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Optimal Spin-up Control of Linear Tether Formation using Electrodynamic Force

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Introduction

background

Compared to conventional single-tether systems, a linear SLETF has better centrifugal stability.

→

Receiving widespread attention in the fields of space observation, weather forecasting and so on.

challenge

Unstability in tether spin-up phase

Less research for the spin-up control of SLETF

→

solution

To solve such problem, this research will establish a synchronous and stable spin-up control scheme for SLETF.

Dynamic model of SLETF

Assumption when establishing approximated Lagrangian model

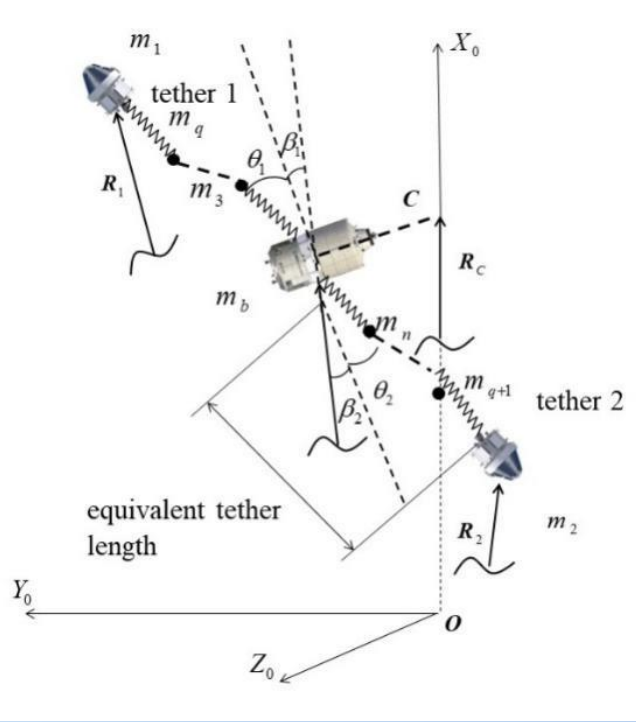
1) The mass center of a SLETF is assumed to be revolving along unperturbed Keplerian orbits, which is modeled by a central gravitational field;

2) As the linear shape is the ideal undeformed tether forms, tethers are modeled as rigid rods;

3) To analyze potential unsynchronized spinning motions of SLETF, coupling influence between two tethers are considered;

4) The base spacecraft and the sub-satellite (end bodies) are regarded as rigid mass points.

Dynamic model of SLETF



The SLETF model

1. Approximated Lagrangian model of SLETF

The motion of a SLETF can be described by the following equations:

$$\boldsymbol{A}\ddot{\boldsymbol{x}} = \boldsymbol{B}, \quad \boldsymbol{x} = [\theta_1, \theta_2, \beta_1, \beta_2]^T$$

$$\boldsymbol{A} = \begin{bmatrix} A_{11} & A_{12} & 0 & A_{14} \\ A_{21} & A_{22} & A_{23} & 0 \\ 0 & A_{32} & A_{33} & A_{34} \\ A_{41} & 0 & A_{43} & A_{44} \end{bmatrix}$$

$$\boldsymbol{B} = \boldsymbol{V}_t + \boldsymbol{Q}_t + \boldsymbol{Q}_I$$

Dynamic model of SLETF

2. Flexible lumped model of SLETF

The dynamic equation including gravitational force, electrodynamic force, tension force, and damping force can be expressed by:

$$m_k \frac{d^2 \boldsymbol{R}_k}{dt^2} = \boldsymbol{G}_k + \boldsymbol{D}_k' + \boldsymbol{T}_k' + \boldsymbol{F}_k, \ k = 1,\ 2,\ b,\ 3, \dots,\ q, \dots n$$

The gravitational force

$$\boldsymbol{G}_k = -\mu_e \frac{m_k \boldsymbol{R}_k}{r_k^3}$$

The tension force

$$T_k = \begin{cases} c_L \dfrac{\left|\boldsymbol{R}_{k+1} - \boldsymbol{R}_k\right| - \Delta L_k}{\Delta L_k}, & \left|\boldsymbol{R}_{k+1} - \boldsymbol{R}_k\right| - \Delta L_k \geq 0 \\ 0, & \left|\boldsymbol{R}_{k+1} - \boldsymbol{R}_k\right| - \Delta L_k < 0 \end{cases}$$

The damping force

$$\boldsymbol{D}_1' = \boldsymbol{D}_1, \boldsymbol{D}_2' = \boldsymbol{D}_2, \boldsymbol{D}_3' = \boldsymbol{D}_3, \boldsymbol{D}_k' = \boldsymbol{D}_k - \boldsymbol{D}_{k-1}, k = 3, \dots n$$

$$D_k = \begin{cases} K_D \dfrac{d\gamma_k}{dt} = K_D \dfrac{1}{\Delta L_{0k}} \dfrac{d(\Delta L_k)}{dt}, & T_k > 0 \\ 0, & T_k \leq 0 \end{cases}$$

The electrodynamic force

$$\boldsymbol{F}_1 = \frac{\Delta \boldsymbol{F}_1}{2}, \boldsymbol{F}_b = \frac{\Delta \boldsymbol{F}_3 + \Delta \boldsymbol{F}_{n-1}}{2}, \quad \boldsymbol{F}_2 = \frac{\Delta \boldsymbol{F}_2}{2}$$

$$\boldsymbol{F}_k = \frac{1}{2}(\Delta \boldsymbol{F}_{k-1} + \Delta \boldsymbol{F}_k), k = 3, \dots n-1$$

$$\Delta \boldsymbol{F}_k = I\left|\boldsymbol{R}_k - \boldsymbol{R}_{k+1}\right|\left(\tau_k \times \boldsymbol{B}_k\right), k = 1, 2, b, \dots n-1$$

Dynamic model of SLETF

3. Modeling method

Obtain the position vectors of three satellites through **flexible lumped model**

$\boldsymbol{R}_1$, $\boldsymbol{R}_c$, $\boldsymbol{R}_2$

→

Obtain equivalent tether length vectors through relative position vectors

$$\boldsymbol{L}_1 = \boldsymbol{R}_1 - \boldsymbol{R}_c$$
$$\boldsymbol{L}_2 = \boldsymbol{R}_2 - \boldsymbol{R}_c$$

→

Introduce equivalent tether lengths $|\boldsymbol{L}_1|, |\boldsymbol{L}_2|$ into **approximate Lagrangian rigid rod model** as $\boldsymbol{L}$

Spin-up controller based on Bellman Dynamic Programming

Simplification of SLETF dynamic model

Considering that the electrodynamic force does not affect the deployed tether length L and it also has less control effectiveness in the out-of-plane motion β, the simplified model will only consider the in-plane spinning motion and neglect the out-of-plane motion, which can be calculated as:

$$\ddot{\boldsymbol{y}} = \boldsymbol{C} + \boldsymbol{D}\boldsymbol{u}_t,\quad \boldsymbol{y} = [\theta_1, \theta_2]^T,\quad \boldsymbol{u}_t = [u_{t1} \quad u_{t2}]^T,\quad u_{ti} = I_i$$

$$\boldsymbol{C} = \boldsymbol{A}_\theta^{-1}\boldsymbol{B}_\theta,$$

$$\boldsymbol{D} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}, \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} = \boldsymbol{A}_\theta^{-1}\begin{bmatrix} J_{t1} \\ J_{t2} \end{bmatrix}$$

$$\boldsymbol{A}_\theta = \begin{bmatrix} L_1^2 m_1 (m_2 + m_b) & L_1 L_2 m_1 m_2 \cos(\theta_1 - \theta_2) \\ L_1 L_2 m_1 m_2 \cos(\theta_1 - \theta_2) & L_2^2 m_2 (m_1 + m_b) \end{bmatrix}$$

$$\boldsymbol{B}_\theta = \begin{bmatrix} V_{t1} + Q_{t1} \\ V_{t2} + Q_{t2} \end{bmatrix}$$

Spin-up controller based on Bellman Dynamic Programming

Simplification of SLETF dynamic model

$$V_{t1}=\frac{-L_1m_1}{(m_1+m_2+m_b)}(L_2m_2\dot{u}_o^2\cos\theta_1\sin\theta_2-L_2m_2\dot{u}_o{}^2\cos\theta_2\sin\theta_1+L_2m_2\dot{\theta}_2^2\cos\theta_1\sin\theta_2-L_2m_2\dot{\theta}_2^2\cos\theta_2\sin\theta_1+2L_2m_2\dot{u}_o\dot{\theta}_2\cos\theta_1\sin\theta_2-2L_2m_2\dot{u}_o\dot{\theta}_2\cos\theta_2\sin\theta_1)$$

$$V_{t2}=\frac{-L_2m_2}{(m_1+m_2+m_b)}(-L_1m_1\dot{u}_o^2\cos\theta_1\sin\theta_2+L_1m_1\dot{u}_o{}^2\cos\theta_2\sin\theta_1-L_1m_1\dot{\theta}_1^2\cos\theta_1\sin\theta_2+L_1m_1\dot{\theta}_1^2\cos\theta_2\sin\theta_1-2L_1m_1\dot{u}_o\dot{\theta}_1\cos\theta_1\sin\theta_2+2L_1m_1\dot{u}_o\dot{\theta}_1\cos\theta_2\sin\theta_1)$$

$$Q_{t1}=\frac{-L_1m_1\dot{u}_o^2}{\nu(m_1+m_2+m_b)}(3L_1m_1\cos\theta_1\sin\theta_1+L_2m_2\cos\theta_1\sin\theta_2+2L_2m_2\cos\theta_2\sin\theta_1+3L_1m_b\cos\theta_1\sin\theta_1)$$

$$Q_{t2}=\frac{-L_2m_2\dot{u}_o^2}{\nu(m_1+m_2+m_b)}(2L_1m_1\cos\theta_1\sin\theta_2+L_1m_1\cos\theta_2\sin\theta_1+3L_2m_1\cos\theta_2\sin\theta_2+3L_2m_b\cos\theta_2\sin\theta_2)$$

$$J_{t1}=B_zL_1(J_{x1}\sin\theta_1+J_{y1}\cos\theta_1),\quad J_{t2}=B_zL_2(J_{x2}\sin\theta_2+J_{y2}\cos\theta_2)$$

$$J_{x1}=\frac{L_1\sin\theta_1(m_1-m_2-m_b)}{2(m_1+m_2+m_b)},\quad J_{y1}=\frac{L_1\cos\theta_1(m_2+m_b-m_1)}{2(m_1+m_2+m_b)}$$

$$J_{x2}=\frac{L_2\sin\theta_2(m_2-m_b-m_1)}{2(m_1+m_2+m_b)},\quad J_{y2}=\frac{L_2\cos\theta_2(m_b+m_1-m_2)}{2(m_1+m_2+m_b)}$$

$$B_x=-2B_0[\sin\gamma\sin i\sin u_o+\cos\alpha\cos\gamma(\cos\Omega_u\cos u_o-\cos i\sin\Omega_u\sin u_o)$$
$$+\cos\gamma\sin\alpha\left(\cos u_o\sin\Omega_u+\cos i\cos\Omega_u\sin u_o\right)]$$

$$B_y=B_0[\cos u_o\sin\gamma\sin i-\cos\alpha\cos\gamma\left(\cos\Omega_u\sin u_o+\cos i\cos u_o\sin\Omega_u\right)$$
$$-\cos\gamma\sin\alpha\left(\sin\Omega_u\sin u_o-\cos i\cos\Omega_u\cos u_o\right)]$$

$$B_z=B_0[\cos i\sin\gamma+\cos\alpha\cos\gamma\sin i\sin\Omega_u-\cos\gamma\cos\Omega_u\sin\alpha\sin i]$$

Spin-up controller based on Bellman Dynamic Programming

Optimal Controller based on Bellman Dynamic Programming

Linearize the simplified model based on the assumption of small perturbations, and then the linearized system has the form:

$$\frac{d\boldsymbol{y}_B}{dt} = \boldsymbol{B}_B(t)\boldsymbol{y}_B + \boldsymbol{M}(t)\Delta\boldsymbol{u}, \quad \boldsymbol{y}_B = \begin{bmatrix}\Delta\theta_1 & \Delta\dot{\theta}_1 & \Delta\theta_2 & \Delta\dot{\theta}_2\end{bmatrix}^T$$

$$\boldsymbol{B}_B(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \mathrm{B}_{21} & \mathrm{B}_{22} & \mathrm{B}_{23} & \mathrm{B}_{24} \\ 0 & 0 & 0 & 1 \\ \mathrm{B}_{41} & \mathrm{B}_{42} & \mathrm{B}_{43} & \mathrm{B}_{44} \end{bmatrix} \Rightarrow \mathrm{B}_{ij}(t) = \frac{\partial F_i(t)}{\partial \mathrm{y}_j}; \quad \mathrm{B}_{43} = \frac{\partial F_4(t)}{\partial \mathrm{y}_3} = \frac{\partial \boldsymbol{C}_{2},:}{\partial \theta_2} + \frac{I_2 * \partial \boldsymbol{D}_{2},:}{\partial \theta_2}$$

$$\boldsymbol{M}(t) = \begin{bmatrix} 0 & 0 \\ \varepsilon_{\theta 1} & 0 \\ 0 & 0 \\ 0 & \varepsilon_{\theta 2} \end{bmatrix} \Rightarrow \varepsilon_{\theta 1} = J_1, \quad \varepsilon_{\theta 2} = J_2$$

Spin-up controller based on Bellman Dynamic Programming

Optimal Controller based on Bellman Dynamic Programming

The optimality criterion:

$$J = \int_0^{t_k} \boldsymbol{y}_B{}^* \boldsymbol{a} \boldsymbol{y}_B + \eta \Delta u^2 dt$$

⬇

The Bellman condition:

$$\min_u \left(\boldsymbol{y}_B{}^* \boldsymbol{a} \boldsymbol{y}_B + \eta \Delta u^2 dt + \frac{d\boldsymbol{W}}{dt} \right) = 0$$

$$\frac{d\boldsymbol{W}}{dt} = \frac{\partial \boldsymbol{W}}{\partial t} + \frac{\partial \boldsymbol{W}}{\partial \boldsymbol{y}_B} \frac{d\boldsymbol{y}_B}{dt}$$

⬇

A solution to this equation is in quadratic form:

$$\boldsymbol{W}(\mathrm{y}, t) = \boldsymbol{y}_B{}^* \boldsymbol{A}_B(t) \boldsymbol{y}_B$$

where

$$\frac{d\boldsymbol{A}_B}{dt} = -a - \boldsymbol{A}_B \boldsymbol{B}_B - \boldsymbol{B}_B{}^T \boldsymbol{A}_B + \boldsymbol{A}_B \boldsymbol{M} \boldsymbol{M}^T \boldsymbol{A}_B$$

⬇

The part of the optimal control amendments (control feedbacks for suppressing errors) is:

$$\Delta u(\boldsymbol{y}_B, t) = \boldsymbol{P}^T(t) \boldsymbol{y}_B, \quad P_k(t) = -1/\eta \sum_{i=1}^{n} A_{ik}(t) M_k, k = 1 \ldots 4$$

⬇

Then the control law of current is defined as:

$$u^o(\boldsymbol{y}_B, t) = u_N(t) + \Delta u(\boldsymbol{y}_B, t)$$

$$I^o(\boldsymbol{y}_B, t) = I_N(t) + P_{\theta_1} \Delta\theta_1 + P_{\dot{\theta}_1} \Delta\dot{\theta}_1 + P_{\theta_2} \Delta\theta_2 + P_{\dot{\theta}_2} \Delta\dot{\theta}_2$$

Spin-up controller based on Bellman Dynamic Programming

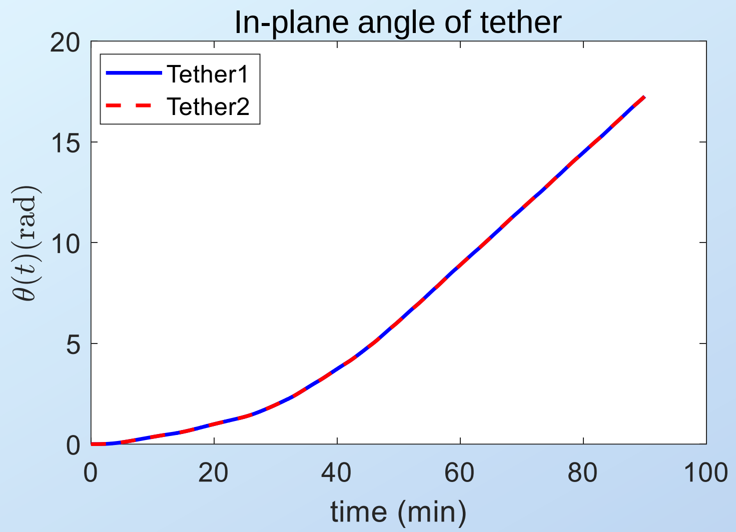
Initial setting of the SLETF

Orbital parameters	
Eccentricity and orbital height e, H	0, 500 km
Right ascension of ascending node and orbital inclination Ω_{u0}, i_0	0, 0

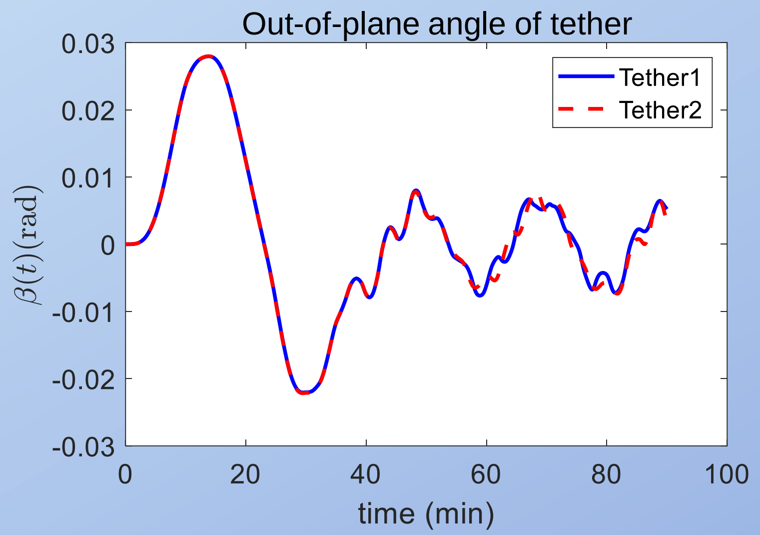
Other Parameters	
Masses m_1, m_2, m_b, m_t	10,10, 60, 2.7 /kg
Tether length L	2.5 /km
Maximum current for closed-loop control	5 /A
Maximum current for open-loop control	1.95 /A
Elastic coefficient of tethers c_L	485476 /(N/m)
Dissipation factors of tethers λ	0.045, 0.335
Moments of inertia	$J_{x1} = 0.267, J_{y1} = 0.267, J_{z1} = 0.267,$ $J_{x2} = 0.267, J_{y2} = 0.267, J_{z2} = 0.267$
Control coefficients of tether $\mathbf{A}(t)$ and η. $\mathbf{A}(t)$ is a diagonal matrix.	$a_{11} = 0.1, a_{22} = 0.3,$ $a_{33} = 0.5, a_{44} = 0.5, \eta = 100$
θ_0, β_0	0, 0 /rad
$\dot{\theta}_0, \dot{\beta}_0$	0, 0 /(rad/s)

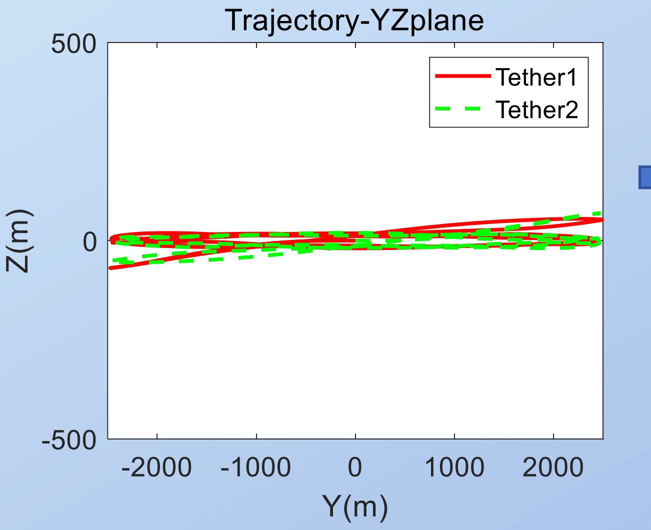
Numerical results

Open-loop spin-up controller for SLETF

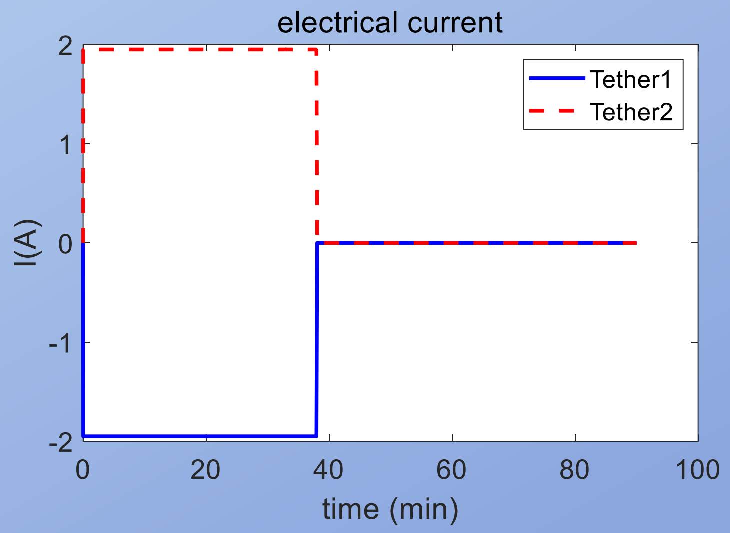


The out-of-plane angle are less than 0.03 rad, which is much smaller than in-plane angle.





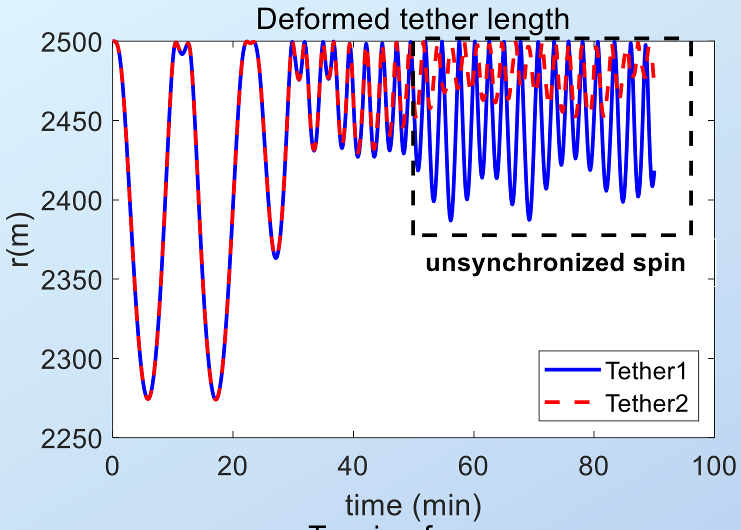
The magnitudes of out-of-plane motion is less than 100m, which is also a small value compared to deformed tether length 2500m.



Therefore the assumption that neglecting out-of-plane motion is validated.

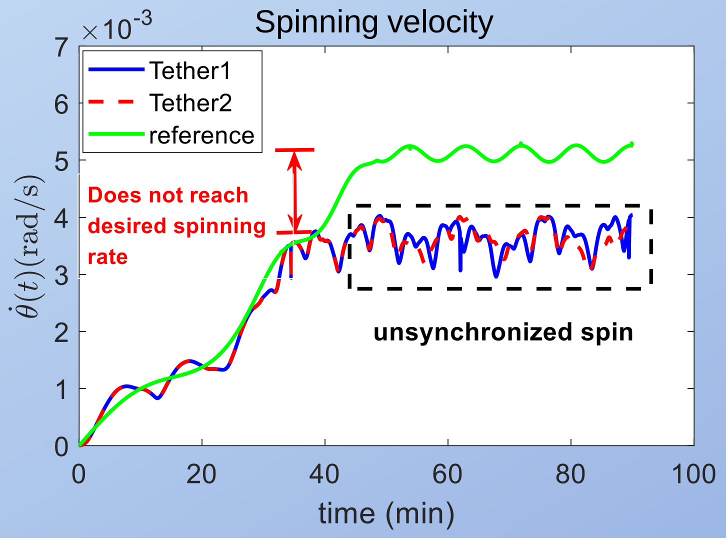
Numerical results

Open-loop spin-up controller for SLETF

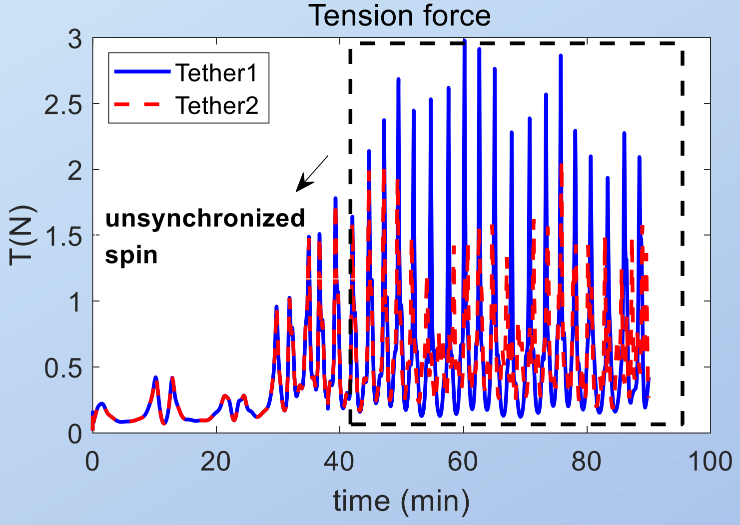


The minimum deformed tether length is about 2280m (91.2% of the linear length).

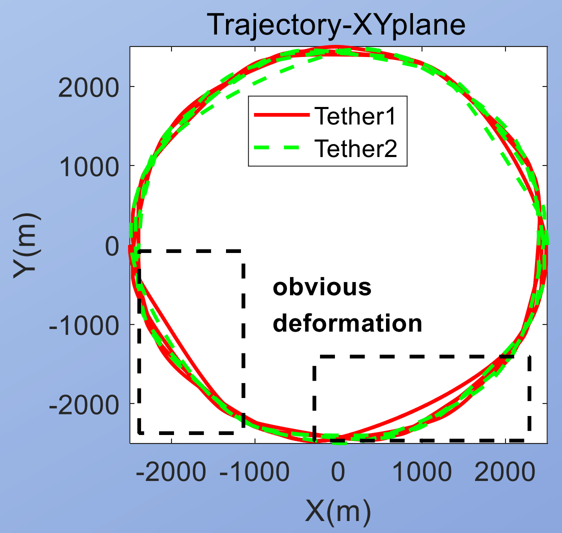
SLETF fails to reach the desired spinning rate.



Significant tether deformations occur and the motion of two tethers is not synchronized

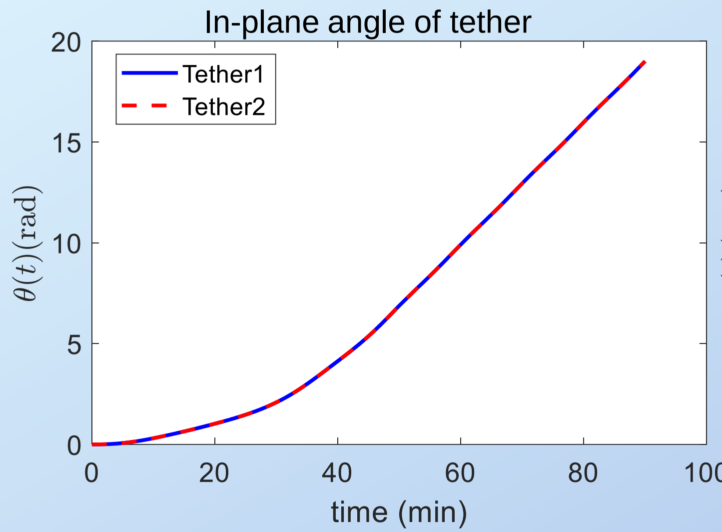


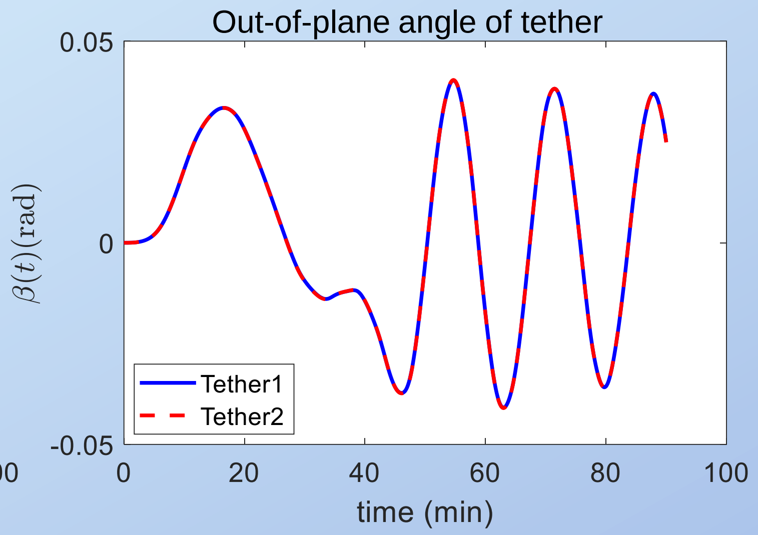
SLETF becomes spinning after about 2100s (35min).

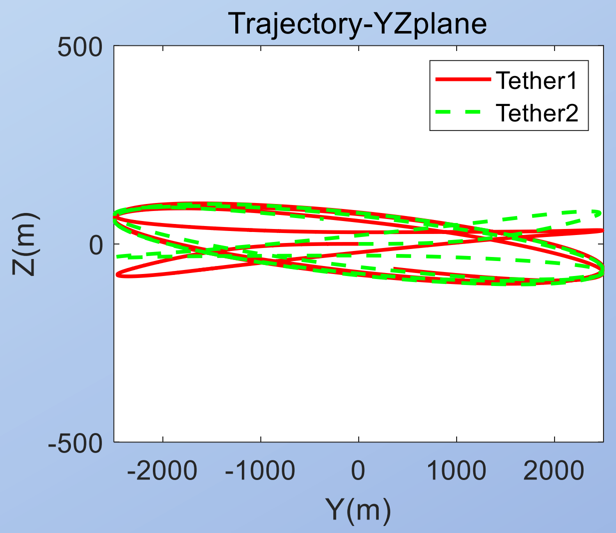


Numerical results

Closed-loop spin-up controller for SLETF



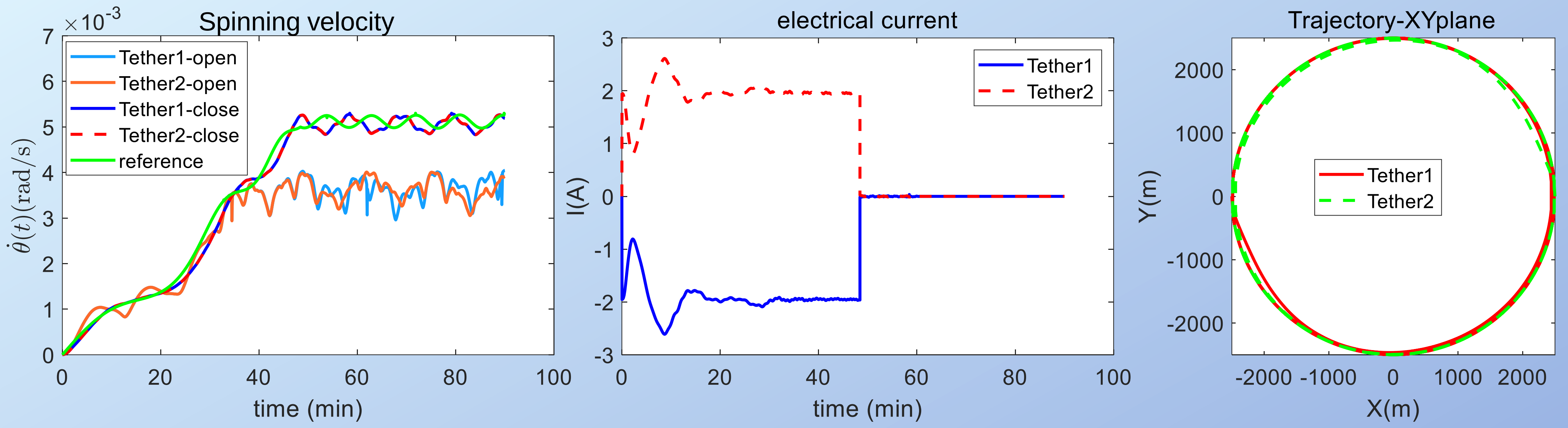




Tether 1 and 2 ultimately achieved synchronous motion.

Numerical results

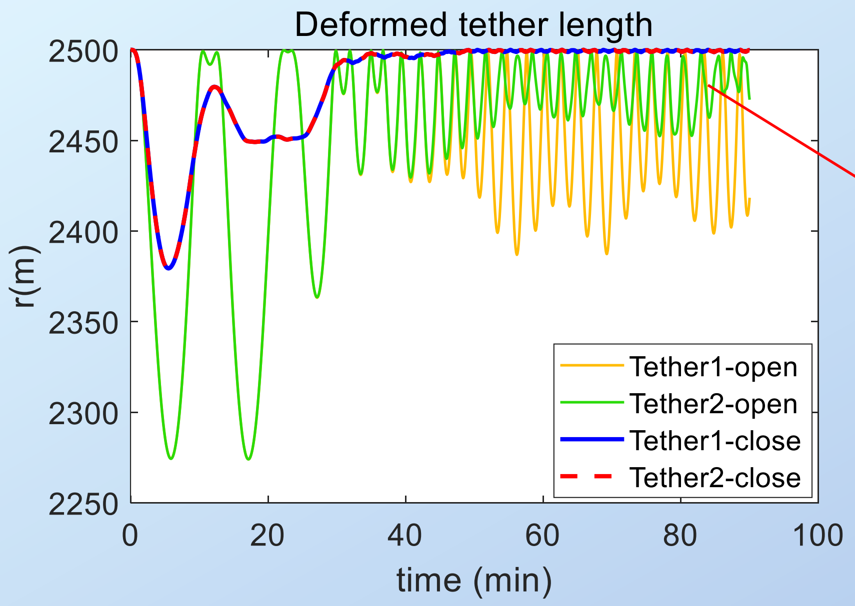
Closed-loop spin-up controller for SLETF



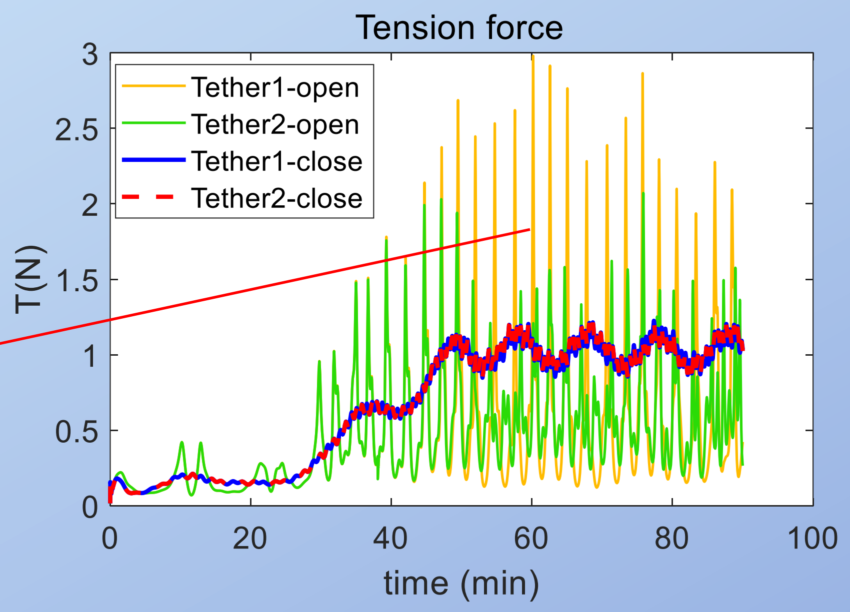
Under the regulation of the optimal current controller, SLETF reaches the desired spinning rate (0.005rad/s) at about 3000s (50min). The final error of the spinning rate is about $3\times10^{-4} rad / s$.

Numerical results

Closed-loop spin-up controller for SLETF



Compared to the open-loop system, oscillations of the closed-loop system is significantly reduced.



The minimum deformation tether length increased to approximately 2380 m (95.2% of the linear length) and two tethers achieve synchronous motion throughout the whole spin-up process (Red and blue curves overlap).

Conclusion

- This paper established the dynamic model of SLETF based on the approximated Lagrangian model and the flexible lumped model, and then studied its spin-up control process.

- The open-loop numerical simulation (current is constant) of the system reveals that the in-plane angular rate cannot track the desired rate, and the two tethers cannot spin synchronously.

- To solve such problem, the optimal Bellman controller is designed. The results show that the spinning rate of the system is almost the same as the desired rate, and the oscillation of the tethers is also effectively suppressed.

Thank you for your attention.