Along-Track Deployment Control of Space Tether System for SAR-GMTI Mission

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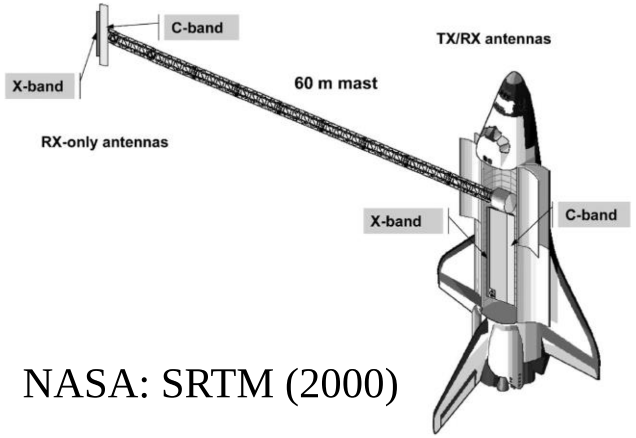
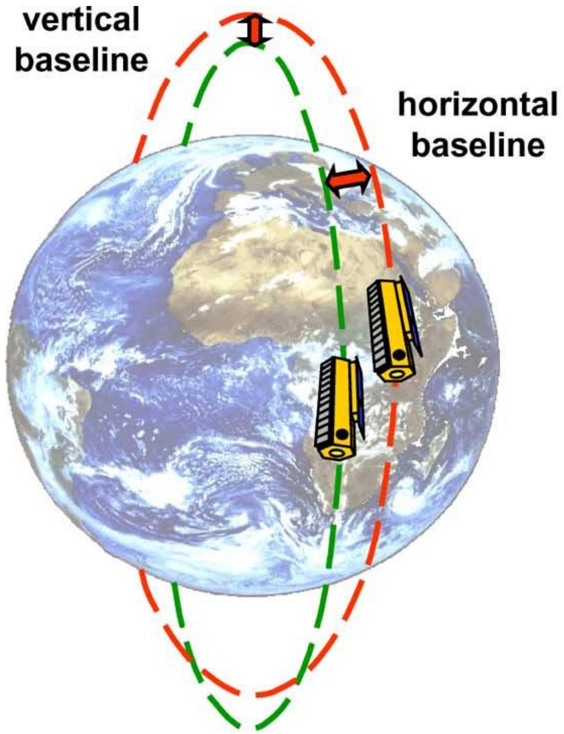
SAR-GMTI Mission



NASA: SRTM (2000)

ESA: Sentinel-1A (2014)
Sentinel-1B (2016)



DLR: TerraSAR-X (2007)
TanDEM-X (2010)
CNSA: TianHui-2 (2021)

Radio detection and ranging (Radar)

↓

Synthetic Aperture Radar (SAR)

↓

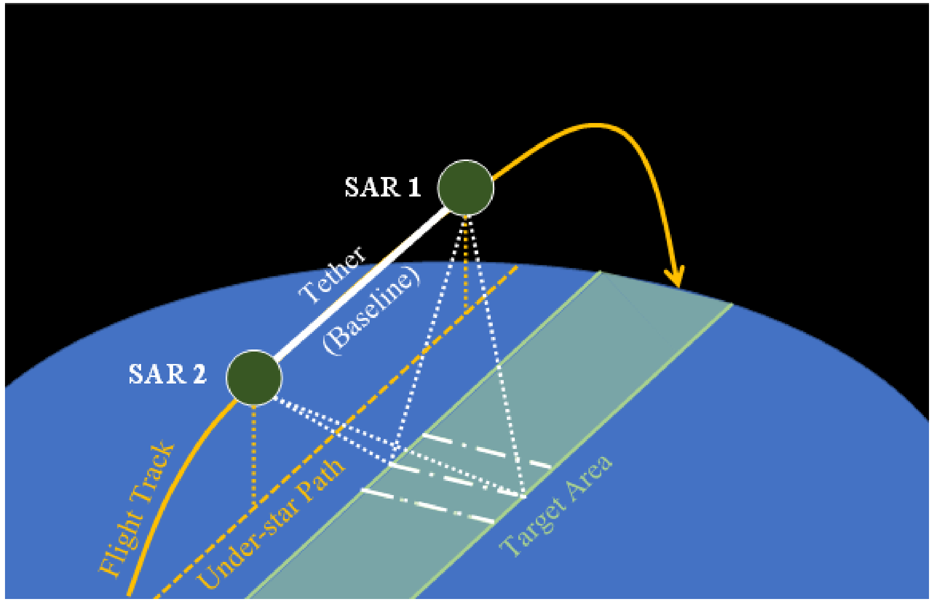
Interferometric SAR (InSAR) mission:

- Digital Elevation models (DEM)
- **Ground Moving Target Indications (GMTI)**

The current distributed InSAR system suffers from problems:

- presence of **periodic variations** in the interference baseline
- **coupling** between different baseline components

Main problems and highlights



1. The horizontal position of STS is far **less studied** than the traditional vertical position

- Two optimal trajectory planning strategies are discussed under vertical and non-vertical initial conditions.

2. The GMTI mission has **accuracy requirements**

- A synthetic criterion of measurement error is proposed to value the deployment accuracy

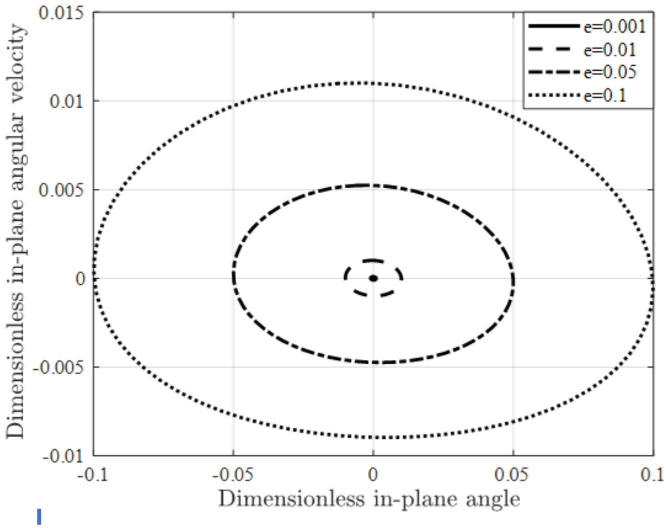
3. There are **initial state errors** and external **disturbances** in the actual situation

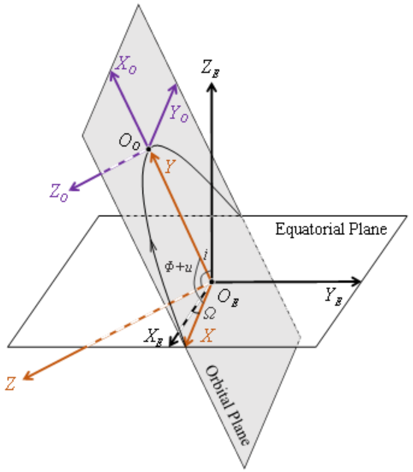
- An adaptive tracking controller is designed based on the backstepping method.

STS dynamic model

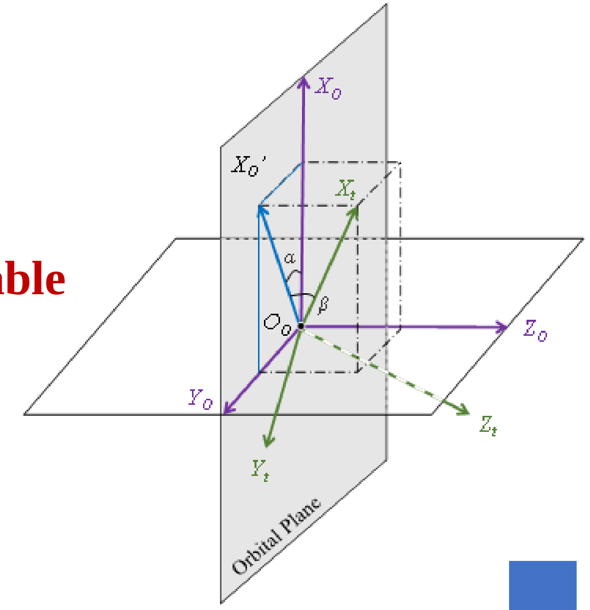
- $\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{g}(\boldsymbol{q})=\boldsymbol{Q}$

$$
\begin{cases}
\boldsymbol{M}(\boldsymbol{q})=\begin{bmatrix} \mathrm{m_e} & 0 \\ 0 & \mathrm{m_e} l^2 \end{bmatrix} \\
\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})=\begin{bmatrix} 0 & -\mathrm{m_e} l(\dot{\alpha}+2\dot{u}) \\ \mathrm{m_e} l(\dot{\alpha}+2\dot{u}) & \mathrm{m_e} l\dot{l} \end{bmatrix} \\
\boldsymbol{g}(\boldsymbol{q})=\begin{bmatrix} -\mathrm{m_e} l\dot{u}^2-\dfrac{\mu \mathrm{m_e} l}{R^3}\left(3\cos^2\alpha-1\right)+\dfrac{kl\delta^2}{(1+\delta)^2} \\ \dfrac{3\mu \mathrm{m_e} l^2}{R^3}\sin\alpha\cos\alpha \end{bmatrix}
\end{cases}
$$



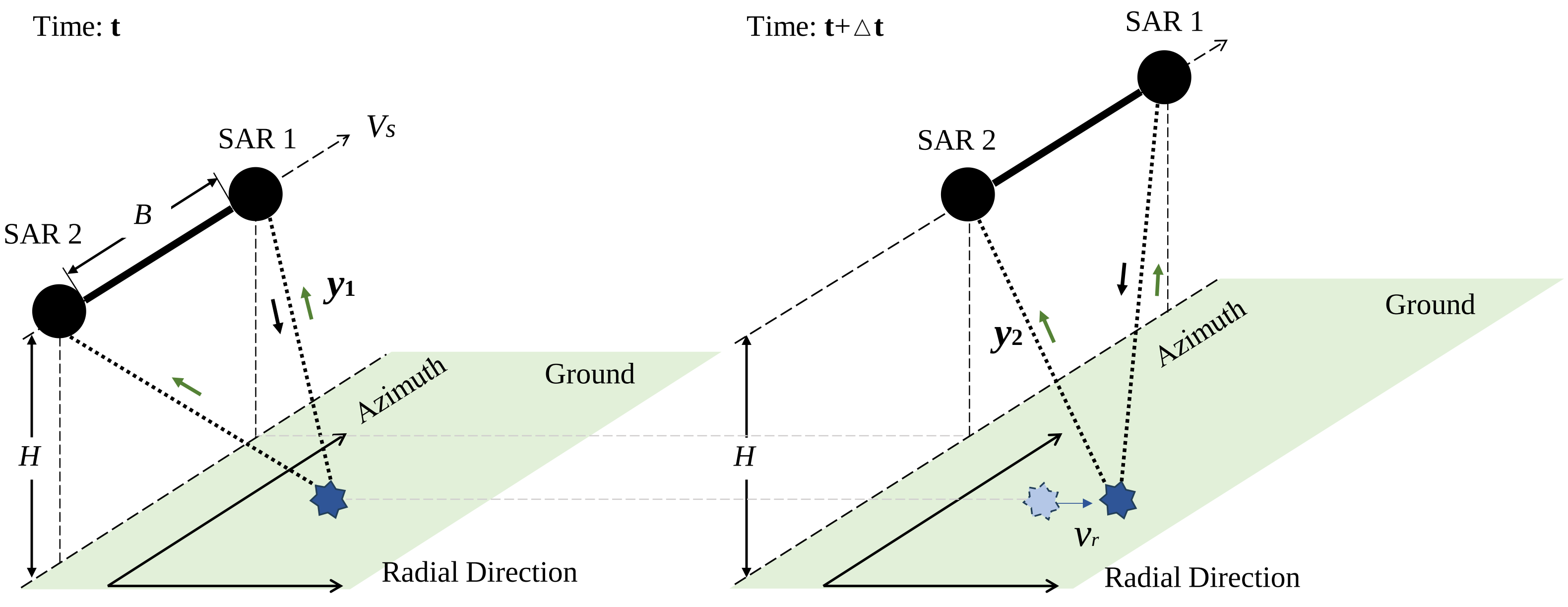


$$
\begin{cases}
\boldsymbol{q_b}=[0,0]^T \quad \textbf{stable} \\
\lambda_{1,2}=\pm j\sqrt{3\mu/R^3} \\
\boldsymbol{q_b}=[\pm\pi/2,0]^T \quad \textbf{unstable} \\
\boxed{\lambda_{1,2}=\pm\sqrt{3\mu/R^3}}
\end{cases}
$$



- $\ddot{\alpha}=-\dfrac{3\mu}{R^3}\sin\alpha\cos\alpha \longrightarrow \dot{\bar{\boldsymbol{q}}}=\begin{bmatrix} 0 & 1 \\ -\dfrac{3\mu}{R^3}\cos 2\alpha_b & 0 \end{bmatrix}\bar{\boldsymbol{q}} \longrightarrow \lambda^2+\dfrac{3\mu}{R^3}\cos 2\alpha_b=0$

Synthetic criterion of measurement error



$$\Delta\varphi = \frac{4\pi}{\lambda_b}(y_1 - y_2) = \frac{4\pi}{\lambda_b}(\frac{B}{V_s}v_r) \longrightarrow v_r = \frac{V_s \lambda_b \Delta\varphi}{4\pi B}$$

- Consider L, α into $\Delta v_r = \frac{V_s \lambda_b \Delta\varphi}{4\pi} s(L,\alpha) \longrightarrow s(L,\alpha) = \frac{1}{L}(\frac{1}{|\sin\alpha|} - 1)$

$\Delta\varphi$: interference phase difference

y_1, y_2 : ranges from the target to the center of SAR1 and SAR2

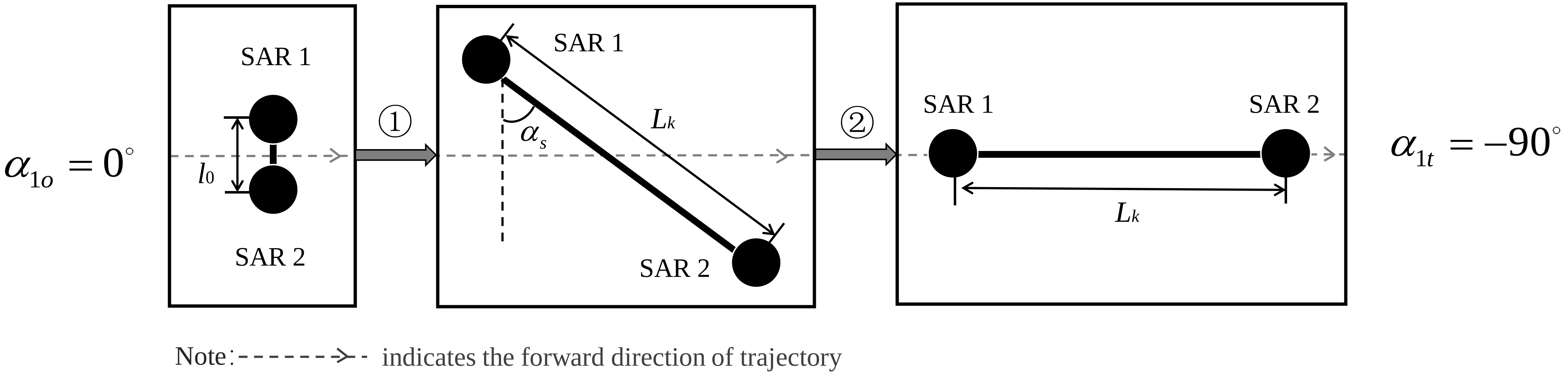
V_s : the speed of satellites

v_r : the radial speed of the target relative to SARs

$\lambda_b = 3$ cm : the wavelength of the SAR in the GMTI mission

B : the length of the interference baseline

Case I: Combined tension and thrust strategy



① Tether deployment:

- the tether is deployed to the expected length L_k under tension control u_L
- the in-plane angle swings to the α_s
- the in-plane angular velocity $\dot{\alpha}_s \neq 0^\circ/\text{s}$

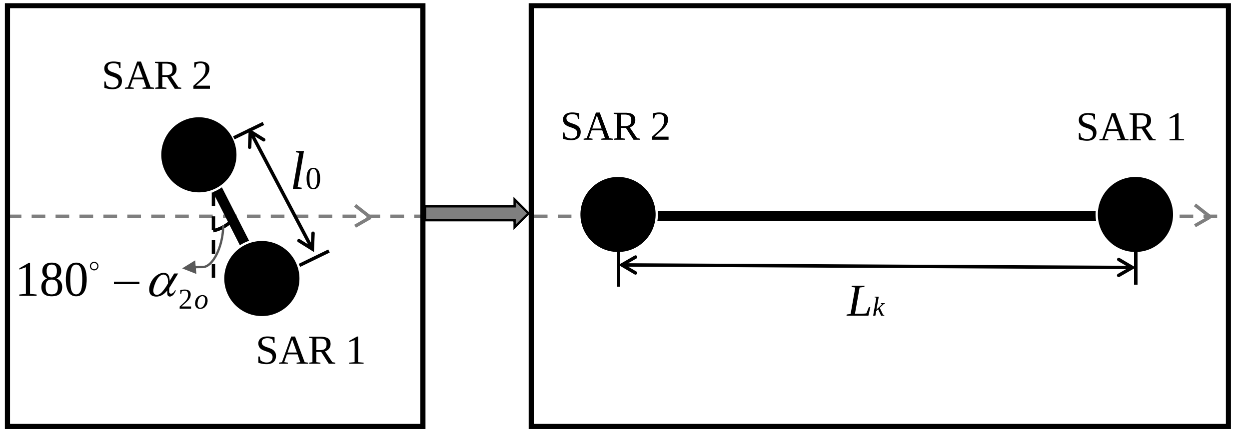
② The in-plane angle adjustment:

- apply control thrust u_α to keep $\dot{\alpha}_s$ constant until α_s to the range around 90°
- apply control thrust u_α to decrease the in-plane angular velocity $\dot{\alpha}_1$ to $0^\circ/\text{s}$

Case II: Optimal tension strategy

Time-optimal tension control law:

$$u_L(t)=\begin{cases} u_{\min} & 0\le t<t_1 \\ u_{\min}+(u_{\max}-u_{\min})\sin^2(k(t-t_1)) & t_1\le t<t_2 \\ u_{\max} & t_2\le t<t_3 \\ u_{\max}(1-\sin^2(k(t-t_3))) & t_3\le t<t_4 \\ u_{\min} & t_4\le t \end{cases}$$



Note: - - - - - - - -> indicates the forward direction of trajectory

Selection of optimum initial position :

- Coriolis force effect should be considered
- The terminate in-plane angle $\alpha_{2t}=90^\circ$ and angular velocity $\dot{\alpha}_{2t}=0^\circ/\text{s}$ should be guaranteed.

$$\alpha_{2o}\ne 0^\circ \xrightarrow{u_L} \alpha_{2t}=90^\circ$$

this strategy **does not introduce any additional thrust**, only by controlling the tension from the optimum initial position.

Design of the closed-loop tracking controller

Theorem 1. Consider the STS (1) controlled by the adaptive backstepping tracking controller (2) with the adaptive law (3). Under assumptions 1 and 2, for any initial conditions satisfying $V(0)\le\sigma$ with a positive constant σ, the control errors converge to an adjustable neighborhood of the origin.

$$\ddot{\boldsymbol{q}} = -\boldsymbol{M}(\boldsymbol{q})^{-1}\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{M}(\boldsymbol{q})^{-1}\boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{M}(\boldsymbol{q})^{-1}\boldsymbol{Q}$$

$$\downarrow$$

$$\begin{cases} \dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2 \\ \dot{\boldsymbol{x}}_2 = -\boldsymbol{M}(\boldsymbol{x}_1)^{-1}\boldsymbol{C}(\boldsymbol{x}_1,\boldsymbol{x}_2)\boldsymbol{x}_2 - \boldsymbol{M}(\boldsymbol{x}_1)^{-1}\boldsymbol{g}(\boldsymbol{x}_1) + \boldsymbol{M}(\boldsymbol{x}_1)^{-1}\boldsymbol{Q} - \boldsymbol{d} \end{cases} \quad \textbf{(1)}$$

- $$\boldsymbol{Q} = \boldsymbol{C}(\boldsymbol{x}_1,\boldsymbol{x}_2)\boldsymbol{x}_2 + \boldsymbol{g}(\boldsymbol{x}_1) + \boldsymbol{M}(\boldsymbol{x}_1)\left[\dot{\boldsymbol{x}}_{2d} - \boldsymbol{e}_1 + \boldsymbol{K}_2\boldsymbol{\delta} + \hat{D}\frac{\boldsymbol{\delta}}{\|\boldsymbol{\delta}\|+\varepsilon}\right] \quad \textbf{(2)}$$

$$\dot{\hat{D}} = \frac{\boldsymbol{\delta}^T\boldsymbol{\delta}}{\|\boldsymbol{\delta}\|+\varepsilon} - \hat{D} \quad \textbf{(3)} \qquad \boldsymbol{e}_1 = \boldsymbol{x}_{1d} - \boldsymbol{x}_1 \quad \boldsymbol{x}_{2d} = \dot{\boldsymbol{x}}_{1d} + \boldsymbol{K}_1\boldsymbol{e}_1 \quad \dot{\boldsymbol{\delta}} = \boldsymbol{e}_1 - \boldsymbol{K}_2\boldsymbol{\delta}$$

$$V = \frac{1}{2}\boldsymbol{e}_1^T\boldsymbol{e}_1 + \frac{1}{2}\boldsymbol{\delta}^T\boldsymbol{\delta} + \frac{1}{2}\tilde{D}^2$$

$$\dot{V} \le -\gamma V + D_d$$

where $D_d = 1/2D^2 + \varepsilon D$ is bounded

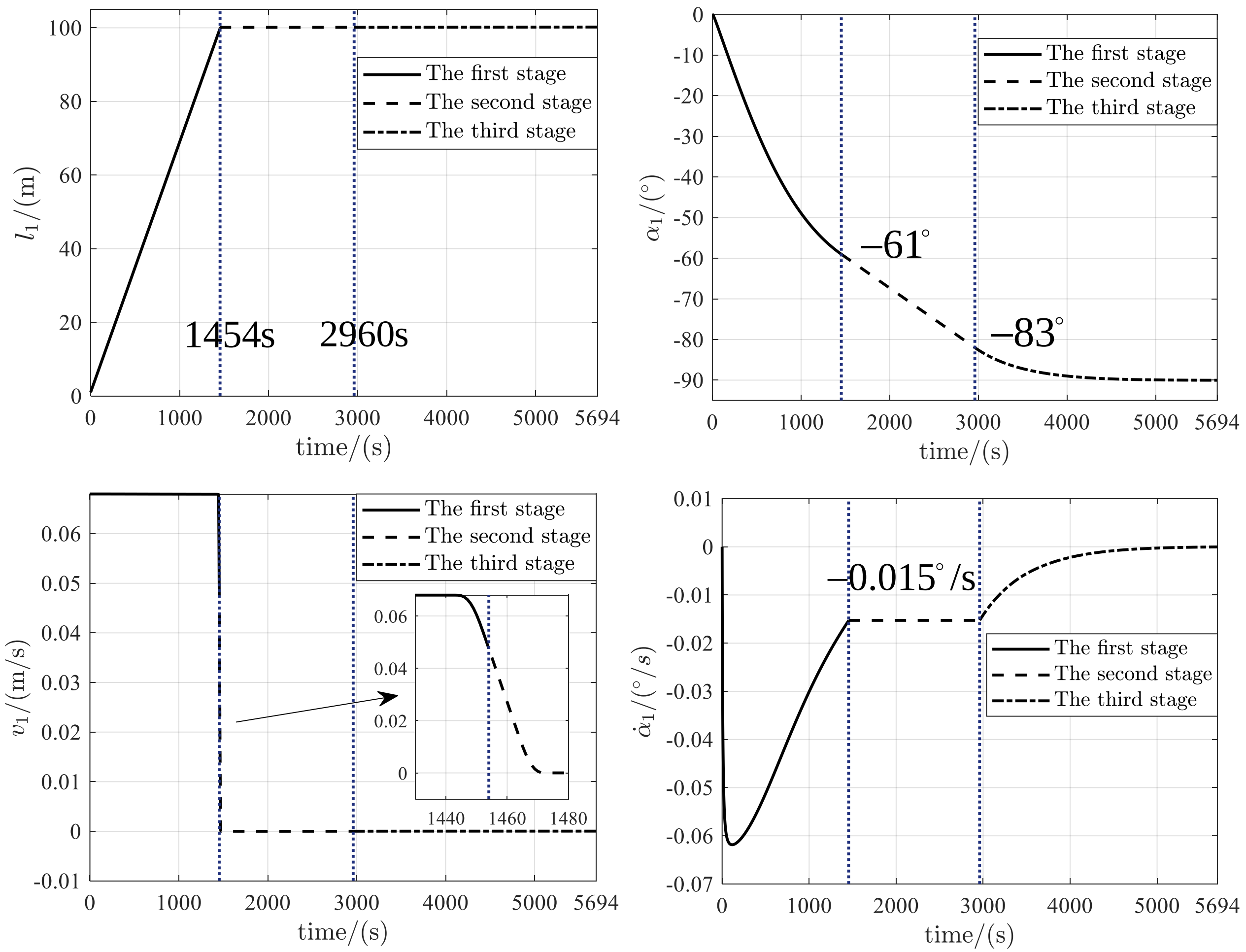
Suppose $\gamma > D_d/\sigma$

$\dot{V} < 0$ when $V = \sigma$

$V(t) \le \sigma$ if $V(0) \le \sigma$

Therefore, all states in the controlled system are semi-globally uniformly ultimately bound.

Case I (the sub-satellite in front of the main satellite)

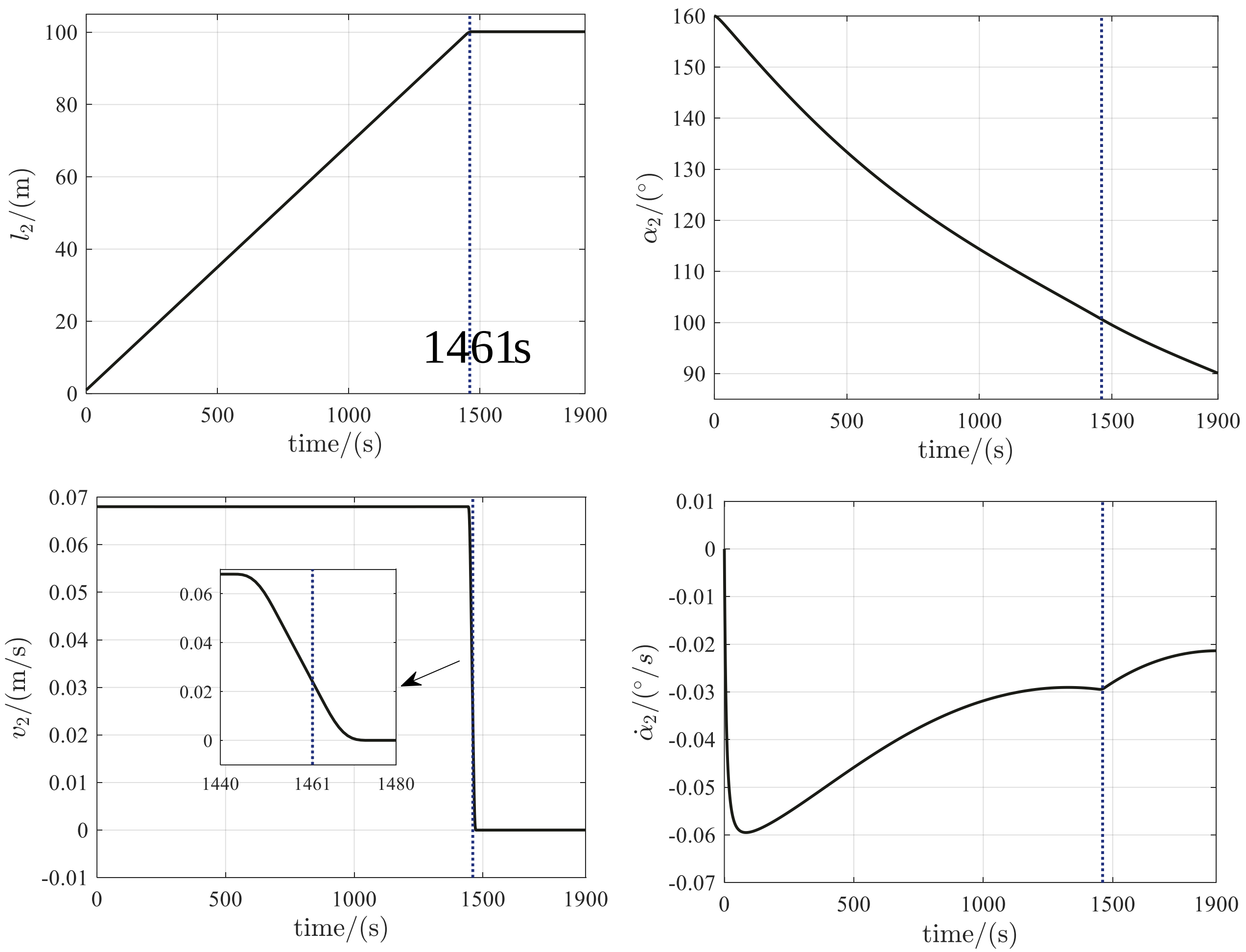


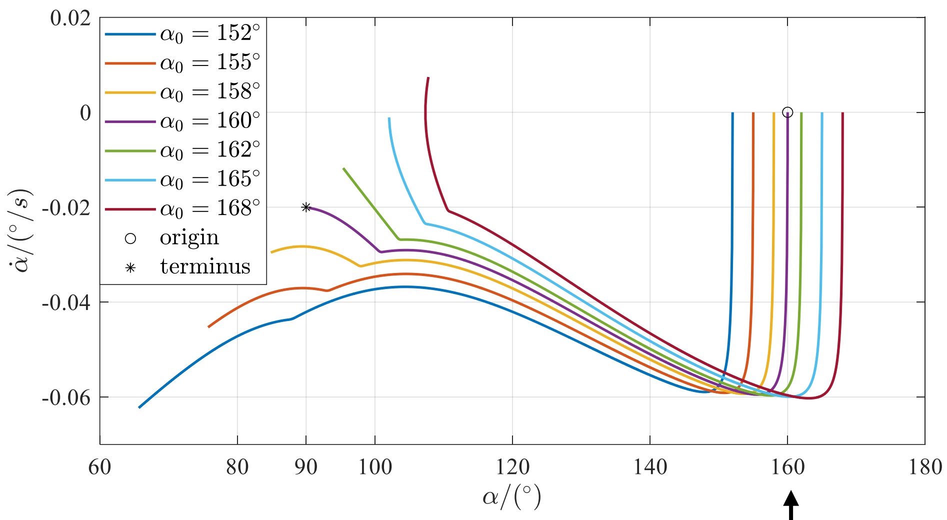
Parameter	Description (Unit)	Value
a	Orbital semimajor axis (km)	6892.6
i	Orbital inclination in degrees (°)	97.4
e	Orbital eccentricity (/)	0.0011
Φ	The argument of perigee in degrees (°)	90
Ω	The right ascension of the ascending node in degrees (°)	0
u_0	The initial true anomaly in degrees (°)	0

Parameter	Description (Unit)	Value
m_A	Mass of the main satellite (kg)	400
m_B	Mass of the sub-satellite (kg)	350
m_t	Mass of tether (kg)	0.486
S_A	The frontal area of the main satellite (m^2)	2
S_B	The frontal area of the sub-satellite (m^2)	2
V_P	Orbital velocity of satellites (km/s)	7.605
L_k	The total length of tether (m)	100
k	Elastic coefficient of tether (N/m)	2.46×10^4

- $[l_{1o}, v_{1o}, \alpha_{1o}, \dot{\alpha}_{1o}] = [1.0, 0.068, 0, 0]$
  $[l_{1t}, v_{1t}, \alpha_{1t}, \dot{\alpha}_{1t}] = [100, 0, -90°, 0]$

Case II (the main satellite in front of the sub-satellite)





- $[l_{2o}, v_{2o}, \alpha_{2o}, \dot{\alpha}_{2o}] = [1.0, 0.068, 160^\circ, 0]$

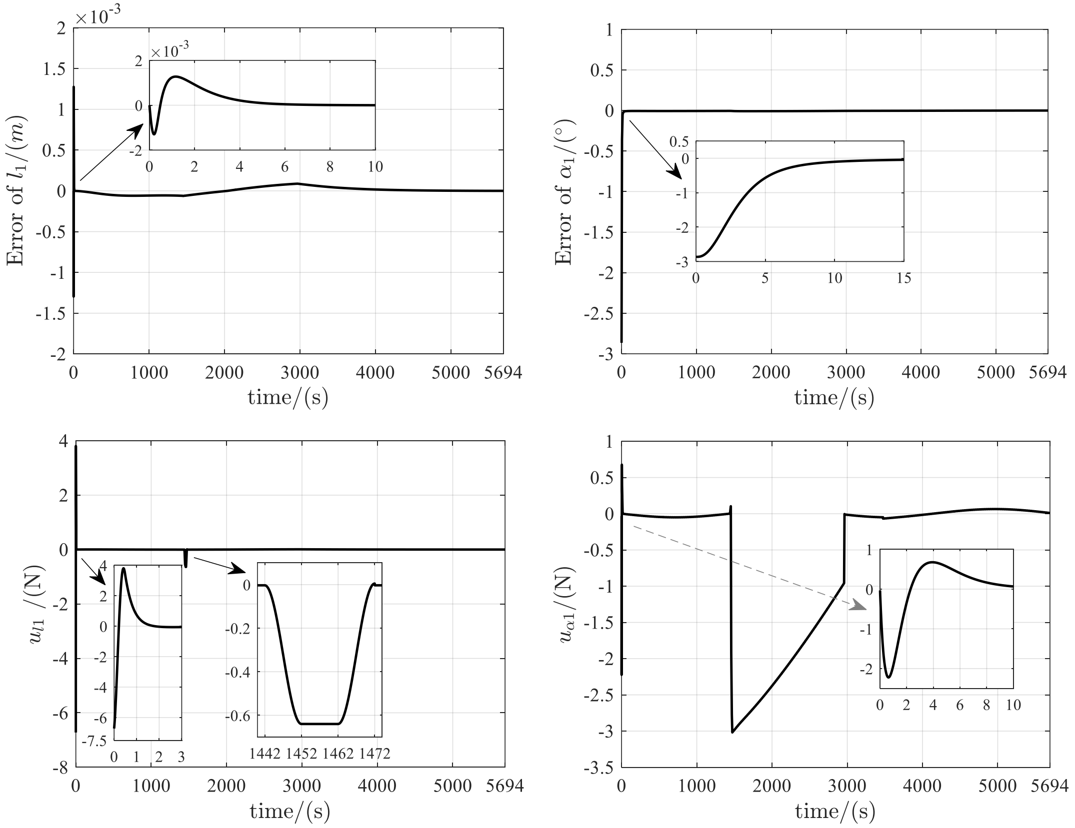
  $[l_{2t}, v_{2t}, \alpha_{2t}, \dot{\alpha}_{2t}] = [100, 0, 90^\circ, -0.021\,^\circ/\mathrm{s}]$

It can be regarded as the small initial state error in the subsequent station-keeping phase

Comparison of case I and II

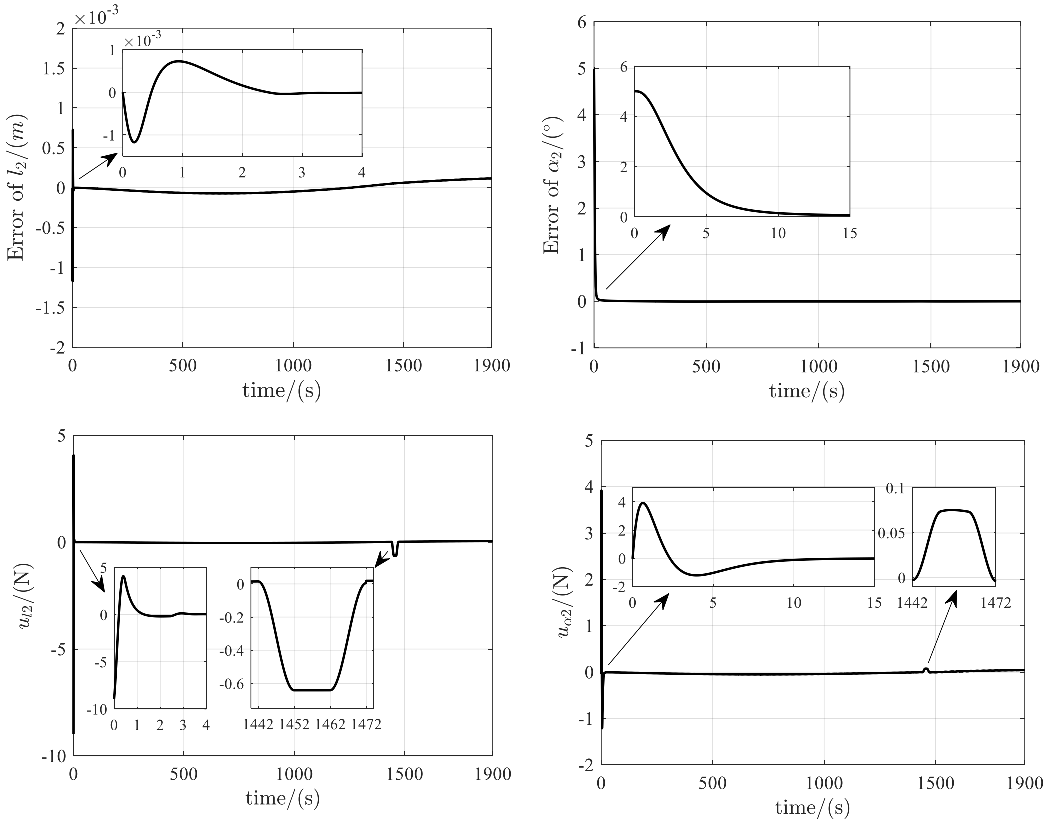
Initial errors: $\Delta \dot{l}_0 = 0.01\text{m/s}$ $\Delta \alpha_0 = -2.86^\circ$

Control Forces: u_L $[-6.8, 3.9]$ u_α $[-3, 0.7]$

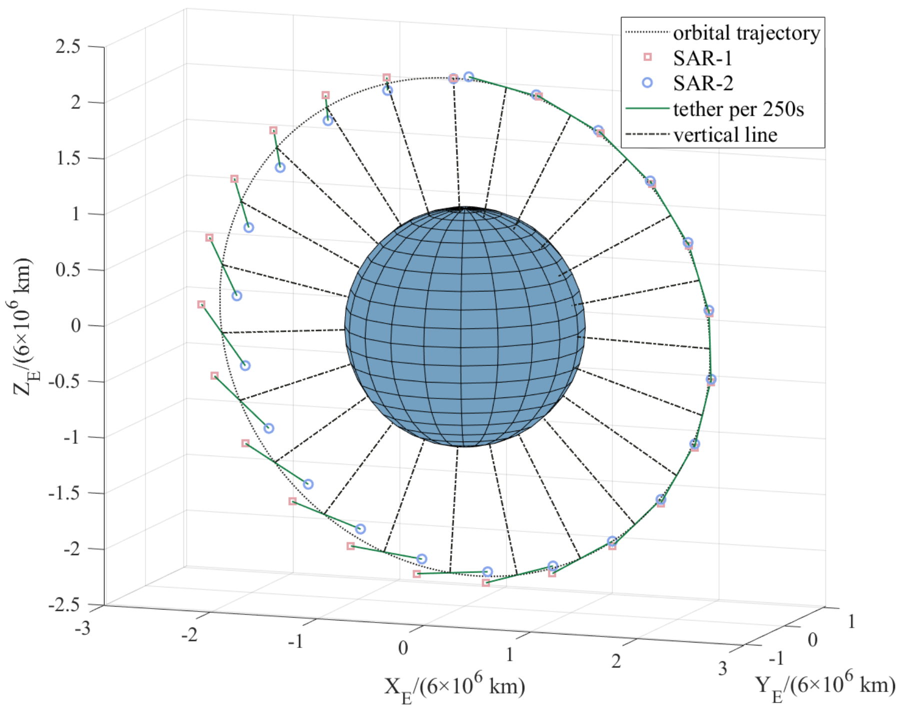


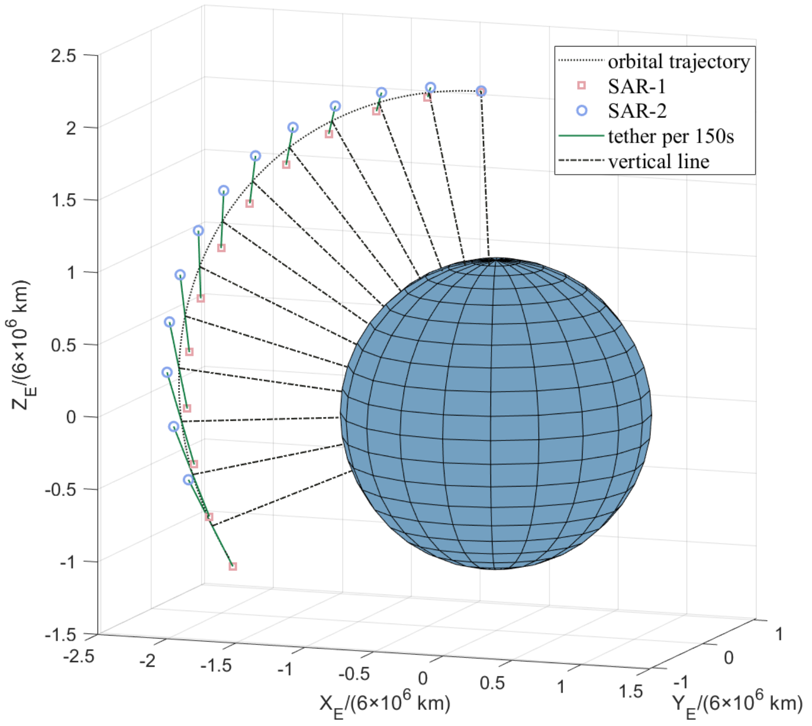
Initial errors: $\Delta \dot{l}_0 = 0.01\text{m/s}$ $\Delta \alpha_0 = -5^\circ$

Control Forces: u_L $[-9, 4]$ u_α $[-1.2, 3.9]$



Comparison of case I and II

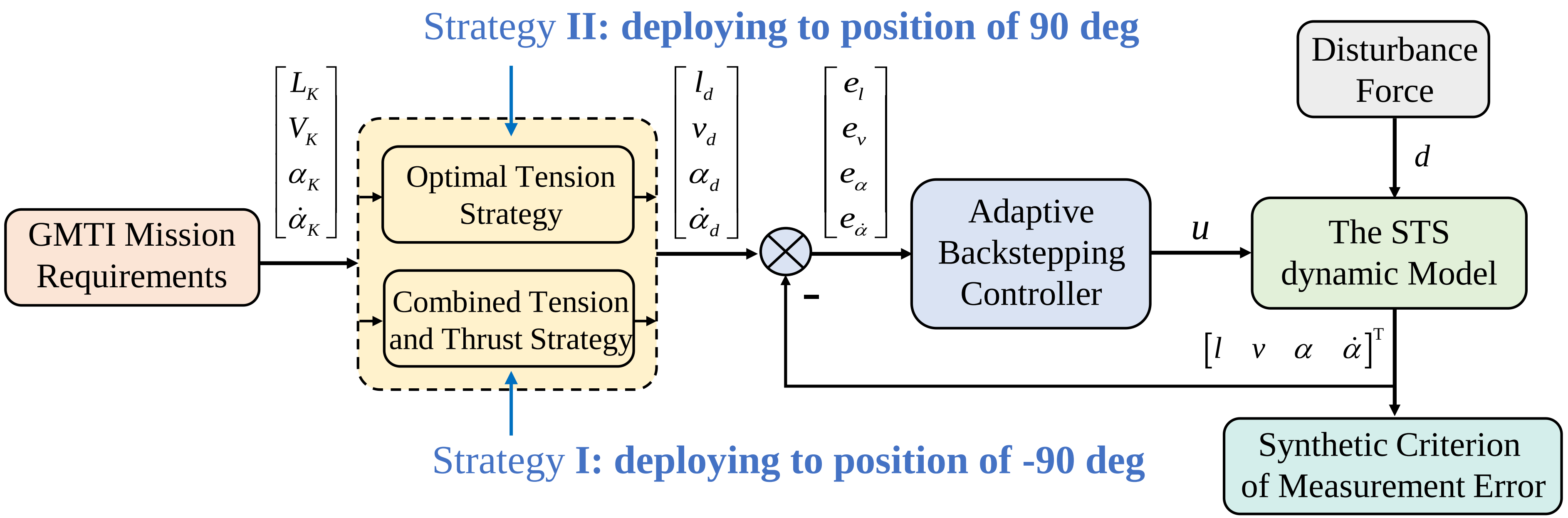




Strategy	Tension	Thrust	Initial in-plane angle	Deployment time	synthetic criterion	Energy Consumption
I	√	√	0 deg	5694 s	6.064×10^{-10}	7064.9
II	√	×	160 deg	1900 s	150.37×10^{-10}	6.8497

Note: $\Delta\alpha = \pm 5^{\circ}$, $L = 100$ ⟶ $s(L,\alpha) = 3.82\times10^{-5}$

5. Conclusion



- Strategy **I** ensures a **stable** deployment with a longer operation time.
  Strategy **II** ensures a **quick** deployment with a larger synthetic criterion.
- When the initial α is **0 or ±180 deg**, the STS requires **thrust** assistance.
  When the initial α falls within **90~180 deg or -180 ~ -90 deg**, the STS only searches for the **optimal state**.
- The results numerical demonstrate that the controller ensures a stable deployment to the operational configuration under **initial state errors** and **external disturbances**.

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