## Partner

# TETHERED ARTIFICIAL GRAVITY ASSISTS FOR CAPTURE ABOUT BINARY ASTEROIDS IN THE CIRCULAR RESTRICTED THREE-BODY PROBLEM 

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Credit: NASA/Johns Hopkins APL [1]

## 5 Conclusion \& Future Work

## INTRODUCTION

Tethered artificial gravity assist: a proposed alternative to traditional methods of propulsion that changes the orbital path of a spacecraft using a tether

Objective: Develop an approach to optimize tethered artificial gravity assist maneuvers in binary asteroid systems using the circular restricted three-body problem (CR3BP)


Credit: Arecibo/GBO/NSF/NASA/JPL-Caltech [2]

## PROBLEM FORMULATION

Ratio of mass between secondary asteroid and total system [3]:

$$
\mu=\frac{m_{2}}{m_{1}+m_{2}}
$$

Normalized position of asteroids in synodic frame [3]:


$$
\begin{aligned}
& \text { Additional Parameters: } \\
& m_{1}=\text { real mass of primary } \\
& m_{2}=\text { real mass of secondary } \\
& R_{1}=\text { real radius of primary } \\
& R_{2}=\text { real radius of secondary } \\
& a_{s}=\text { real semi-major axis of secondary }
\end{aligned}
$$

$$
\vec{r}_{1}=\left[\begin{array}{c}
-\mu \\
0
\end{array}\right], \quad \vec{r}_{2}=\left[\begin{array}{c}
1-\mu \\
0
\end{array}\right]
$$

Normalized radii of asteroids:

$$
R_{p}=\frac{R_{1}(1-\mu)}{a_{s}}, \quad R_{s}=\frac{R_{2}(1-\mu)}{a_{s}}
$$



## TETHER DYNAMICS



Tether Attachment [4]:

$$
\begin{gathered}
\overrightarrow{\mathbf{r}}_{i}=\left[\begin{array}{l}
r_{x_{i}} \\
r_{y_{i}}
\end{array}\right]=\left[\begin{array}{c}
l \cos (\psi+\delta)+R_{s} \cos \psi+(1-\mu) \\
l \sin (\psi+\delta)+R_{s} \sin \psi
\end{array}\right] \\
\vec{v}_{i}=\left[\begin{array}{l}
v_{x_{i}} \\
v_{y_{i}}
\end{array}\right]=\left[\begin{array}{c}
v_{\infty} \sin (\psi+\beta) \\
-v_{\infty} \cos (\psi+\beta)
\end{array}\right]
\end{gathered}
$$

Tether Detachment [4]:

$$
\begin{gathered}
\overrightarrow{\mathbf{r}}_{i}=\left[\begin{array}{l}
r_{x_{i}} \\
r_{y_{i}}
\end{array}\right]=\left[\begin{array}{c}
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l \sin (\psi-\delta)+R_{S} \sin \psi
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\vec{v}_{i}=\left[\begin{array}{l}
v_{x_{i}} \\
v_{y_{i}}
\end{array}\right]=\left[\begin{array}{c}
v_{\infty} \sin (\psi-\beta) \\
-v_{\infty} \cos (\psi-\beta)
\end{array}\right]
\end{gathered}
$$

## CR3BP DYNAMICS



Jacobi Constant: The only conserved value in CR3BP dynamics

Jacobi Constant [3]:

$$
C_{j}=(1-\mu) r_{p}^{2}+\mu r_{s}^{2}+\frac{2(1-\mu)}{r_{p}}+\frac{\mu}{r_{s}}-v_{\infty}^{2}
$$

Distance between spacecraft and each asteroid [3]:

$$
\begin{gathered}
r_{p}^{2}=\left(r_{x}+\mu\right)^{2}+r_{y}^{2} \\
r_{p}^{2}=\left(r_{x}-1+\mu\right)^{2}+r_{y}^{2}
\end{gathered}
$$

## OPTIMIZATION

Genetic algorithm (GA): evolutionary algorithm for complex optimization problems with high modality

- Select a planar periodic capture orbit as a desired final orbit to insert the spacecraft into (described by $\overrightarrow{\mathbf{r}}_{d}$ and $\overrightarrow{\mathbf{v}}_{d}$ )
- Design variables: $\psi, \delta, l, \overrightarrow{\mathbf{r}}_{d}$ and $\overrightarrow{\mathbf{v}}_{d}$

Objective Function:

$$
J=-\omega_{1}\left|C_{j_{d}}-C_{j_{i}}\right|+\omega_{2}\left\|\overrightarrow{\mathbf{r}}_{d}-\overrightarrow{\mathbf{r}}_{f}\right\|+\omega_{3}\left\|\overrightarrow{\mathbf{v}}_{d}-\overrightarrow{\mathbf{v}}_{f}\right\|
$$



$$
\left\|\vec{r}_{i_{t r a j}}-\vec{r}_{2}\right\|-R_{p}>0
$$

## SIMULATION PARAMETERS

Criteria for selecting systems:

1. Enough data known to accurately simulate systems
2. Low orbital eccentricity observed in secondary
3. Categorize systems into different mass ratio ranges (i.e., small $\mu$, medium $\mu$, and large $\mu$ )
4. Largest distance between surface of primary to $L_{1}$ stability point

| Observed Characteristics [5-9] |  |  |  |
| :---: | :--- | :--- | :--- |
| Parameter | 2002 CE26 | Dionysus \& S/1997 | 2000 DP107 \& S/2000 |
| $\mu$ | 0.0800 | 0.1667 | 0.2908 |
| $m_{T}$ | $1.95 \times 10^{13} \mathrm{~kg}$ | $2.48 \times 10^{12} \mathrm{~kg}$ | $4.6 \times 10^{11} \mathrm{~kg}$ |
| $d_{P}$ | 3.46 km | 1.43 km | 0.8 km |
| $e_{S}$ | 0.00 | 0.07 | 0.01 |
| $a_{s}$ | 4.7 km | 3.4 km | 2.62 km |
| $d_{s}$ | 0.3 km | 0.29 km | 0.3 km |

## SIMULATION SCENARIOS






A S T R D A B


RESULTS

| Optimized Tethered Maneuvers |  |  |  |
| :---: | :---: | :---: | :---: |
| Design Variables | 2002 CE26 | Dionysus \& S/1997 | $2000 \mathrm{DP} 107 \& \mathrm{~S} / 2000$ |
| $\psi$ | $210.12^{\circ}$ | $206.11^{\circ}$ | $206.64^{\circ}$ |
| $\delta$ | $31.73^{\circ}$ | $26.62^{\circ}$ | $25.51^{\circ}$ |
| $l$ | $2.67 \times 10^{3} \mathrm{~m}$ | $2.49 \times 10^{3} \mathrm{~m}$ | $2.46 \times 10^{3} \mathrm{~m}$ |





## RESULTS



- Minimal difference found between desired orbit and final orbit in 2002 CE26 and Dionysus \& S/1997
- Larger distance between position vectors for 2000 DP107 \& S/2000 caused a more circular final orbit shape

| Difference between desired orbit and final orbit |  |  |  |
| :---: | :--- | :--- | :--- |
| Parameter | 2002 CE26 | Dionysus \& S/1997 | 2000 DP107 \& S/2000 |
| $\left\\|\overrightarrow{\mathbf{r}}_{f}-\overrightarrow{\mathbf{r}}_{d}\right\\|$ | 0.1275 m | 5.9600 m | 259.2802 m |
| $\left\\|\overrightarrow{\mathbf{v}}_{f}-\overrightarrow{\mathbf{v}}_{d}\right\\|$ | $0.0120 \mathrm{~m} / \mathrm{s}$ | $0.0370 \mathrm{~m} / \mathrm{s}$ | $0.0191 \mathrm{~m} / \mathrm{s}$ |
| $\left\|C_{j_{f}}-C_{j_{d}}\right\|$ | 0.0022 | 0.0144 | 0.6775 |

## DISCUSSION



- GA successfully optimized tethered artificial gravity assists in each binary system
- Initial trajectory orbited in same direction as final desired orbit and rotation of system
- Tether at detachment point was optimized to be nearly aligned with $x$-axis
- Positive trend observed between $\mu$ and $\Delta C_{j}$

| Change in Jacobi Constant |  |  |  |
| :---: | :---: | :---: | :---: |
| Parameter | 2002 CE26 | Dionysus and S/1997 | 2000 DP107 and S/2000 |
| $\mu$ | 0.0800 | 0.1667 | 0.2908 |
| $C_{j_{i}}$ | 2.0166 | 1.6960 | 1.8426 |
| $C_{j_{f}}$ | 3.4581 | 3.8112 | 4.3924 |
| $\Delta C_{j}$ | 1.4415 | 2.1152 | 2.5498 |

## CONCLUSION \& FUTURE WORK

- Successfully optimized tethered artificial gravity assist maneuvers in three binary asteroid systems using CR3BP dynamics
- Future work:
- Incorporate external perturbations into model
- Utilize varying-length tether in maneuver


Credit: NASA/Goddard/SwRI/ASU [10]
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## Any Questions?

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## Disclaimer Statement:

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## A S T R D A B

## BACKUP SLIDES

## TETHER DYNAMICS

Tether Attachment [4]:

$$
\begin{gathered}
\overrightarrow{\mathbf{r}}_{i}=\left[\begin{array}{l}
r_{x_{x}} \\
r_{y_{i}}
\end{array}\right]=\left[\begin{array}{c}
l \cos (\psi+\delta)+R_{s} \cos \psi+(1-\mu) \\
l \sin (\psi+\delta)+R_{s} \sin \psi
\end{array}\right] \\
\vec{v}_{i}=\left[\begin{array}{l}
v_{x_{i}} \\
v_{y_{i}}
\end{array}\right]=\left[\begin{array}{c}
v_{\infty} \sin (\psi+\beta) \\
-v_{\infty} \cos (\psi+\beta)
\end{array}\right]
\end{gathered}
$$

Tether Detachment [4]:

$$
\begin{gathered}
\overrightarrow{\mathbf{r}}_{i}=\left[\begin{array}{l}
r_{x_{i}} \\
r_{y_{i}}
\end{array}\right]=\left[\begin{array}{c}
l \cos (\psi-\delta)+R_{s} \cos \psi+(1-\mu) \\
l \sin (\psi-\delta)+R_{s} \sin \psi
\end{array}\right] \\
\vec{v}_{i}=\left[\begin{array}{l}
v_{x_{i}} \\
v_{y_{i}}
\end{array}\right]=\left[\begin{array}{c}
v_{\infty} \sin (\psi-\beta) \\
-v_{\infty} \cos (\psi-\beta)
\end{array}\right]
\end{gathered}
$$

## 2002 CE26



| Design Variables | Values |  |
| :---: | :---: | :---: |
| $\psi$ | $210.12^{\circ}$ |  |
| $\delta$ | $31.73{ }^{\circ}$ |  |
|  | Dimensionless | Dimensional |
| $l$ | 0.5221 | $2.67 \times 10^{3} \mathrm{~m}$ |
| $\overrightarrow{\mathbf{r}}_{d}$ | $\left[0.3730,-3.98 \times 10^{-5}\right]^{T}$ | $\left[1.91 \times 10^{\wedge} 3,-0.2033\right]^{\mathrm{T}} \mathrm{m}$ |
| $\overrightarrow{\mathbf{v}}_{d}$ | $[-0.0211,1.0540]^{\mathrm{T}}$ | $[-0.0121,0.6020]^{\mathrm{T}} \mathrm{m} / \mathrm{s}$ |
| Output Parameters | Values |  |
| $C_{j_{i}}$ | 2.0166 |  |
| $C_{j_{f}}$ | 3.4581 |  |
| $\Delta C_{j}$ | 1.4415 |  |
|  | Dimensionless | Dimensional |
| $\overrightarrow{\mathbf{r}}_{i}$ | [0.6482, -0.4751] ${ }^{\text {T }}$ | $\left[3.31 \times 10^{3},-2.43 \times 10^{3}\right]^{\mathrm{T}} \mathrm{m}$ |
| $\overrightarrow{\mathbf{v}}_{i}$ | $[-0.9151,0.5234]^{\mathrm{T}}$ | $[-0.5227,0.2990]^{\mathrm{T}} \mathrm{m} / \mathrm{s}$ |
| $\overrightarrow{\mathbf{r}}_{f}$ | $\left[0.3726,-6.48 \times 10^{-5}\right]^{\mathrm{T}}$ | $\left[1.91 \times 10^{3},-0.3308\right]^{\mathrm{T}} \mathrm{m}$ |
| $\overrightarrow{\mathbf{v}}_{f}$ | $\left[-1.25 \times 10^{-4}, 1.0542\right]^{\mathrm{T}}$ | $\left[-7.12 \times 10^{-5}, 0.6021\right]^{\mathrm{T}} \mathrm{m} / \mathrm{s}$ |

Optimized tethered maneuver in 2002 CE26

## DIONYSUS \& S/1997



| Optimized tethered maneuver in 2002 CE26 |  |  |
| :---: | :---: | :---: |
| Design Variables | Values |  |
| $\psi$ | $206.11^{\circ}$ |  |
| $\delta$ | $26.62^{\circ}$ |  |
|  | Dimensionless | Dimensional |
| $l$ | 0.6099 | $2.49 \times 10^{3} \mathrm{~m}$ |
| $\overrightarrow{\mathbf{r}}_{d}$ | $\left[0.1927,-9.36 \times 10^{-3}\right]^{\mathrm{T}}$ | $[786.33,-38.20]^{\mathrm{T}} \mathrm{m}$ |
| $\overrightarrow{\mathbf{v}}_{d}$ | $[-0.1635,1.2280]^{\mathrm{T}}$ | $[-0.0420,0.3152]^{\mathrm{T}} \mathrm{m} / \mathrm{s}$ |
| Output Parameters | Values |  |
| $C_{j}$ | 1.6960 |  |
| $C_{j_{f}}$ | 3.8112 |  |
| $\Delta C_{j}$ | 2.1152 |  |
|  | Dimensionless | Dimensional |
| $\overrightarrow{\mathbf{r}}_{i}$ | [0.4320, -0.5010] ${ }^{\text {T }}$ | $\left[1.76 \times 10^{3},-2.04 \times 10^{3}\right]^{\mathrm{T}} \mathrm{m}$ |
| $\overrightarrow{\mathbf{v}}_{i}$ | $[-0.9669,0.7745]^{\mathrm{T}}$ | $[-0.2482,0.1988]^{\mathrm{T}} \mathrm{m} / \mathrm{s}$ |
| $\overrightarrow{\mathbf{r}}_{f}$ | $[0.1915,-0.0102]^{\mathrm{T}}$ | $[781.47,-41.65]^{\mathrm{T}} \mathrm{m}$ |
| $\overrightarrow{\mathbf{v}}_{f}$ | $[-0.0197,1.2387]^{\mathrm{T}}$ | $\left[-5.06 \times 10^{-3}, 0.3180\right]^{\mathrm{T}} \mathrm{m} / \mathrm{s}$ |

## 2000 D107 \& S/2000



Optimized tethered maneuver in 2002 CE26

| Design Variables | Values |  |
| :---: | :---: | :---: |
| $\psi$ | $206.64{ }^{\circ}$ |  |
| $\delta$ | $25.51{ }^{\circ}$ |  |
|  | Dimensionless | Dimensional |
| $l$ | 0.6662 | $2.46 \times 10^{3} \mathrm{~m}$ |
| $\overrightarrow{\mathbf{r}}_{d}$ | $\left[0.0671,4.74 \times 10^{-3}\right]^{T}$ | $[247.70,17.50]^{\mathrm{T}} \mathrm{m}$ |
| $\overrightarrow{\mathbf{v}}_{d}$ | $[-0.1765,1.1621]^{\mathrm{T}}$ | $[-0.0270,0.1776]^{\mathrm{T}} \mathrm{m} / \mathrm{s}$ |
| Output | Values |  |
| $C_{j_{i}}$ | 1.8426 |  |
| $C_{j_{f}}$ | 4.3924 |  |
| $\Delta C_{j}$ | 2.5498 |  |
|  | Dimensionless | Dimensional |
| $\overrightarrow{\mathbf{r}}_{i}$ | [0.2641, -0.5442] ${ }^{\text {T }}$ | $\left[975.61,-2.01 \times 10^{3}\right]^{\mathrm{T}} \mathrm{m}$ |
| $\overrightarrow{\mathbf{v}}_{i}$ | $[-0.9099,0.7442]^{\mathrm{T}}$ | $[-0.1390,0.1137]^{\mathrm{T}} \mathrm{m} / \mathrm{s}$ |
| $\overrightarrow{\mathbf{r}}_{f}$ | $\left[6.84 \times 10^{-3},-0.0313\right]^{\mathrm{T}}$ | $[25.28,-115.75]^{\mathrm{T}} \mathrm{m}$ |
| $\overrightarrow{\mathbf{v}}_{f}$ | $[-0.0524,1.1743]^{\mathrm{T}}$ | $\left[-8.00 \times 10^{-3}, 0.1794\right]^{\mathrm{T}} \mathrm{m} / \mathrm{s}$ |

