Calculation and Control of Equilibrium Position of Bare Electrodynamic Tether System

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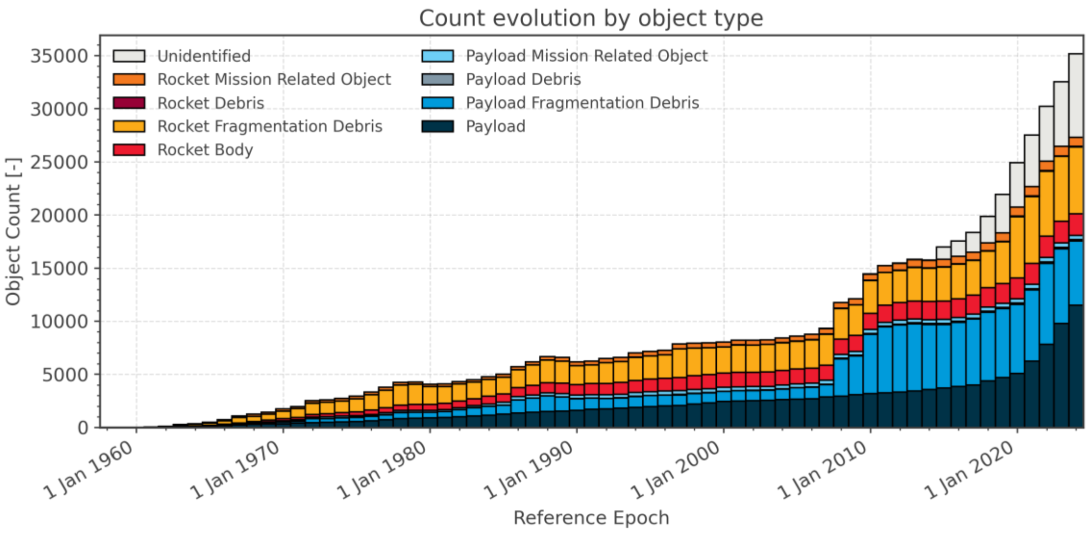
Outline

1. Background
2. Dynamic Model
3. Method
4. Conclusions

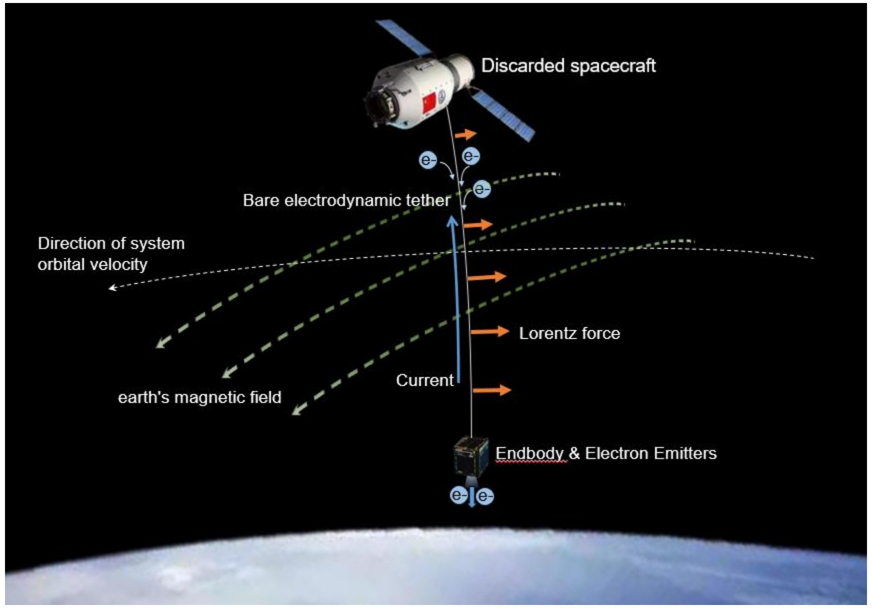
1 Background

- Background

  - According to incomplete statistics, the number of man-made objects in space today exceeds 35,000, the vast majority of which are space debris.



  - Among these de-orbiting technologies, the electrodynamic tether(EDT) is an attractive technology because provides a propellant-free technology for de-orbiting.



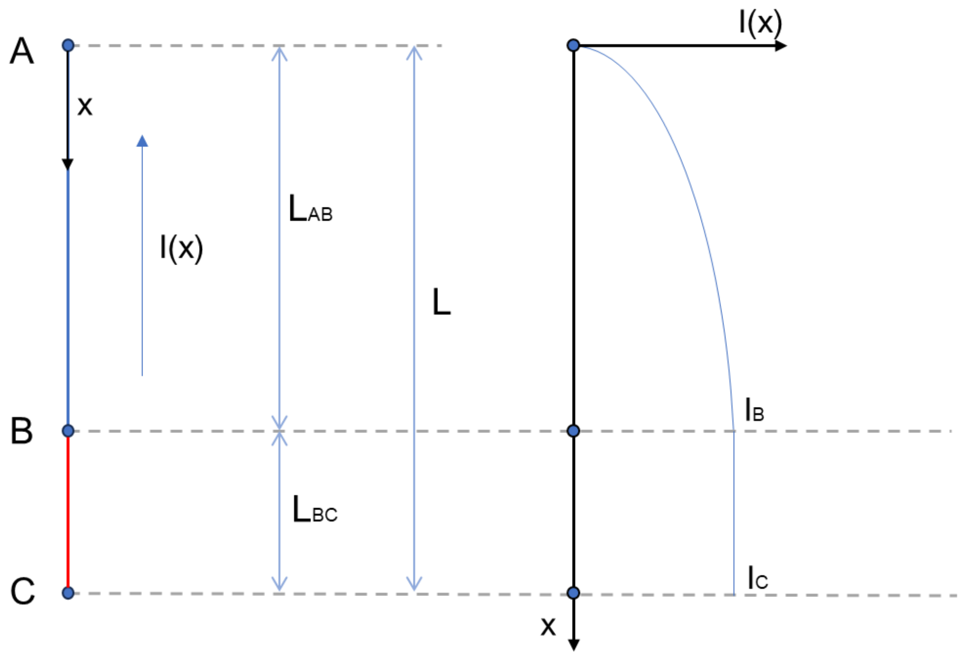
1 Background

- Research content

1.Because of the nonlinearity of the current leads to problems in the calculation of equilibrium position and designing stable control strategy.

2.A method for calculating the equilibrium position of BEDT based on the substitution of integral variables.

3. A stable sliding mode control strategy based on adjusting the length of the tether.

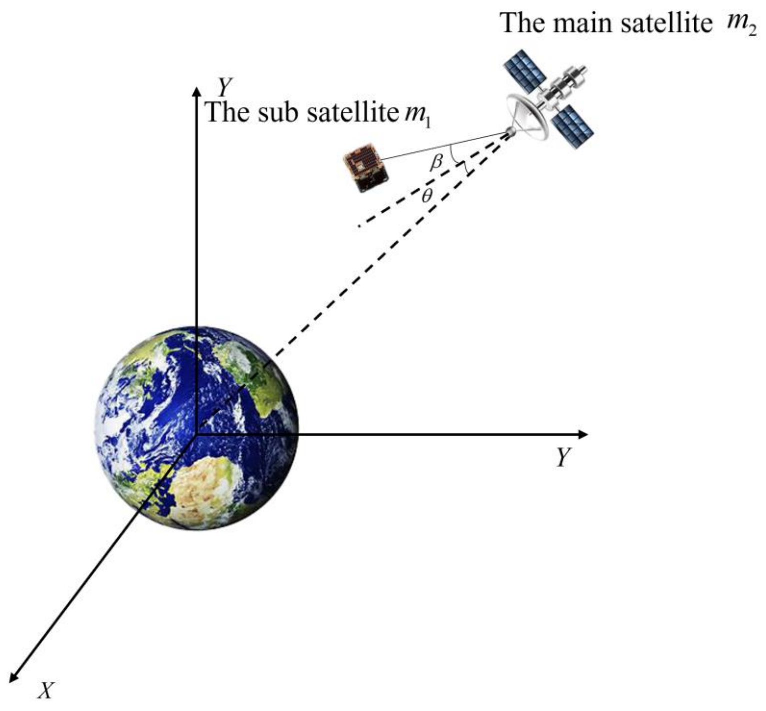


2 Dynamic Model

Assumptions:

1. The tether is considered as a rigid rod neglecting mass;
2. The end bodies are considered as a mass point;
3. The tether is fully deployed.

$$\begin{cases} \ddot{l}-l\left[\dot{\beta}+\left(\dot{\theta}+\omega\right)^{2}\cos^{2}\beta+v^{-1}\omega^{2}\left(3\cos^{2}\theta\cos^{2}\beta-1\right)\right]=\dfrac{T}{m_{e}} \\ \ddot{\theta}+\dot{\omega}+2\left(\dot{\theta}+\omega\right)\left(\dfrac{\dot{l}}{l}-\dot{\beta}\tan\beta\right)+1.5v^{-1}\omega^{2}\sin 2\theta=\dfrac{Q_{\theta}}{m_{e}l^{2}\cos^{2}\beta} \\ \ddot{\beta}+2\dot{\beta}\dfrac{\dot{l}}{l}+\left(0.5(\dot{\theta}+\omega)^{2}+1.5v^{-1}\omega^{2}\cos^{2}\theta\right)\sin 2\beta=\dfrac{Q_{\beta}}{m_{e}l^{2}} \end{cases}$$

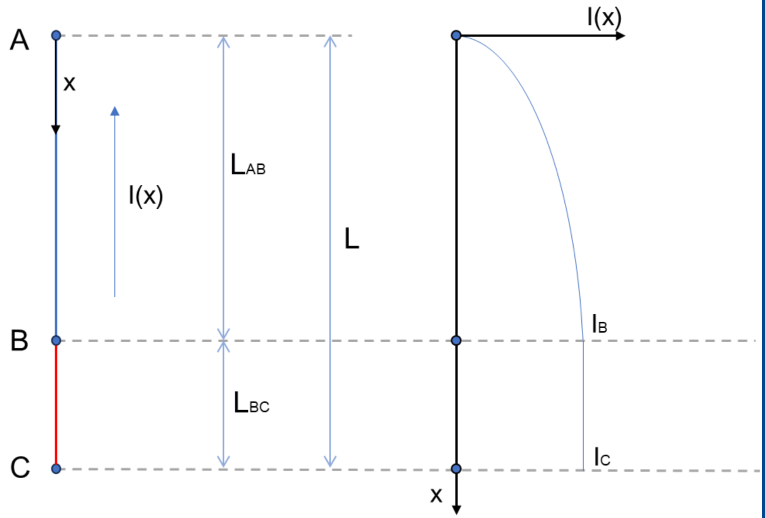


2 Dynamic Model

Current Model

$$\begin{cases} \dfrac{dj}{d\xi} = \dfrac{3}{4}\sqrt{\phi} \\ \dfrac{d\phi}{d\xi} = j - 1 \end{cases}$$

$$\begin{cases} j(\phi) = 1 - \left[1 + \left(\phi^{\frac{3}{2}} - \phi_a^{\frac{3}{2}}\right)\right]^{\frac{1}{2}} \\ \xi(\phi) = \int_{\phi}^{\phi_a} \left[1 + \left(\phi^{\frac{3}{2}} - \phi_a^{\frac{3}{2}}\right)\right]^{-\frac{1}{2}} \\ \phi_a = \left(2j_0 - j_0^2\right)^{\frac{2}{3}} \end{cases}$$



Orbital Motion Model

$$\begin{cases} \dfrac{dA}{dt} = 2\nu\sqrt{\dfrac{A^3}{K(1-q^2-k^2)}}\left[a_s\dfrac{q\sin u - k\cos u}{\nu} + a_t\right] \\ \dfrac{dq}{dt} = \sqrt{\dfrac{p}{K}}\left[a_s\sin u + a_t\left(1+\dfrac{1}{\nu}\right)\cos u + \dfrac{1}{\nu}\left(qa_t + ka_w\cot i\sin u\right)\right] \\ \dfrac{dk}{dt} = \sqrt{\dfrac{p}{K}}\left[-a_s\cos u + a_t\left(1+\dfrac{1}{\nu}\right)\sin u + \dfrac{1}{\nu}\left(ka_t - qa_w\cot i\sin u\right)\right] \\ \dfrac{di}{dt} = \dfrac{a_w}{\nu}\sqrt{\dfrac{p}{K}}\cos u \\ \dfrac{d\Omega_u}{dt} = \dfrac{a_w}{\nu}\sqrt{\dfrac{p}{K}}\dfrac{\sin u}{\sin i} \\ \dfrac{du}{dt} = \dfrac{1}{\nu}\sqrt{\dfrac{p}{K}}\left[\nu^3\dfrac{k}{p^2} - a_w\cot i\sin u\right] \end{cases}$$

A – semi-major axis, e – eccentricity,

$q = e\cos\Omega_u, k = e\sin\Omega_u, e = \sqrt{q^2+k^2}$,

ϑ – true anomaly, Ω_u – argument of perigee.

3 Calculation of equilibrium position

When the system is working in the retention state:

$$\frac{3}{2}\omega^2 \sin 2\theta = \frac{Q_\theta}{m_e l^2}$$

Where

$$Q_\theta = \begin{cases} B_\theta I_* L_* \int_0^{\xi_0} j\Delta_1 d\xi + B_\theta j_0 I_* \left(l - \xi_0 L_*\right)\Delta_2 & \xi_0 \le \frac{l}{L_*} \\ B_\theta I_* L_* \int_0^{\xi_l} j\Delta d\xi & \xi_0 > \frac{l}{L_*} \end{cases}$$

Substitution of integral variables

$$d\xi = \frac{1}{j-1} d\phi$$

Then

$$Q_\theta = \begin{cases} B_\theta I_* L_* \int_{\phi_a}^{0} \frac{j}{j-1}\Delta_1 d\phi + B_\theta j_0 I_* \left(l - \xi_0 L_*\right)\Delta_2 & \xi_0 \le \frac{l}{L_*} \\ B_\theta I_* L_* \int_{\phi_a}^{\phi_l} \frac{j}{j-1}\Delta d\phi & \xi_0 > \frac{l}{L_*} \end{cases}$$

Equilibrium position

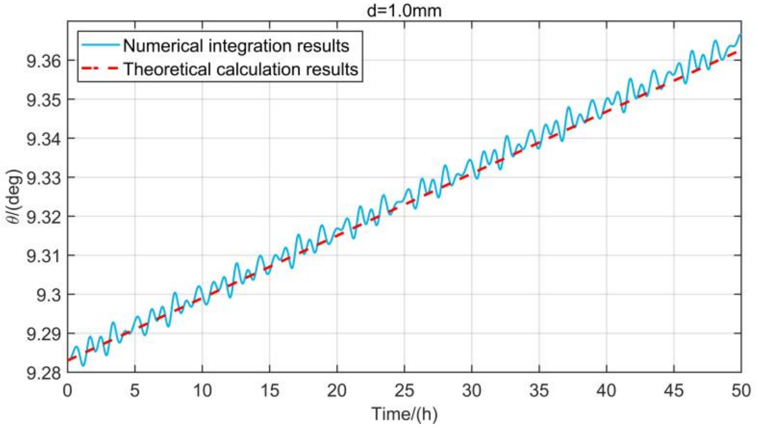
$$\theta_{eq} = \frac{1}{2}\arcsin\left(2\mu_m \chi \cos i\right)$$

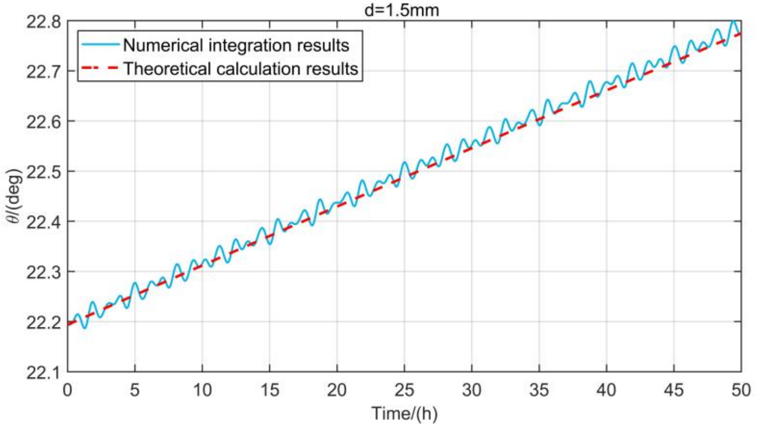
Where

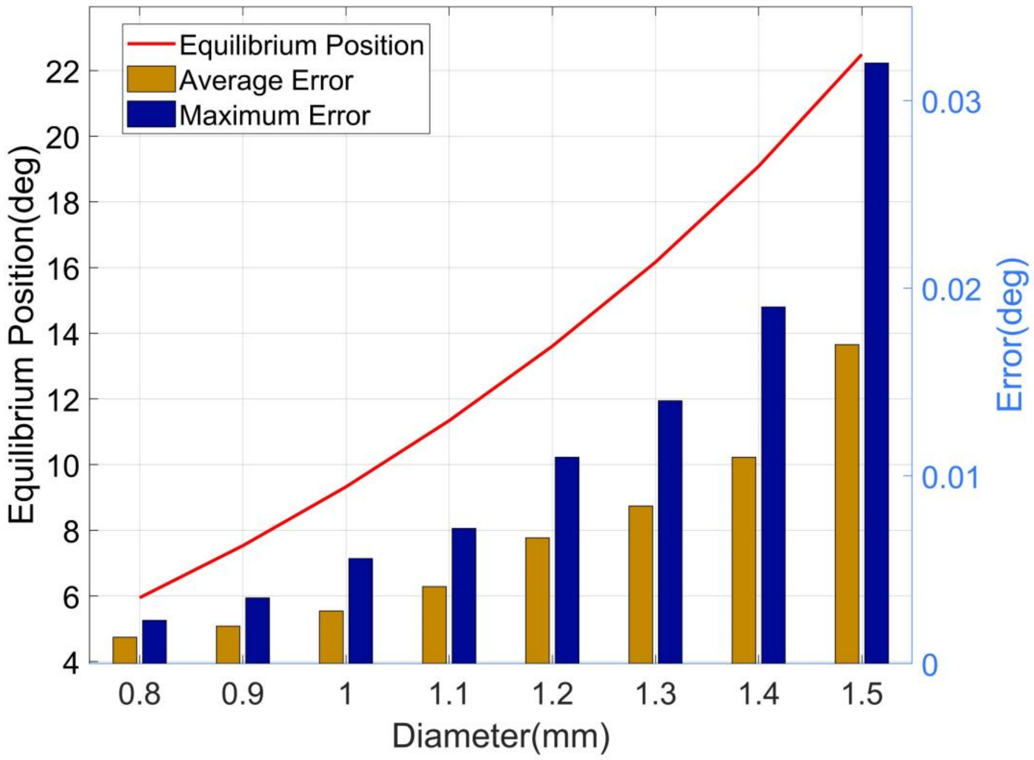
$$\chi == \begin{cases} \dfrac{I_* L_* \int_{\phi_a}^{0} \frac{j}{j-1}\Delta_1 d\phi + j_0 I_* \left(l - \xi_0 L_*\right)\Delta_2}{3 m_e l^2 K} & \xi_0 \le \frac{l}{L_*} \\ \dfrac{I_* L_* \int_{\phi_a}^{\phi_l} \frac{j}{j-1}\Delta d\phi}{3 m_e l^2 K} & \xi_0 > \frac{l}{L_*} \end{cases}$$

3 Calculation of equilibrium position

- Calculation error under different tether diameters



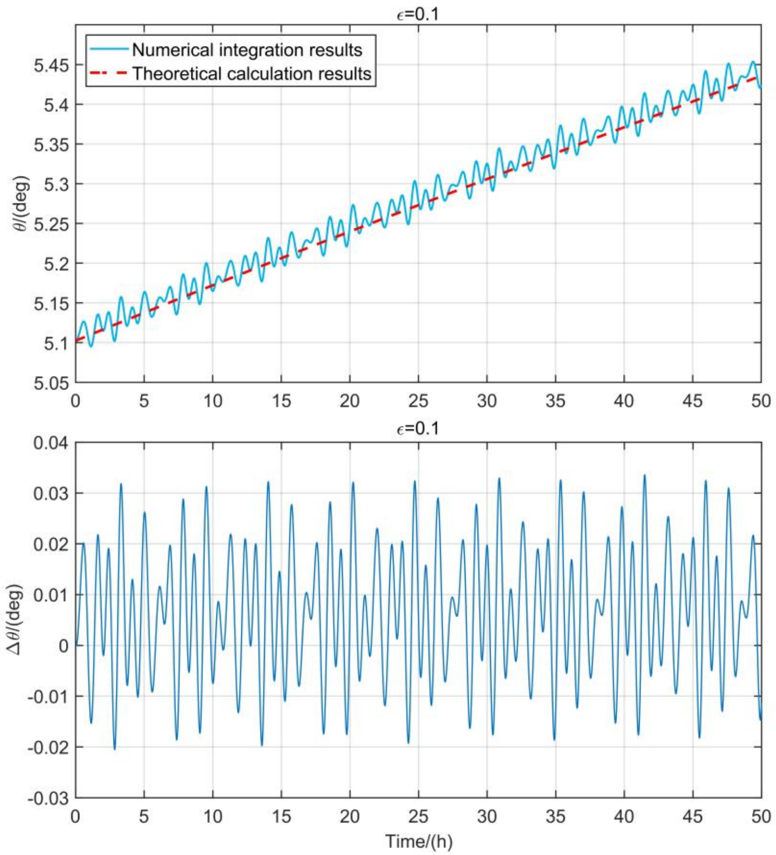


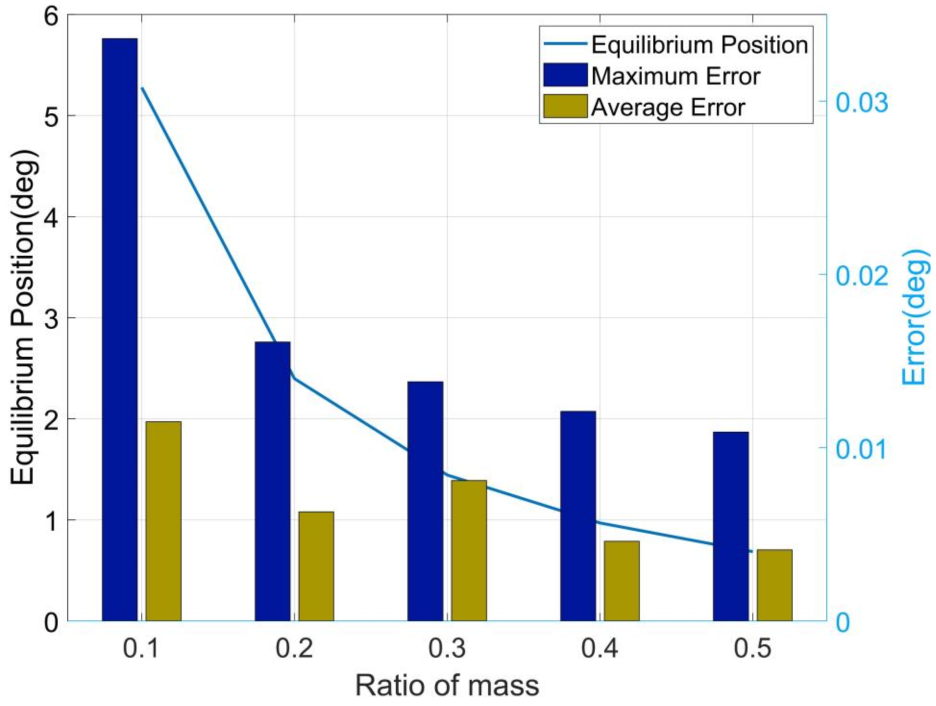


The equilibrium position of the system increases with the increase of the tether diameter the average errors are still kept within 0.02 deg

Calculation of equilibrium position

- Calculation error under different ratio of mass

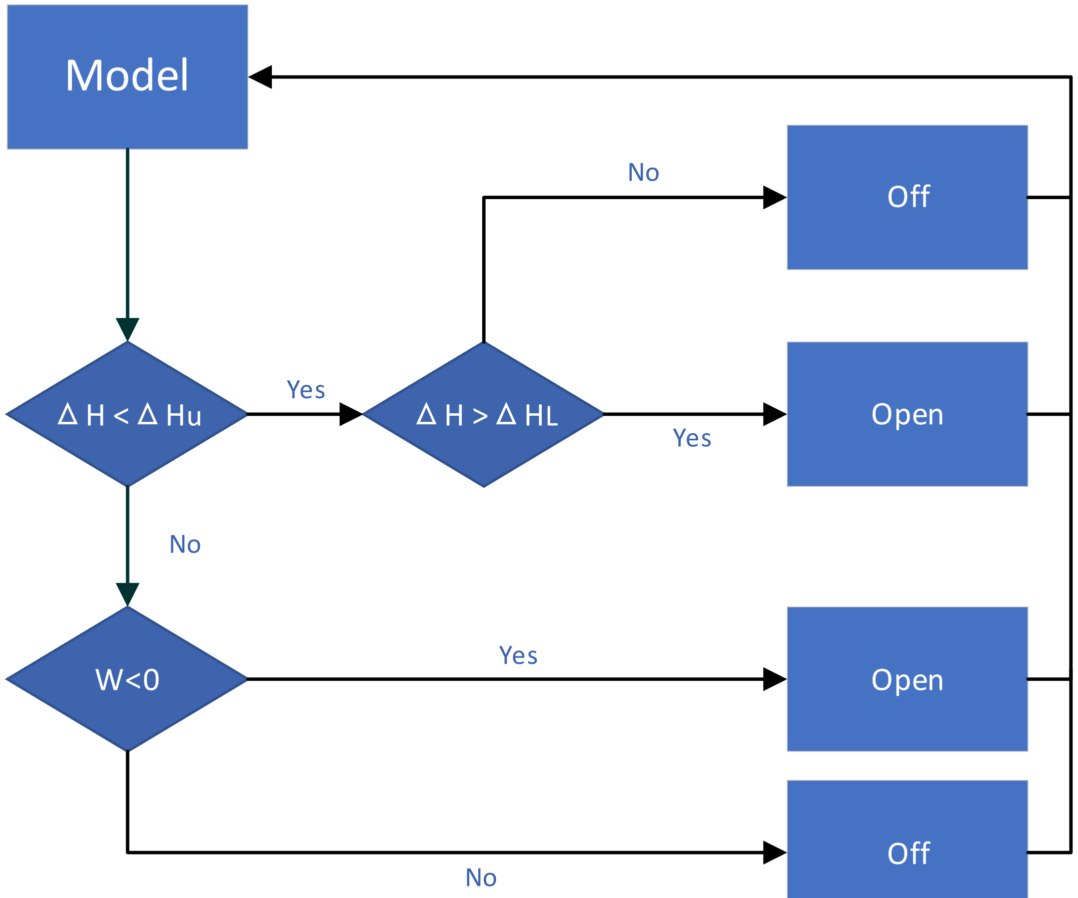




The simulation results show that the average error of the equilibrium position calculation method proposed in this paper is kept within 1%, which has a good accuracy and stability.

3 Stable control strategy

- Traditional current-switching control strategy



$$H = \frac{1}{2}\boldsymbol{\omega}_{rel} \cdot \boldsymbol{J}\boldsymbol{\omega}_{rel} + \frac{3}{2}(\boldsymbol{\omega}_{orb} \cdot \boldsymbol{\omega}_{orb})\boldsymbol{R} \cdot \boldsymbol{J}\boldsymbol{R} - \frac{1}{2}(\boldsymbol{\omega}_{orb} \cdot \boldsymbol{\omega}_{orb})\boldsymbol{W} \cdot \boldsymbol{J}\boldsymbol{W}$$

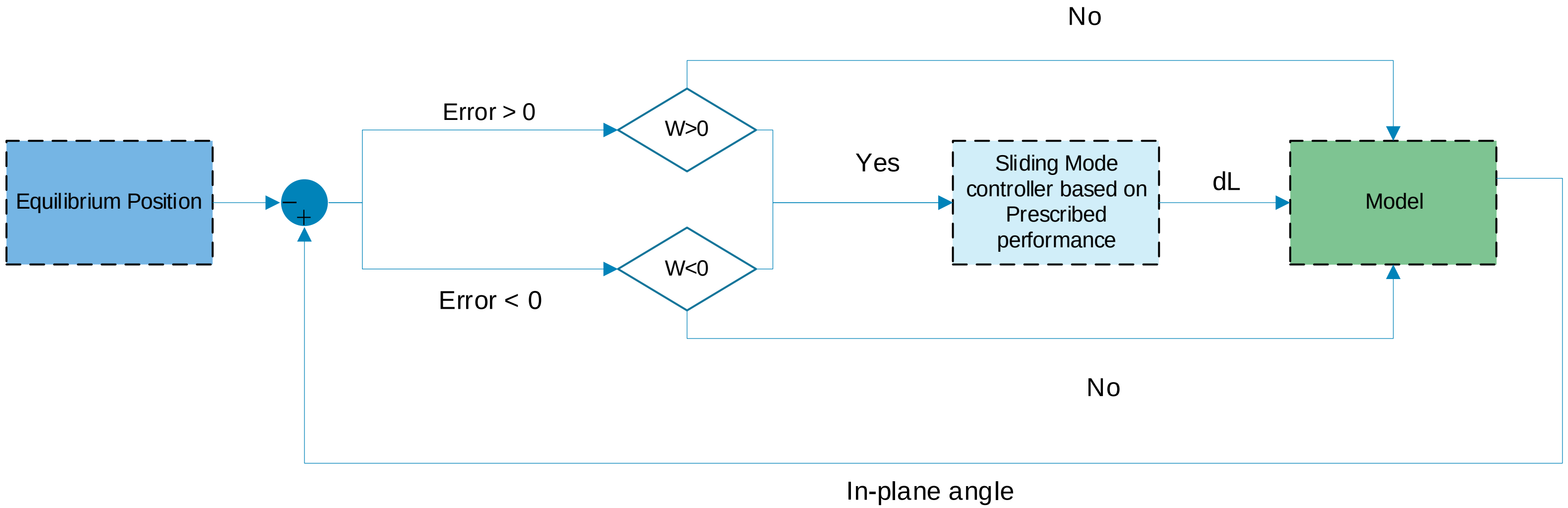
$$\Delta\bar{H} = 4 - 3\cos^2(\theta - \theta_{eq})\cos^2\varphi + \left[\dot{\varphi}^2 + \left((\dot{\theta} - \dot{\theta}_{eq}) - \dot{\vartheta}\right)\cos^2\varphi\right]\frac{r^3}{\mu_g}$$

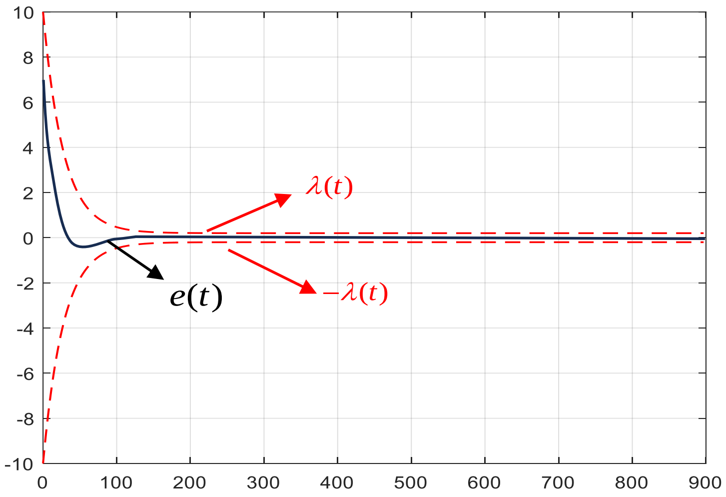
$$(\Delta\bar{H}_L, \Delta\bar{H}_U) \in (0, 3.0)$$

The switching on and off of the current will introduce undesired transient responses to the system and decreases the deorbit efficiency of the system.

3 Stable control strategy

- Stable control strategy based on adjusting the length of tether





This paper proposes to change the moment of inertia of the system as well as the Lorentzian torque by adjusting the length of the tether, thus suppressing the libration of the system

Designed a sliding mode controller based on prescribed performance.

3 Stable control strategy

- Stable control strategy based on adjusting the length of tether

Definition error and Sliding surface
and prescribed performance function:

$$\begin{cases} e = \theta - \theta_d \\ s = \dot{e} + ce \\ \lambda(t) = (\lambda_0 - \lambda_\infty) e^{-kt} + \lambda_\infty \end{cases}$$

Definition the error transformation function:

$$S(\varepsilon) = \frac{e^\varepsilon - e^{-\varepsilon}}{e^\varepsilon + e^{-\varepsilon}} \longrightarrow \varepsilon = \frac{1}{2}\ln\frac{1+S}{1-S} = \frac{1}{2}\ln\frac{\lambda+e}{\lambda-e}$$

Therefore

$$\dot{s} = \ddot{\varepsilon} + c\dot{\varepsilon} = M_1 + M_2 + M_3\ddot{e}$$
$$= M_1 + M_2 + M_3\left(\ddot{\theta} - \ddot{\theta}_d\right)$$

Controller

$$\begin{cases} \dot{l} = \dfrac{\left(-k_1 s - k_2 \tanh\left(\dfrac{s}{k_3}\right) - M_1 - M_2 - M_3 f_1 - c\dot{\varepsilon} + M_3\ddot{\theta}_d\right) l}{-2M_3\left(\dot{\theta} + \omega\right)} \\ f_1 = Q_\theta / m_e l^2 \cos\beta^2 - \dot{\omega} + 2\left(\dot{\theta} + \omega\right)\dot{\beta}\tan\beta - 1.5\nu^{-1}\omega^2 \sin 2\theta \\ k_1 > 0, k_2 > 0, k_3 > 0 \end{cases}$$

3 Stable control strategy

- Proof of stability

Arrival segment

The Lyapunov function is chosen as follows:

$$V_1 = \frac{1}{2}s^2$$

Derive

$$\dot{V}_1 = s\dot{s} = -k_1 s^2 - k_2 s \tanh\left(\frac{s}{k_3}\right)$$
$$\le -k_1 s^2 - k_2 |s| + k_2 k_3 \mu$$
$$\le -k_1 s^2 + k_2 k_3 \mu = -2k_1 V_1 + k_2 k_3 \mu$$

→

$$V_1 \le e^{-2k_1(t-t_0)} V_1(t_0) + k_2 k_3 \mu e^{-2k_1 t} \int_{t_0}^{t} e^{-2k_1 \tau} d\tau$$
$$= e^{-2k_1(t-t_0)} V_1(t_0) + \frac{k_2 k_3 \mu e^{-2k_1 t}}{2k_1}\left(e^{2k_1 t} - e^{2k_1 t_0}\right)$$
$$= e^{-2k_1(t-t_0)} V_1(t_0) + \frac{k_2 k_3 \mu}{2k_1}\left(1 - e^{2k_1(t-t_0)}\right)$$

When $t \to 0$

$$\lim_{t\to\infty} V_1(t) \le \frac{k_2 k_3 \mu}{2k_1}$$

Therefore

$$\dot{V}_1 \le -2k_1 V_1 + k_2 k_3 \mu \le 0$$

Sliding segment

When $s=0$

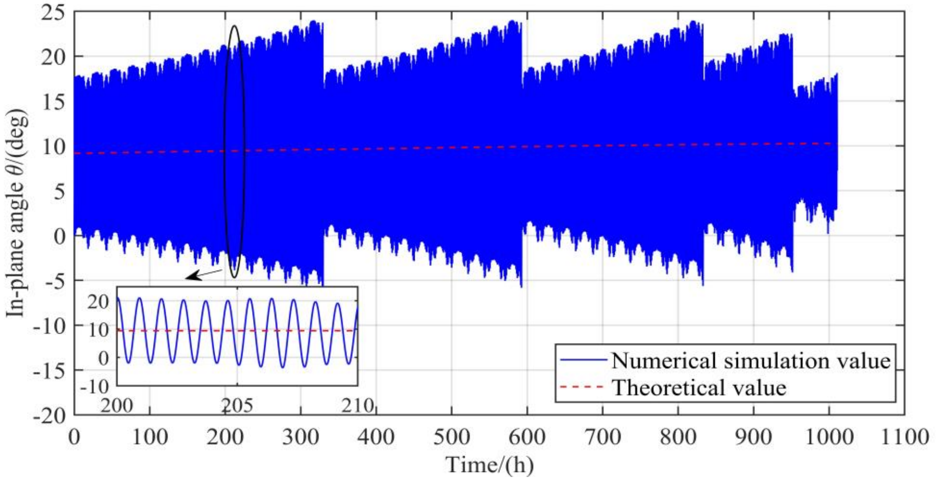
$$\dot{\varepsilon} + c\varepsilon = 0 \quad \to \quad \varepsilon = Ce^{-ct}$$

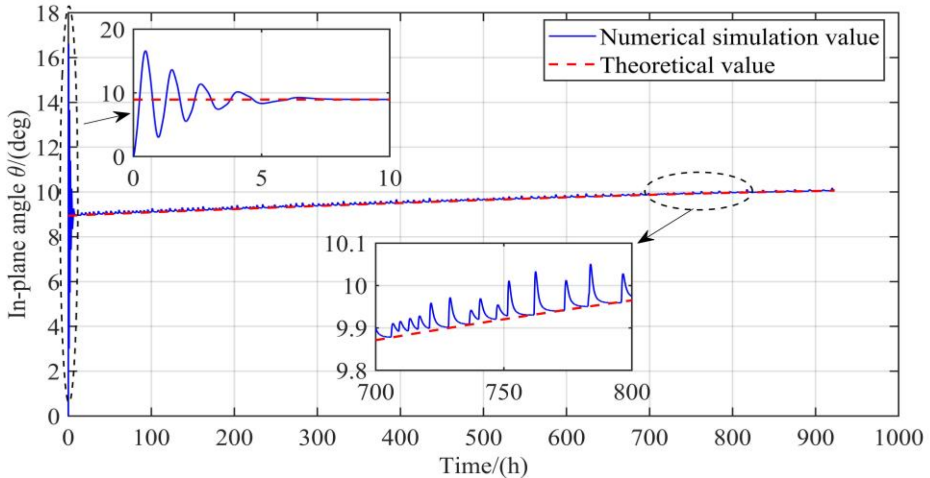
When $t \to \infty, \varepsilon \to 0,$ thus $e \to 0.$

Therefore, the sliding mode controller enables the system to achieve desired state and guarantees the global asymptotic stability of the closed-loop control system.

3 Stable control strategy

- Numerical results



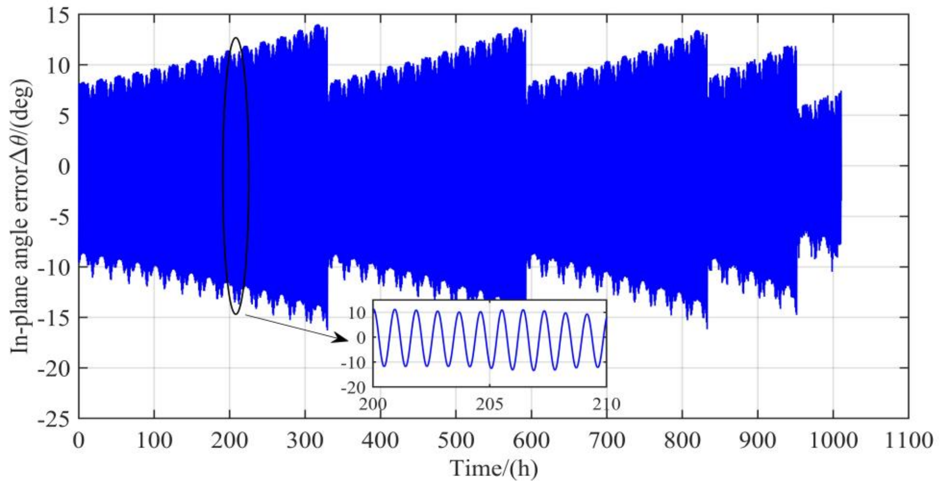


Parameters	Value
Mass of the main satellite	5000kg
Mass of the sub satellite	30kg
Orbital Inclination	10deg
Dimensionless current	0.3(1.4A)
Length of tether	5km
Initial in-plane angle	0deg

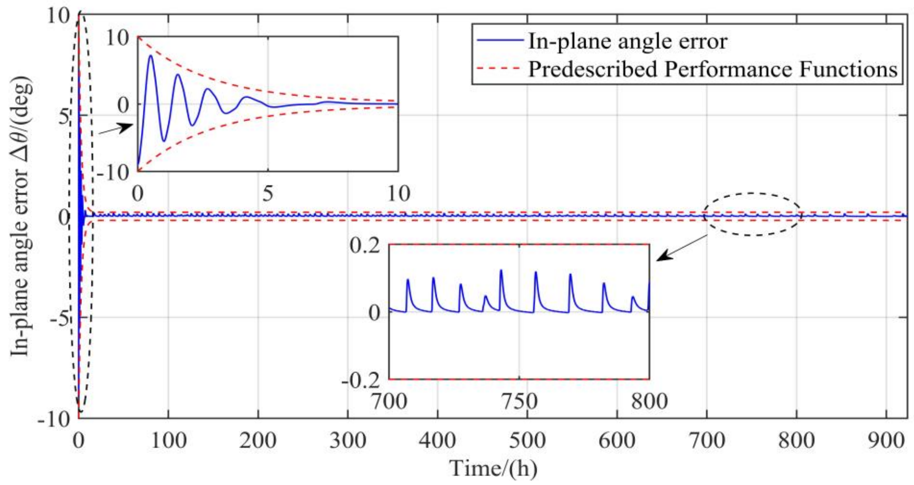
With the controller, the system took 6h to reach the equilibrium position, after the system reaches stable state, the libration of the in-plane angle of the system is very small.

3 Stable control strategy

- Numerical results

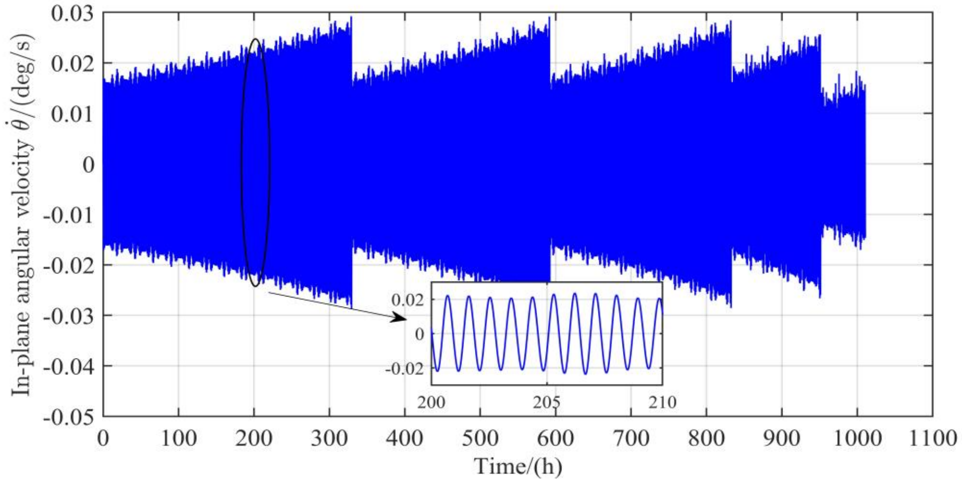


The in-plane angle error of the system under the current switching control strategy, and the error is bounded within ±16 deg.

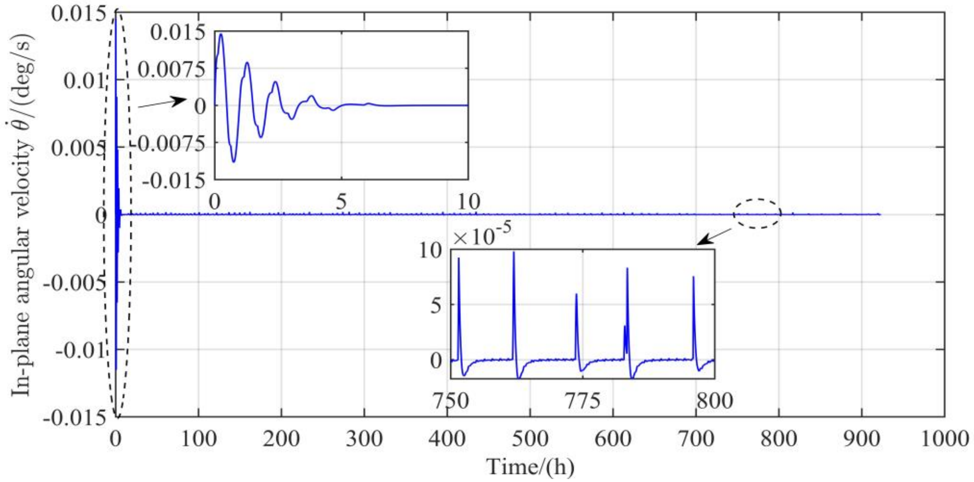


The maximum error of the system is 7.8 deg and the in-plane angle error is less than 0.2 deg after reaching the stable state

- Numerical results



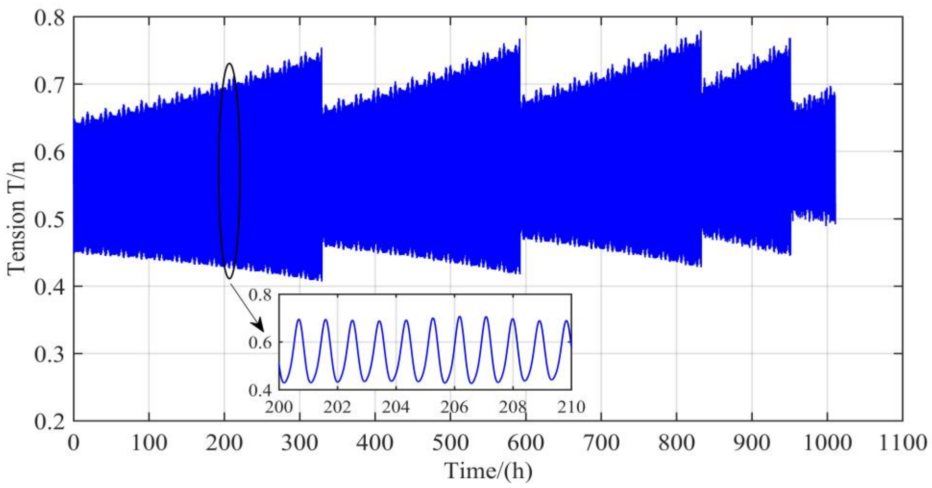
The in-plane angular velocity of the tether system is constrained to be within -0.03~0.03 deg/s



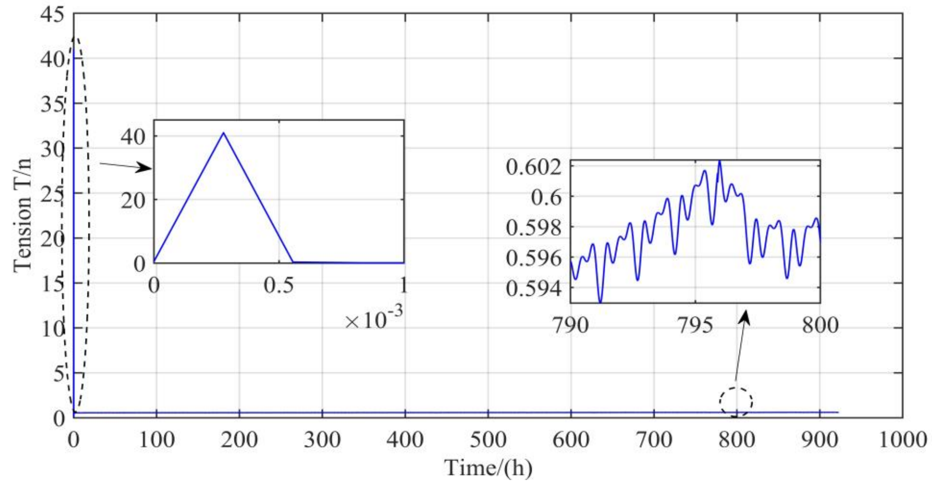
The maximum angular velocity is about 0.015 deg/s, which quickly converges to 0 under the effect of the controller

3 Stable control strategy

- Numerical results



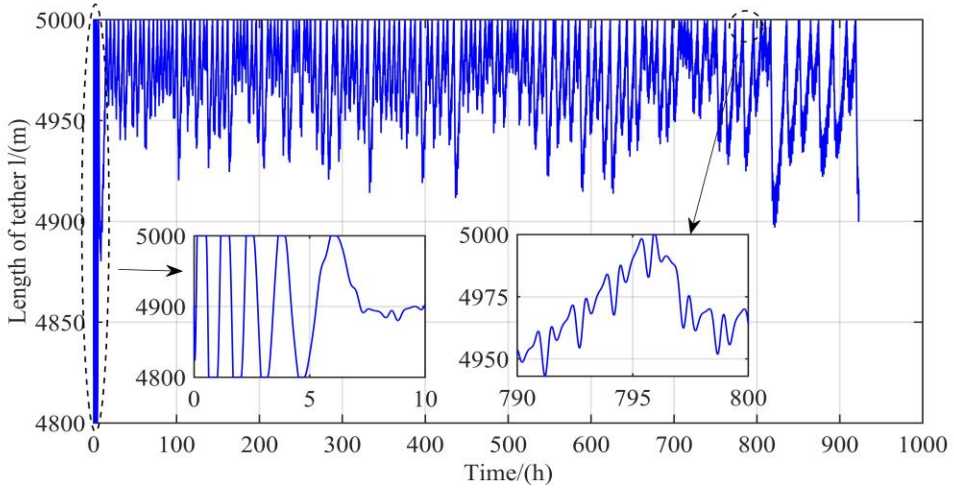
The tension on the tether often shows a step change, due to the fact that the switching on and off of the current introduces undesired transient responses to the system.

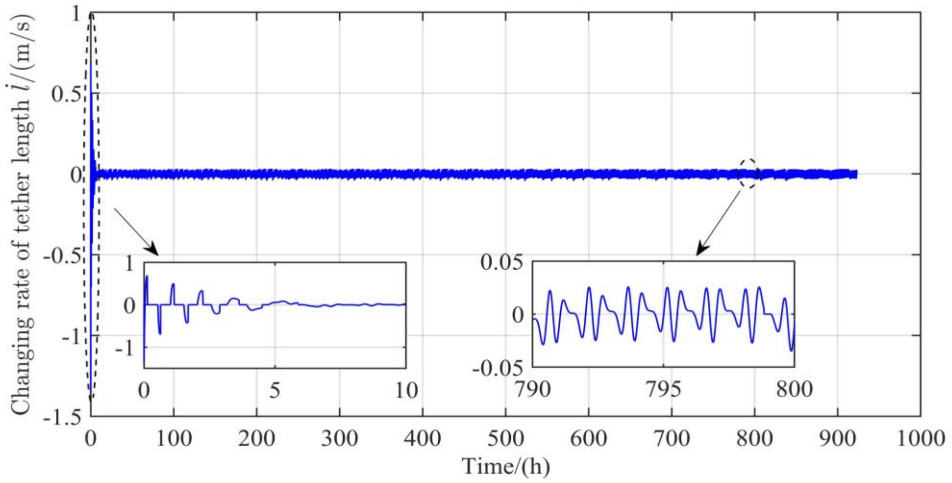


The tether tension stabilized at approximately 0.6 N.

3 Stable control strategy

- Numerical results

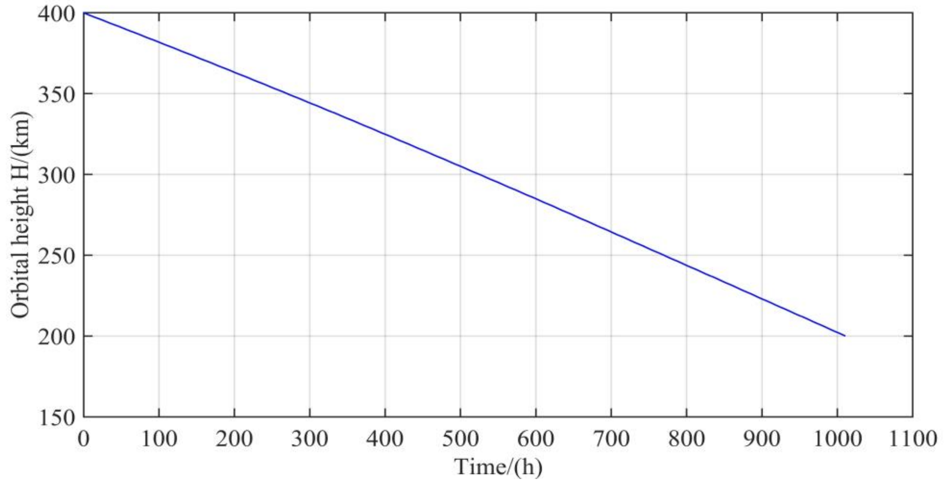


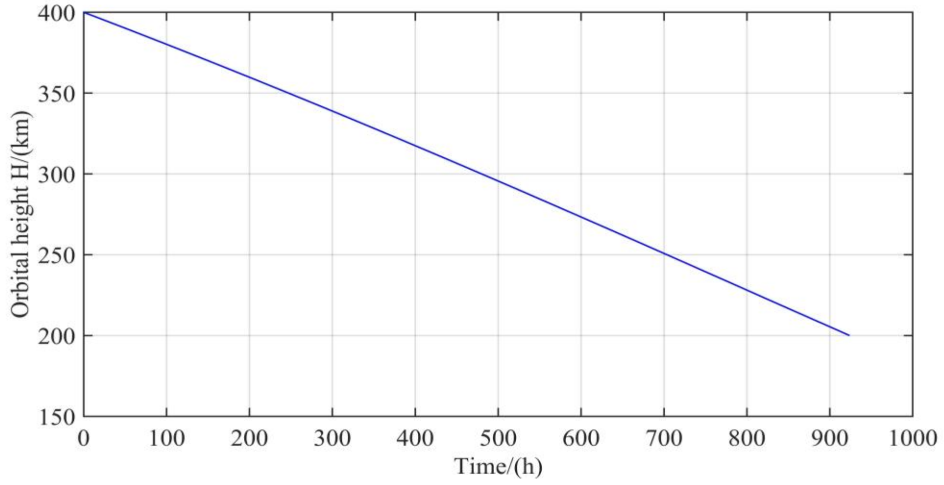


In the initial 5h, the tether's length in the upper and lower limits of the continuous change under the effect of the controller, and when the system reaches stable state, the length of the tether varies between 4870m and 5000m.

3 Stable control strategy

- Numerical results

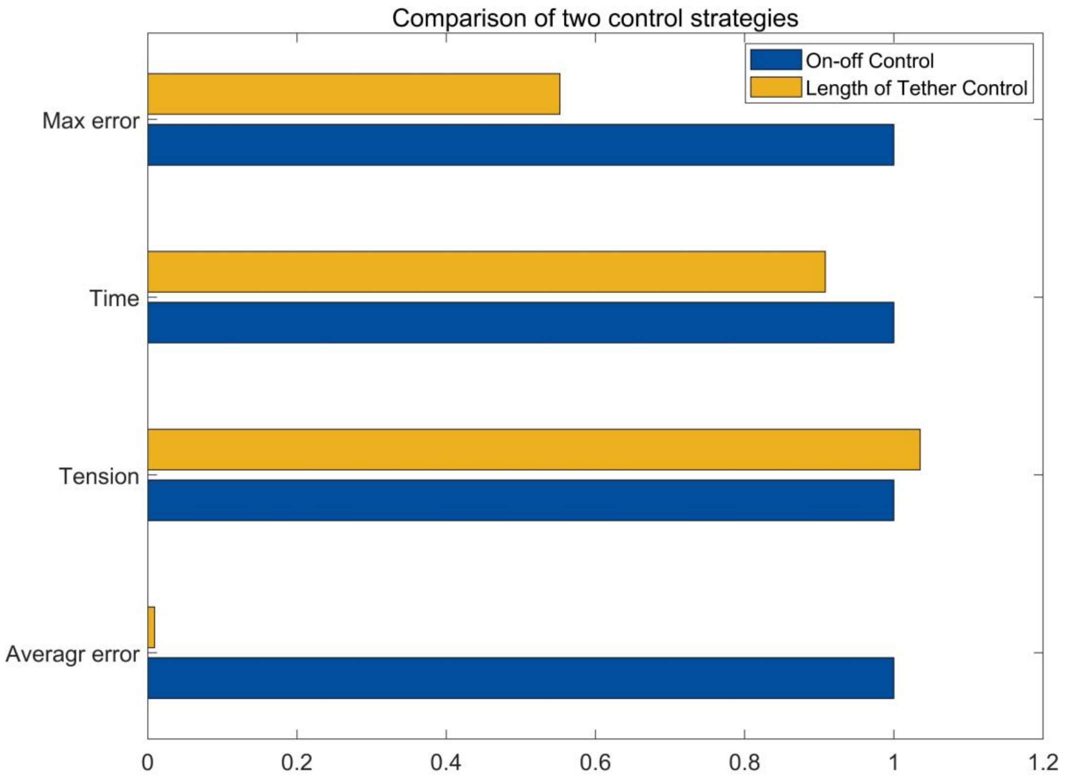




Reduces the deorbiting time of the system by nearly 100h compared with the energy control strategy. It enhances the deorbiting efficiency of the system.

3 Stable control strategy

- Numerical results



A comparison of the simulation results reveals that the stabilization control strategy proposed in this paper is more effective than the traditional current switching control strategy in terms of control accuracy and deorbit efficiency.

4 Conclusions

- A new method of calculating the equilibrium position of the system is proposed based on the substitution of integral variables. The method facilitates the design of subsequent controllers by allowing the system to be calculated in real time throughout the deorbit process

- Comparison with the switching control strategy proves the effectiveness and superiority of control strategy based on adjusting the length of tether, which can stabilize the in-plane angle of the system near the equilibrium position, with a libration amplitude of less than 0.2deg, and at the same time improve the deorbit efficiency of the system to a large extent.