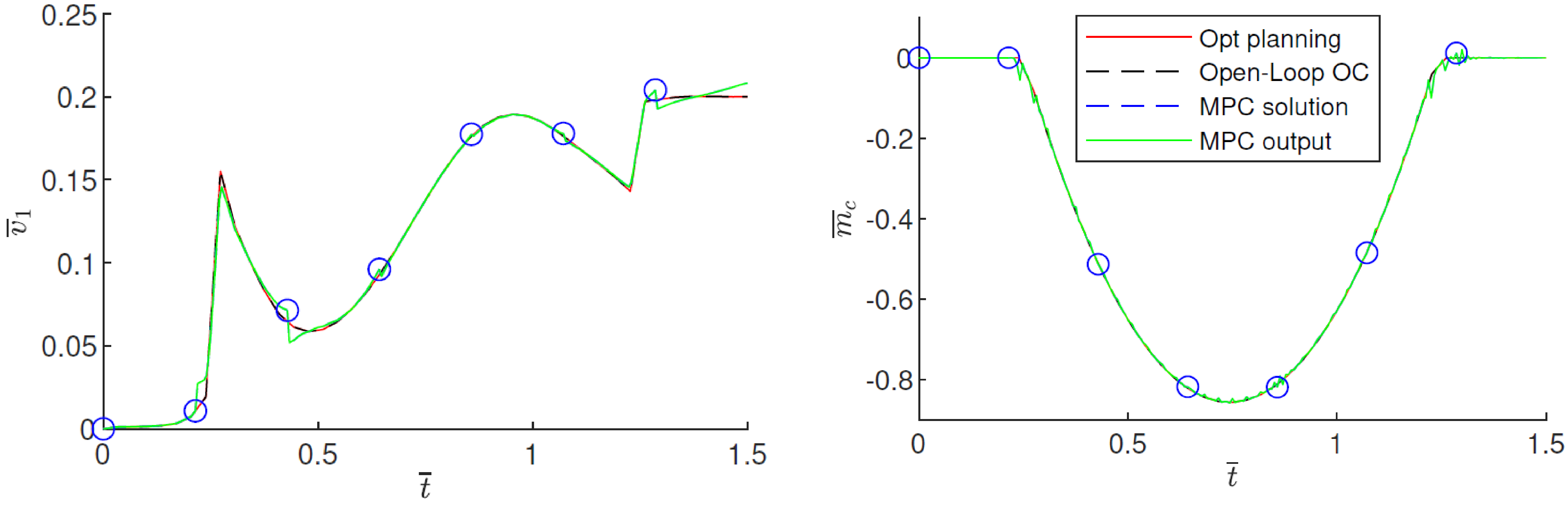
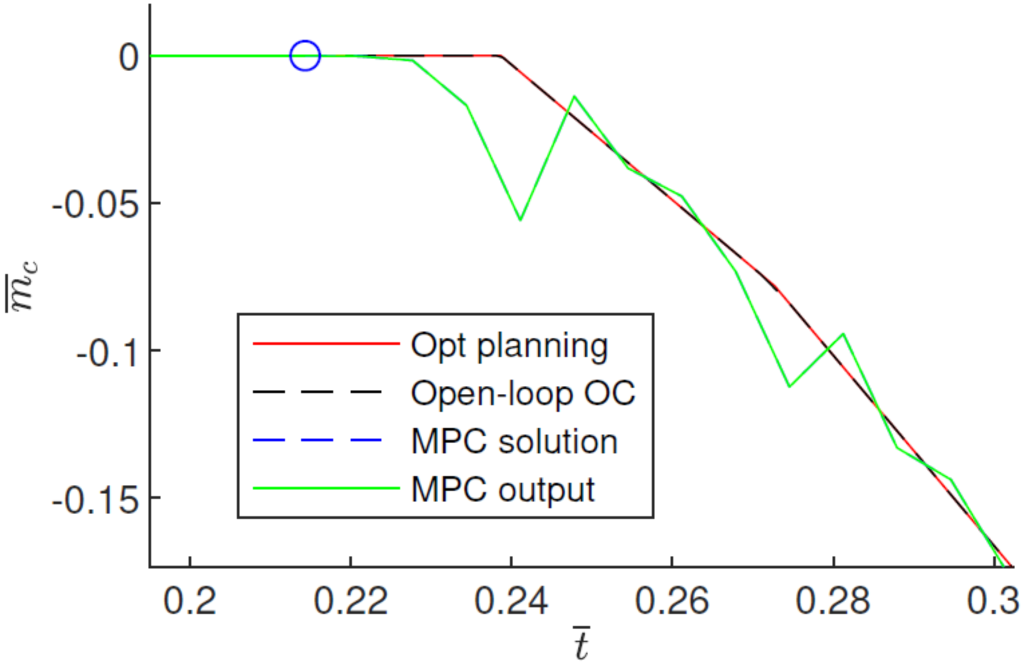
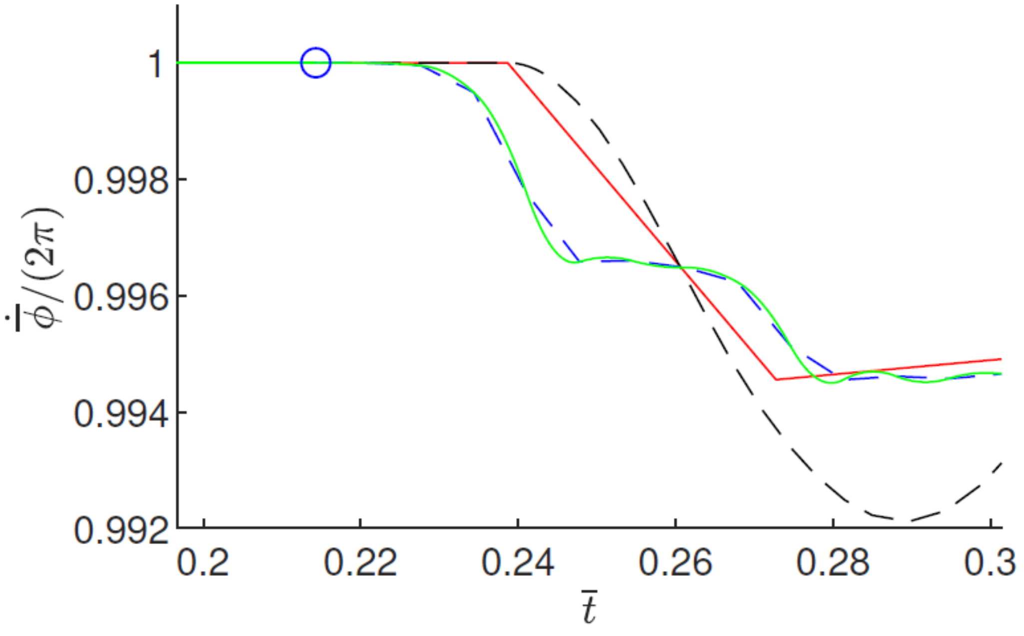


Simulation results - Tracking





Conclusions and future works

- The results obtained suggest the feasibility of stable transitions using underactuated control for E-sail without secondary tethers.
- The proposed planning and tracking approaches yield satisfactory results in both problems.

- This study needs to be extended to more detailed E-sail models and operating scenarios.

Thank you for your attention!