

Analysis of fly-around mission with spinning tether system for space station observation

CONTENTS

Introduction

Small fly-around satellites?

devices to perform a periodical surrounding Nanosatellites are equipped with monitoring relative to the space station $[1-2]$.

Limitation

Fast controlled fly-around

reling on impulsive thrust, leading to high fuel consumption and an inability to sustain long-term monitoring^[16,17].

Natural fly-around.

complex orbit changes and gradual approach control, prolonged period of natural fly-around mode^[6,11].

Fly-around mission process with STS

I. A tethered satellites system in undeployed state is launched into orbit **Ⅱ.** docking with the space station module. **Ⅲ.** The satellites are deployed to designated positions around the space station, **Ⅳ.** STS starts spin up to desired spinning rate **Ⅴ.** Form a stable fly-around configuration.

Introduction

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Two different fly-around schemes

During the whole fly-around process, **maintaining a stable spinning configuration** in these planes is

crucial for the tether system to effectively prevent entanglement with space station's solar panels.

(a) Planar fly-around

(b) Vertical fly-around

Methods

Dynamic model

$$
\boldsymbol{m}_{i} \frac{d^{2} \boldsymbol{R}_{i}}{dt^{2}} = \boldsymbol{G}_{i} + \boldsymbol{D}_{i} + \boldsymbol{T}_{i} + \boldsymbol{F}_{i}, i = 1, 2
$$
 (1)

$$
L = |\Delta \boldsymbol{R}|, \sin \theta = \frac{\Delta y}{\sqrt{\Delta x^{2} + \Delta y^{2}}}, \sin \beta = \frac{\Delta z}{|\Delta \boldsymbol{R}|}
$$
 (2)

Ⅰ. The lumped model avoids the problem of singularities.

Ⅱ. When TFSF is spinning spatially, the coupling definition of two angles leads to incorrect calculation of angles, which makes it difficult to study STS spinning motion.

A new spinning coordinate system

The transition matrix from the orbital motion coordinate system to the spinning plane coordinate system is represented as

$$
L_{cs} = L_{\psi} L_{\eta} L_{\lambda} \tag{1}
$$

where

$$
\boldsymbol{L}_{\psi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{bmatrix}, \boldsymbol{L}_{\eta} = \begin{bmatrix} \cos \eta & 0 & -\sin \eta \\ 0 & 1 & 0 \\ \sin \eta & 0 & \cos \eta \end{bmatrix}, \boldsymbol{L}_{\lambda} = \begin{bmatrix} \cos \lambda & \sin \lambda & 0 \\ -\sin \lambda & \cos \lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

the positions and velocity of fly-around satellites in $Cx_r y_r z_r$ are :

$$
\begin{cases}\nP_1 = \begin{bmatrix}\n0 & L_b \sin \psi_b & -L_b \cos \psi_b\n\end{bmatrix}^T \\
P_2 = \begin{bmatrix}\n0 & -L_b \sin \psi_b & L_b \cos \psi_b\n\end{bmatrix}^T \\
V_1 = \begin{bmatrix}\n0 & \psi_b L_b \cos \psi_b & \psi_b L_b \sin \psi_b\n\end{bmatrix}^T \\
V_2 = \begin{bmatrix}\n0 & -\psi_b L_b \cos \psi_b & -\psi_b L_b \sin \psi_b\n\end{bmatrix}^T\n\end{cases}
$$
\n(2)

Reference trajectories transition from $Cx_r y_r z_r$ to $OXYZ$

The spinning motion in the orbital motion coordinate system is calculated as follows:

$$
\boldsymbol{P}_{i}^{Cxyz} = \boldsymbol{L}_{cs}^{-1} \boldsymbol{P}_{i}, \boldsymbol{U}_{i}^{Cxyz} = \boldsymbol{L}_{cs}^{-1} \boldsymbol{V}_{i}, i = 1, 2.
$$
 (3)

The spinning velocity of the fly-around satellites in the orbital motion coordinate system is calculated as:

$$
V_i^{Cxyz} = U_i^{Cxyz} + V_q^{Cxyz} \tag{4}
$$

where
$$
\mathbf{V}_q^{Cxyz} = \mathbf{V}_c^{Cxyz} + \mathbf{V}_{ei}^{Cxyz} + \mathbf{V}_{rei}^{Cxyz}, \mathbf{V}_c^{Cxyz} = \begin{bmatrix} 0 & r_c \omega_c & 0 \end{bmatrix}^T, \mathbf{V}_{ei}^{Cxyz} = \boldsymbol{\omega}_{ci}^{Cxyz} \times \boldsymbol{P}_i^{Cxyz} = \boldsymbol{\omega}_{ri}^{Cxyz} \times \boldsymbol{P}_i^{Cxyz} \times \boldsymbol{P}_i^{Cxyz}
$$

$$
\boldsymbol{\omega}_{ci}^{Cxyz} = \begin{bmatrix} 0 & 0 & \omega_c \end{bmatrix}^T, \boldsymbol{\omega}_{ri}^{Cxyz} = L_{cs}^T \begin{bmatrix} 0 & \dot{\eta} & 0 \end{bmatrix}^T + L_{\lambda}^T \begin{bmatrix} 0 & \dot{\lambda} & 0 \end{bmatrix}^T
$$

the positions and velocities of the two fly-around satellites in the inertial system *OXYZ* are calculated as :

$$
\boldsymbol{R}_{i} = \boldsymbol{L}_{co} \boldsymbol{P}_{i}^{Cxyz} + \boldsymbol{L}_{co} \boldsymbol{R}_{c}
$$
\n
$$
\boldsymbol{V}_{i} = \boldsymbol{L}_{co} \boldsymbol{V}_{i}^{Cxyz} \tag{5}
$$

where $\mathbf{R}_c = \begin{bmatrix} r_c & 0 & 0 \end{bmatrix}^T$

Reference trajectories transition from $Cx_r y_r z_r$ to $OXYZ$

The spinning plane vector is $\boldsymbol{n}_r = \begin{bmatrix} n_r & n_r \end{bmatrix}^T$, $\Delta \boldsymbol{P}_i^{c_{x_r y_r z_r}} = \begin{bmatrix} x_{\Delta P} & y_{\Delta P} & z_{\Delta P} \end{bmatrix}^T$ is the absolute vector difference between fly-around satellites in the $Cx_r y_r z_r$ frame $n_x = n_y$ n_y n_z $\boldsymbol{n}_r = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix}$ *r r r* i \sim Δt_i Δt_i $\Delta \boldsymbol{P}_i^{Cx_r y_r z_r} = \begin{bmatrix} x_{\Delta P_i} & y_{\Delta P_i} & z_{\Delta P_i} \end{bmatrix}^T$

 η and λ can be solved as follows:

$$
\eta = -\arcsin\left(\frac{n_z}{\sqrt{n_x^2 + n_y^2 + n_z^2}}\right), \lambda = \begin{cases}\n\frac{\pi}{2} & n_x = 0, n_y = 0 \\
\arcsin\left(\frac{n_y}{\sqrt{n_x^2 + n_y^2}}\right) & \text{otherwise}\n\end{cases}
$$
\n
$$
\psi_b = \begin{cases}\n-\arcsin\left(\frac{y_{\Delta P_i}}{\sqrt{y_{\Delta P_i^2} + z_{\Delta P_i^2}}}\right) + \pi & (z_{\Delta P_i} > 0) \\
\arcsin\left(\frac{y_{\Delta P_i}}{\sqrt{y_{\Delta P_i^2} + z_{\Delta P_i^2}}}\right) & (z_{\Delta P_i} \le 0, y_{\Delta P_i} \ge 0)\n\end{cases}
$$
\n(7)

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Controller design

Dynamic eqution of STS can be rewritten as

$$
\dot{\xi}_1 = \xi_2
$$
\n
$$
\dot{\xi}_2 = f(\xi) + g(\xi)u
$$
\nwhere $\xi = [R, v]^T$, $f(\xi) = \frac{1}{m}(G + D + T)$, $g(\xi) = \frac{1}{m}$ (1)

The error function is defined as follows

$$
\tilde{\xi} = \xi - \xi_d \tag{2}
$$

where $\zeta_d = [\mathbf{R}_d, \mathbf{v}_d]^T$ is reference trajectory. The tracking error dynamics is expressed as

$$
\tilde{\xi}_1 = \xi_1 - \xi_{d1}
$$
\n
$$
\tilde{\xi}_2 = \xi_2 - \alpha
$$
\n(3)

where $\tilde{\xi}_1$, $\tilde{\xi}_2$ are respectively the error of position and velocity, $\alpha = -k_a \tilde{\xi}_1 + \dot{\xi}_{d1}$ is virtual controls, k_a is the control coefficient

The control law is defined as

$$
\mathbf{u} = -\mathbf{f}(\xi) + m\dot{\alpha} - m k_b \dot{\tilde{\xi}}_2 \tag{4}
$$

Controller design

A positive definite Lyapunov function is defined as

$$
\boldsymbol{V} = \frac{1}{2} \tilde{\boldsymbol{\xi}}_1^T \tilde{\boldsymbol{\xi}}_1 + \frac{1}{2} \tilde{\boldsymbol{\xi}}_2^T \tilde{\boldsymbol{\xi}}_2 \tag{5}
$$

The derivative of the Lyapunov function is

$$
\dot{V} = \tilde{\xi}_{1}^{T} \dot{\tilde{\xi}}_{1} + \tilde{\xi}_{2}^{T} \dot{\tilde{\xi}}_{2} \n= \tilde{\xi}_{1}^{T} (\xi_{2} - \dot{\tilde{\xi}}_{d1}) - k_{b} \tilde{\xi}_{2}^{T} \tilde{\xi}_{2} \n= \tilde{\xi}_{1}^{T} (-k_{a} \tilde{\xi}_{1} + \tilde{\xi}_{2}) - k_{b} \tilde{\xi}_{2}^{T} \tilde{\xi}_{2} \n\leq \left(\frac{1}{2} - k_{a}\right) \tilde{\xi}_{1}^{T} \tilde{\xi}_{1} + \left(\frac{1}{2} - k_{b}\right) \tilde{\xi}_{2}^{T} \tilde{\xi}_{2}
$$
\n(6)

when the control coefficients k_a and k_b are both greater than 1/2, \dot{V} < 0. the law in (4) ensures asymptotic stability of system.

Results

Planar fly-around

Fig. 1. Trajectories of satellites motion

Planar fly-around

Fig. 2. Trajectories of fly-around satellites during the planar fly-around process

Vertical fly-around

Fig. 1. Trajectories of satellites motion

Fig. 2. Main characteristics of tether length and tension

Vertical fly-around

Fig. 1. variations in the angle λ , η and ν _b Fig. 2. Trajectories of fly-around satellites during the vertical fly-around process

Fuel consumption analysis

Table 1 Comparison of impulse with/without tether within 1 orbital period

During planar spin, 62.8% saving of impulse can be achieved compared to the traditional untethered fly-around, while the vertical fly-around scheme results in 42.9% decrease in impulse.

Conclusion

- The STS is modeled based on Newton-Euler method with a novel description of spinning motion, to overcome the singularity and coupling issues of commonly used models.
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Considering the structural constraints of the space station, the study designs two spinning flyaround schemes and reference fly-around trajectories. Additionally, a backstepping controller is proposed for the tracking of fly-around satellites motion, ensuring s atable fly-around configuration during the whole spinning process.

Comparisons of fuel consumption among different fly-around schemes within one orbital period demonstrate that the STS fly-around scheme significantly reduces energy consumption compared to the untethered fly-around scheme.

During planar spin, 62.8% saving of impulse can be achieved compared to the traditional untethered fly-around, while the vertical fly-around scheme results in 42.9% decrease in impulse.

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