Simple Velocity Planning Control for Space Tether

Junjie Kang, *Jinjun Shan*

York University, Canada

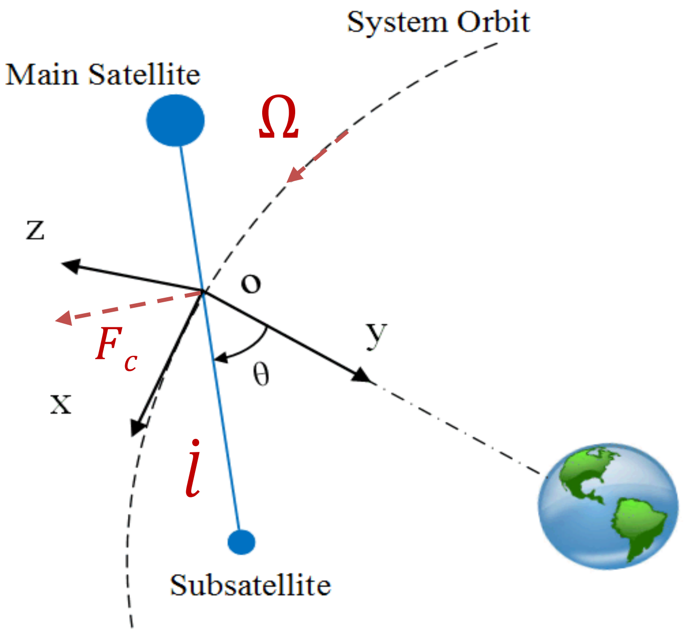
June 9, 2024

LASSONDE SCHOOL OF ENGINEERING | YORK UNIVERSITÉ UNIVERSITY

Department of Earth & Space Science & Engineering

creative passionate rational confident ingenious

1. System description and problem



$$Coriolis\ Force \quad Fc = -2m\left(\vec{\Omega} \times \vec{l}\,\right)$$

Fig. 1(a) Sketch of two-body tethered system.

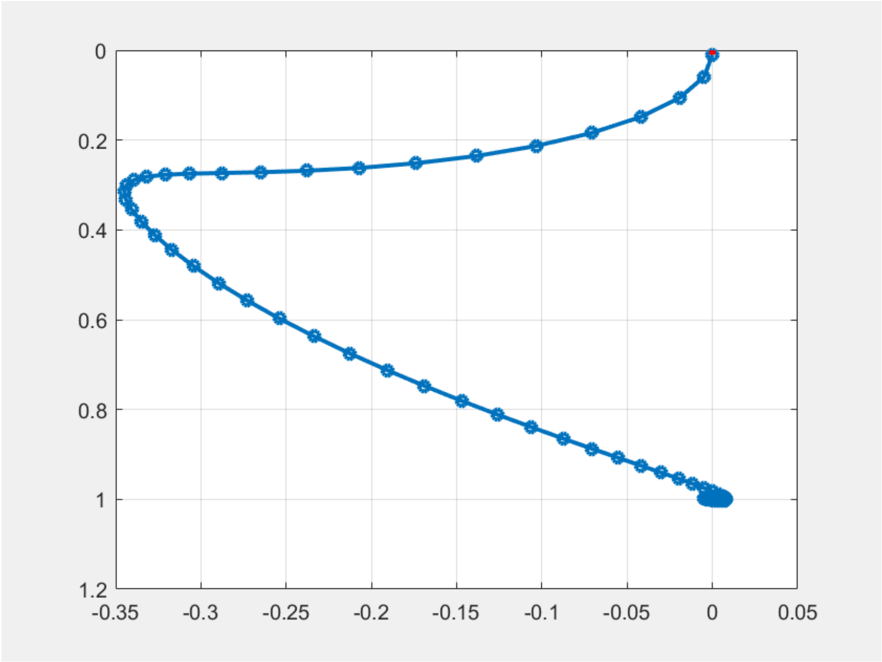


Fig. 1(b) Trajectory in the configuration space.

1. System description and problem

Dynamics equations (dimensionless dumbbell model in 2D):

$$\ddot{l} - l[(1+\dot{\theta})^2 - 1 + 3\cos^2\theta] = -T$$

$$\ddot{\theta} + 2\frac{\dot{l}}{l}(1+\dot{\theta}) + 3\sin\theta\cos\theta = 0$$

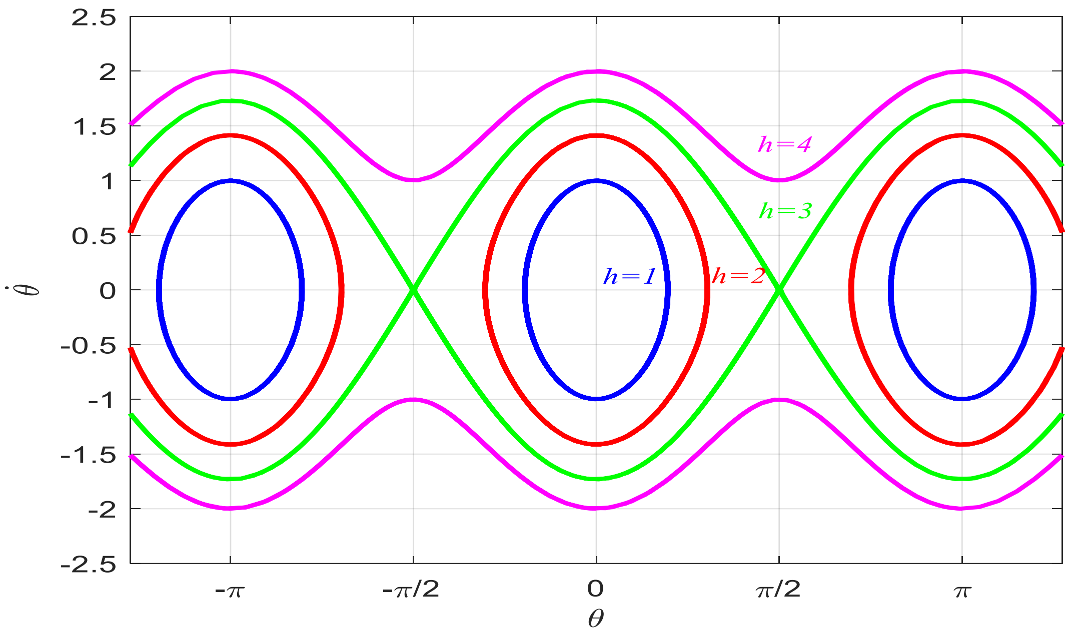
Underactuated

Usually, the control aim is to **simultaneously stabilize *l* and *θ*** at the desired equilibria. Unlike fully actuated system, the ***underactuated system cannot tracking arbitrary trajectories*** in the configuration space. However, the length tracking is possible.

1. System description and problem

Relative equilibria (circular orbit):

$$\begin{cases} \theta = k\pi \\ \theta = (2k+1)\pi / 2 \end{cases}, \quad k \in Z$$



Hamiltonian metric: $h = \dot{\theta}^2 + 3\sin^2\theta$

- h>= 3, the system will possibly spin-up
- **h<3, locally stable around (0,0).**

LASSONDE SCHOOL OF ENGINEERING | YORK UNIVERSITÉ UNIVERSITY

Department of Earth & Space Science & Engineering

creative passionate rational confident ingenious

1. System description and problem

Revisit of control strategies:

- **Length control:** uniformly, exponentially laws, or combinations…
- **Velocity control**(length rate control)**:** similar to length control
- **Tension control :** Kissel's control law, mission control, PD plus gravity…

Length /Velocity control: open-loop controller achieves **bounded stability**.

Tension control: closed-loop controller can achieve **asymptotic stability**. (AS)

1. System description and problem

The considered problem:

a). design an analytical velocity control law, **achieving asymptotic stabilization**. (effectively suppress the residual oscillations)

b). consider the maximum libration angle constraints during deployment/retrieval. (constraint the maximum libration angle)

LASSONDE SCHOOL OF ENGINEERING | YORK UNIVERSITÉ UNIVERSITY
Department of Earth & Space Science & Engineering

2. Simple velocity planning

We propose a simple velocity planning approach:

$$\dot{l}(t) = \dot{l}_n(t) \quad + \quad f(\theta, \dot{\theta})$$

(Nominal velocity) (Libration suppressor)

- Needs the feedback of system information
- Guarantee the stability and has a simple form
- Feasible to convert into the tension controller if differentiable.

2. Simple velocity planning

Step-1. Design of the nominal part: (guiding the tether to the desired value.)

a). The nominal trajectory is sufficiently smooth and bounded, and

$$\lim_{t\to\infty} l_n(t) = l_d$$

b). The nominal velocity is bounded and L_2-convergence,

$$\dot{l}_n(t) \in \mathbb{L}_2 \bigcap \mathbb{L}_\infty$$

Lemma 1: $\lim_{t\to\infty} \dot{l}_n(t)=0.$ Proof: *This is a consequence by the Barbalat's lemma*.

2. Simple velocity planning

Step-2. Design of the libration suppression: (damping injection.)

Recall the libration equation and take the linearized approximation:

$$\ddot{\theta} + 2\frac{\dot{l}}{l}(1+\dot{\theta}) + 3\sin\theta\cos\theta = 0$$

$$\downarrow$$

$$\ddot{\theta} + 2\dot{l} + 3\theta = 0 \qquad \text{(with } l_d\text{=1)}$$

A simple candidate is can be designed as

$$\dot{l} = f(\theta, \dot{\theta}) := k\dot{\theta}, \quad k > 0$$

Libration angle will be asymptotically stabilized at the origin.

LASSONDE SCHOOL OF ENGINEERING | YORK UNIVERSITÉ UNIVERSITY

Department of Earth & Space Science & Engineering

creative passionate rational confident ingenious

2. Simple velocity planning

By combining the two parts, the first velocity controller is

$$\dot{l}(t) = \dot{l}_n(t) + k\dot{\theta} \qquad \text{Controller-I}$$

Theorem 1.

Given simple velocity controller can guarantee $\lim_{t\to\infty} l(t) = l_d, \ \lim_{t\to\infty} \theta = 0.$

Proof: Take a candidate Lyapunov function $V = \frac{1}{2}\dot{\theta}^2 + \frac{3}{2}\theta^2$

its derivative $\dot{V} = -2\dot{l}_n\dot{\theta} - 2\dot{\theta}^2 \le \dot{l}_n^{\,2} - \dot{\theta}^2.$ Then, one has $\dot{\theta} \in \mathbb{L}_2 \cap \mathbb{L}_\infty$, indicating $\lim_{t\to\infty} \dot{\theta} = 0.$

From Lemma 1, $\lim_{t\to\infty} \dot{l}_n = 0$, we have $\lim_{t\to\infty} \dot{l}(t) = 0.$ Finally, by integration, can obtain $\lim_{t\to\infty} l(t) = l_d$ ■ .

Notes: Integration constants should match with the desired length, which can be ensured by design the initial conditions.

2. Simple velocity planning

Furthermore, consider the explicit constraint on maximum libration angle, $|\theta| < \theta_{\max}$.

$$\dot{l}(t) = \dot{l}_n(t) + \underbrace{f_\theta(\theta, \dot{\theta})}_{safety\ part} \qquad \text{Controller-II}$$

Step-2. Design the libration suppression with the safety critical guarantee.

A candidate is designed as

$$\dot{l}(t) = \dot{l}_n(t) + k\,\dot{\theta} / (\theta^2_{\max} - \theta^2),\ k>0. \qquad \text{Controller-II}$$

Theorem 2.

Controller-II can guarantee that $\lim_{t\to\infty} l(t) = l_d$, $\lim_{t\to\infty} \theta = 0$, and $|\theta| < \theta_{\max}$.

LASSONDE SCHOOL OF ENGINEERING | YORK UNIVERSITÉ UNIVERSITY
Department of Earth & Space Science & Engineering

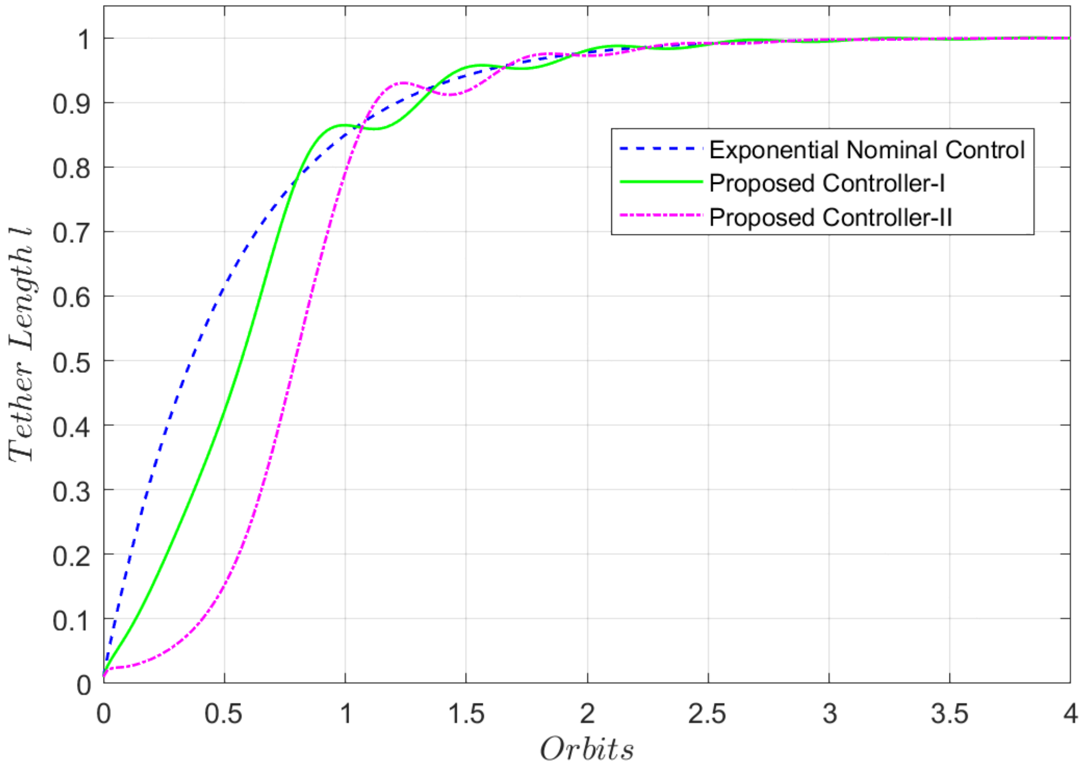
Proof: Similar as in Theorem 1.

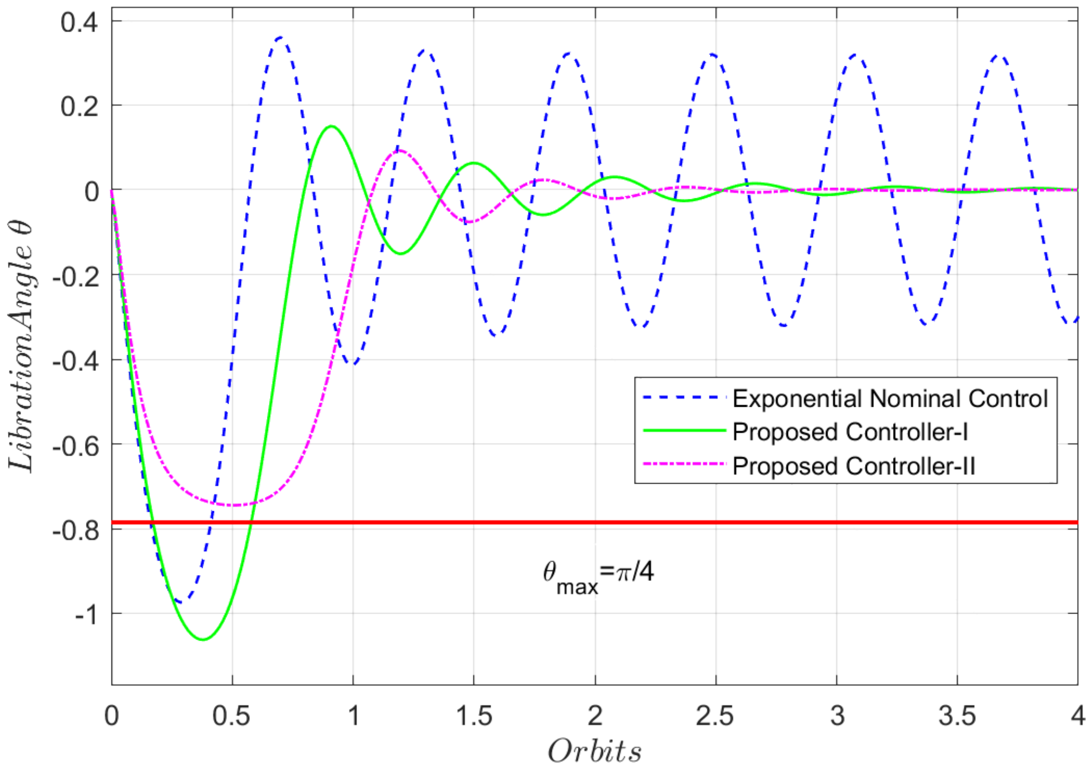
creative passionate rational confident ingenious

3. Simulation Results

Case 1: Deployment Simulation

Exponential nominal velocity function is selected an example. $\dot{l}_n(t) = c l_d \, e^{-ct}$

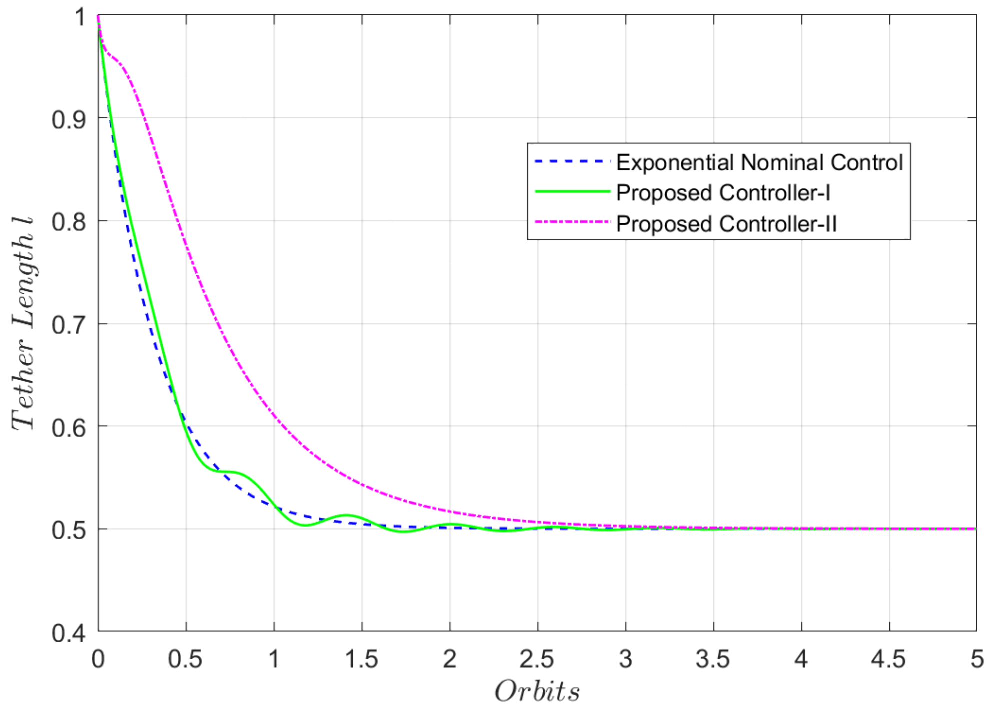


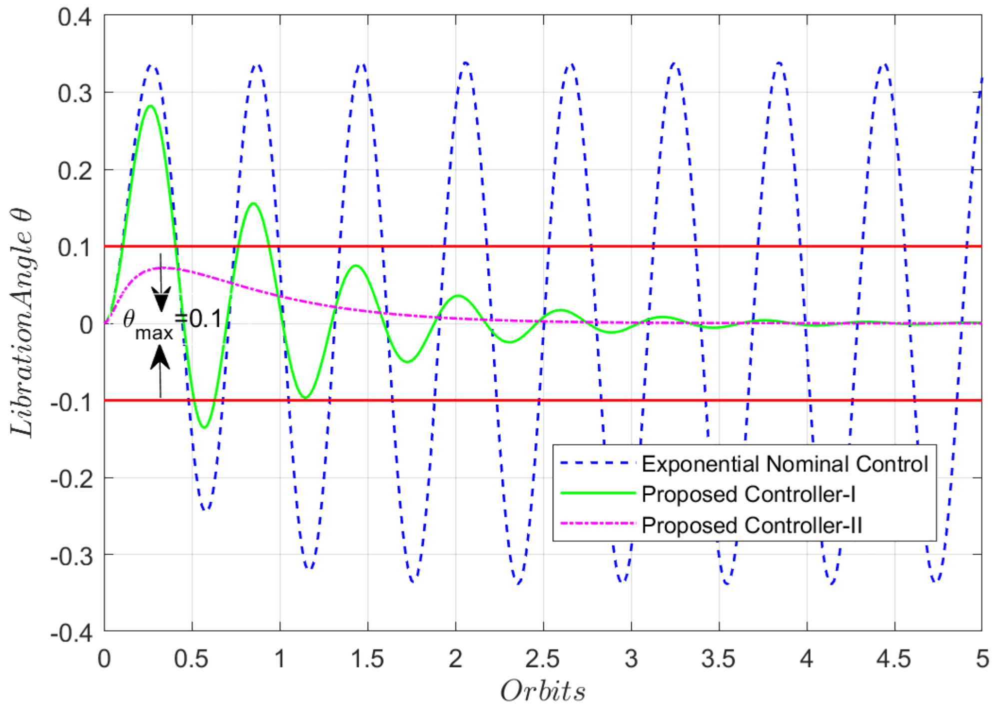


3. Simulation Results

Case 2: Retrieval Simulation (from l_0=1 to l_d=0.5)

Exponential nominal velocity function: $\dot{l}_n(t) = -cl_d\, e^{-ct}$.





4. Conclusions

In this study, we present simple velocity planning control for TSS, where the following two main remarks can be summarized.

1. By simple analytical design of the libration suppressor, **the proposed velocity control approach achieves the AS**. Effetely remove the residual oscillations .
2. Further consider the safety guarantee in the libration suppressor, the proposed approach can **constraint the maximum libration angle,** while ensuring the AS.

Thank You
&
Queries

LASSONDE SCHOOL OF ENGINEERING | YORK UNIVERSITÉ UNIVERSITY

Department of Earth & Space Science & Engineering

creative
passionate
rational
confident
ingenious