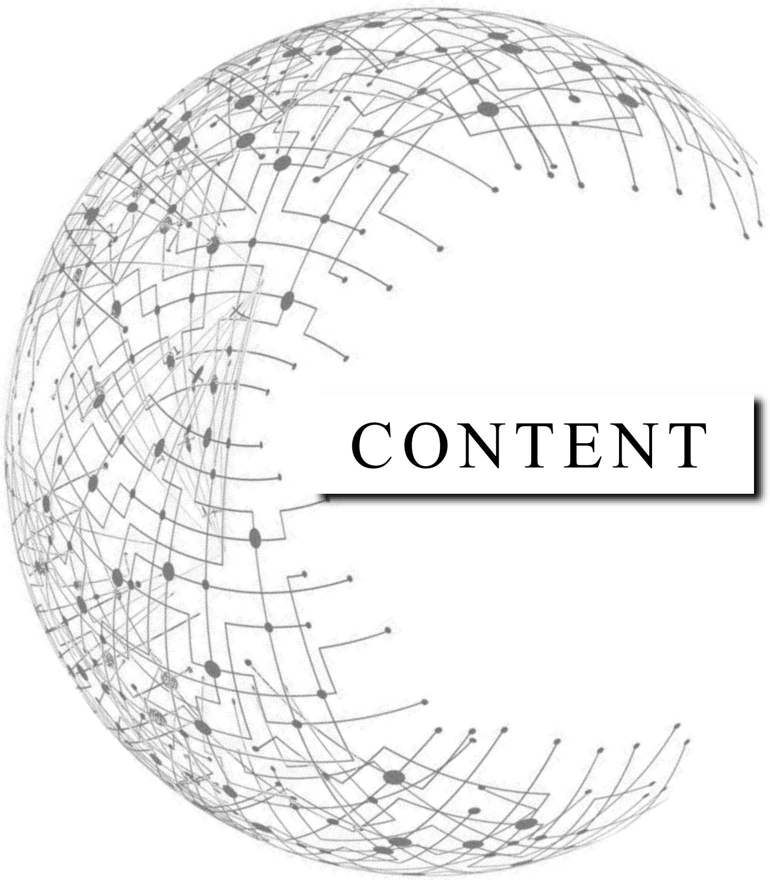
Adaptive fault-tolerant control of spinning tether system for space debris removal

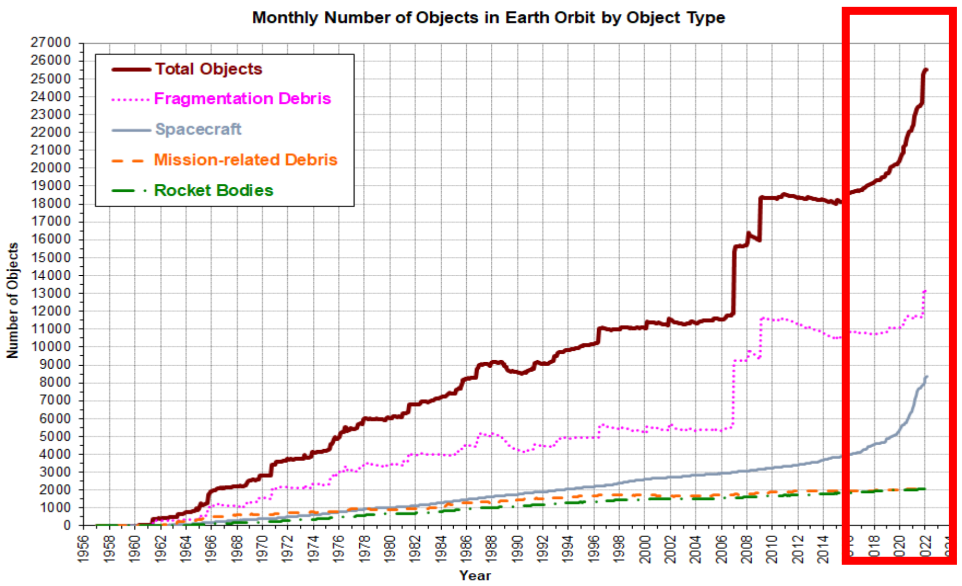
Authors: PeiJie Sun, HongShi Lu, ChangQing Wang, AiJun Li, Yuriy Zabolotnov



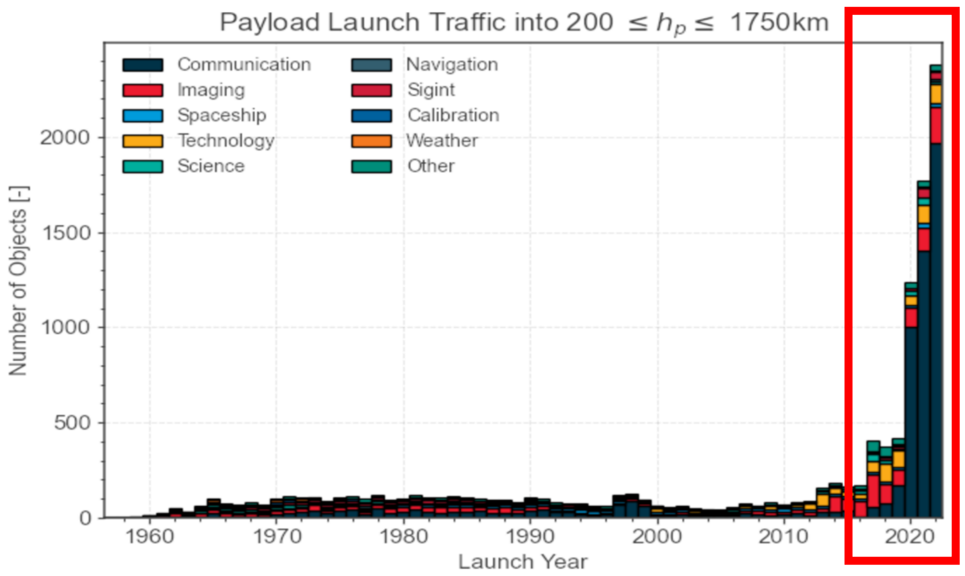
CONTENT

PART 1 Introduction

1 Introduction



ESA



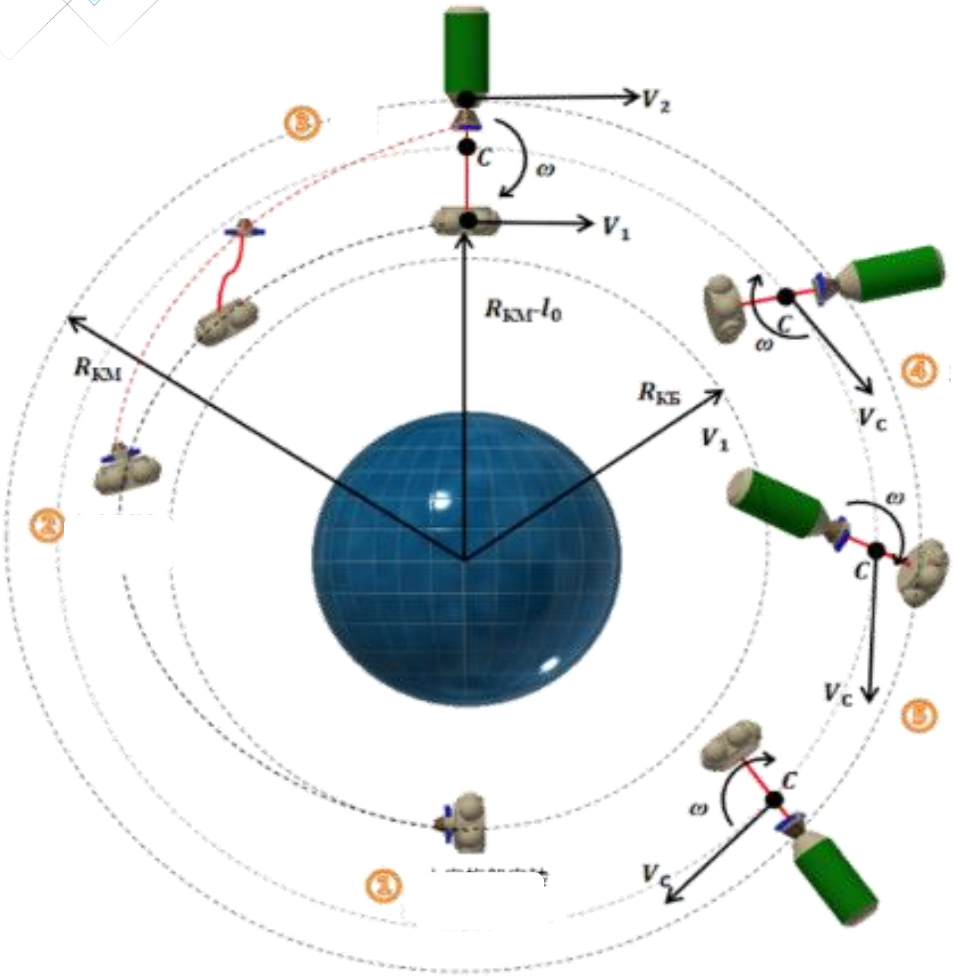
Changes in the quantity of space debris

Trends of Payload Launch Traffic in Low Earth orbit

Table1 ESA statistics related to space debris

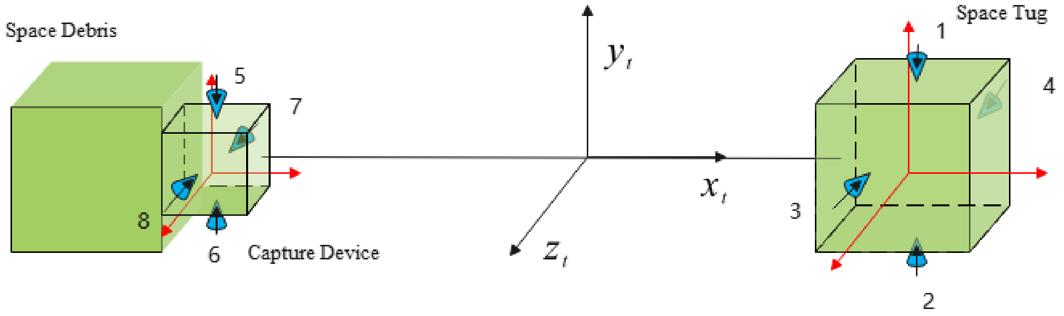
Rocket launches	Deployment of satellites	Satellites in orbit	Functional satellites	Number of debris	Total mass in orbit	Anomalous events resulting in fragmentation
6250	13630	8840	6200	31460	10000	630

1 Introduction



The study of scientific issues such as the dynamics and control of the space debris de-orbit removal process of space spin tether systems can provide important theoretical support for space debris mitigation and is of great research significance.

By selecting appropriate system parameters, such as system configuration, tether length, system mass and spinning angular velocity, researchers can use the system to provide a wide range of velocity variables to the payload to satisfy a variety of payload transfer tasks.

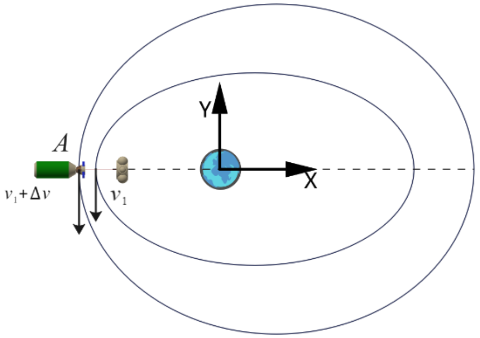


Because the components of the on-orbit STS is not repairable, in order to increase the STS's autonomy of the operation, it is required that STS can effectively deal with component failures without any maintenance to be able to deal with component failures efficiently.

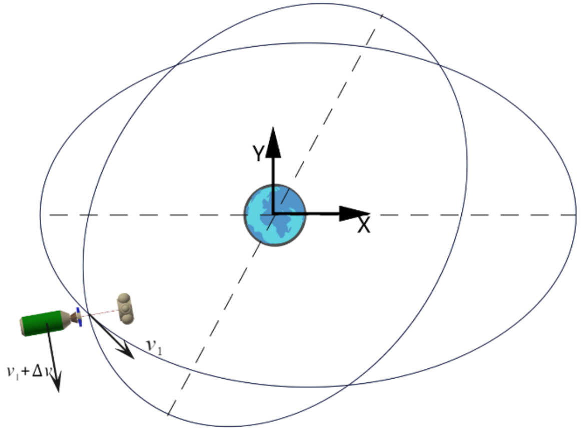
PART 2 Research Contents

2 Research Contents

Mechanistic analysis of space debris removal through the spinning tether system

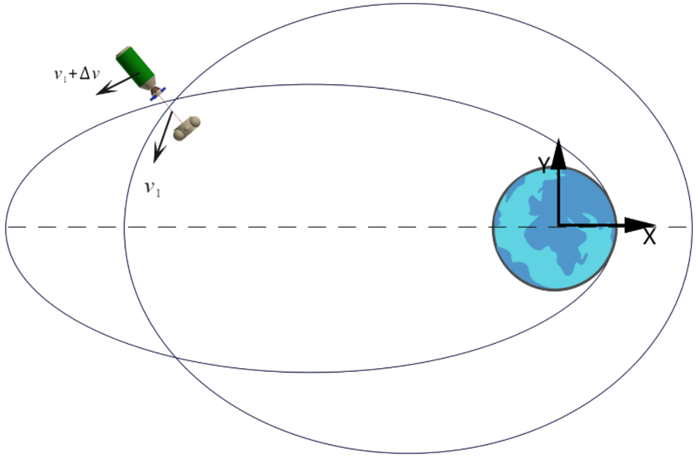


Debris removal at perigee with common apse line

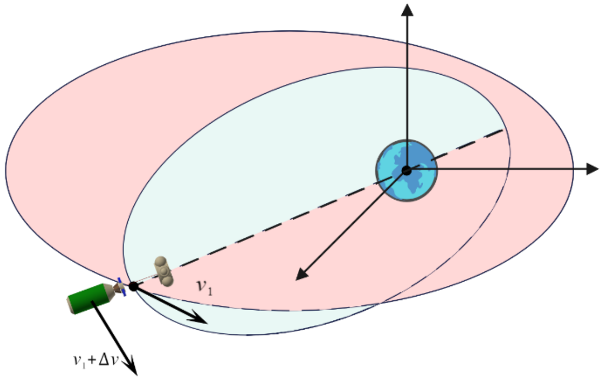


Debris removal without common apse line

At the instant of space debris ejection, its orbital velocity is synthesized by the system's orbital motion velocity and the linear velocity relative to the center of mass motion, which is equivalent to generating an instantaneous velocity impulse to complete the orbital maneuver relative to the orbital velocity of the previous free on-orbit operation, and is capable of realizing debris removal.

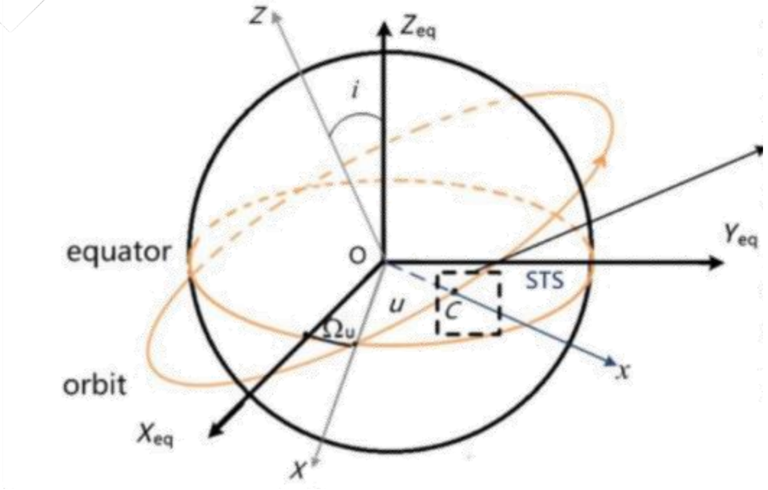


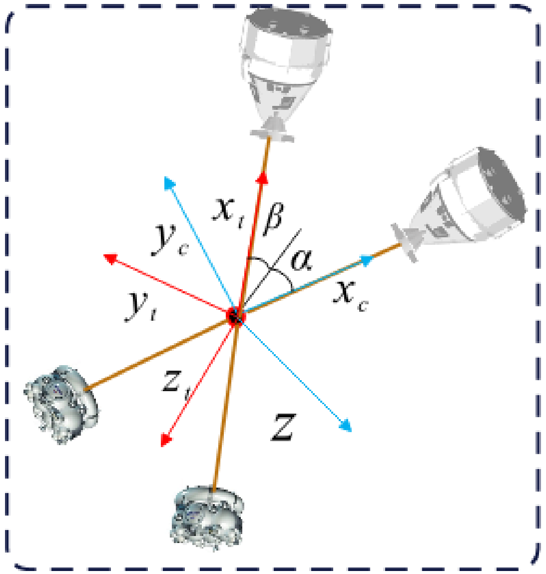
Debris removal with common apse line

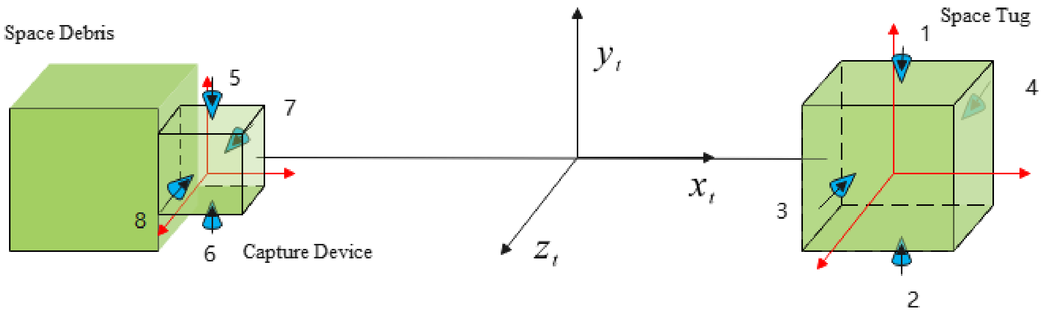


Noncoplanar debris removal

Dynamic model







$$\ddot{r}_c = \frac{3\mu m_* l^2 (1 - 3\cos^2(\alpha)\cos^2(\beta))}{2 m {r_c}^4} + r_c \dot{\delta}^2 - \frac{\mu}{{r_c}^2} + \frac{Q_{rc}}{m}$$

$$\ddot{\delta} = \frac{3\mu m_* l^2 \sin(\alpha)\cos(\alpha)\cos^2(\beta)}{m {r_c}^5} - \frac{2\dot{\delta}\dot{r}_c}{r_c} + \frac{Q_\delta}{m r_c^2}$$

$$\ddot{\alpha} = 2\dot{\beta}\tan(\beta)\dot{\alpha} - \frac{3\mu m_* l^2 \sin(\alpha)\cos(\alpha)\cos^2(\beta)}{m {r_c}^5} - \frac{3\mu \sin(\alpha)\cos(\alpha)}{{r_c}^3} + \frac{2\dot{\delta} r_c}{r_c} + \frac{Q_\alpha}{m r_c^2}$$

$$\ddot{\beta} = -\frac{\sin(2\beta)\left({r_c}^3\left(\dot{\alpha} + \dot{\delta}\right)^2 + 3\mu\cos^2(\alpha)\right)}{2{r_c}^3} + \frac{Q_\beta}{m r_c^2}$$

$$Q_{ai} = \sum_{i=1}^{N} F_i \frac{\partial r_i}{\partial q_a}$$

$$Q_i = \rho_i Q_{ei} + \bar{Q}_{ei}$$

2 Research Contents

Generalized coordinate transformation

$$\ddot{r}_c=\frac{3\mu m_* l^2\left(1-3\cos^2(\alpha)\cos^2(\beta)\right)}{2mr_c^{\ 4}}+r_c\dot{\delta}^2-\frac{\mu}{r_c^{\ 2}}+\frac{Q_{rc}}{m}$$

$$\ddot{\delta}=\frac{3\mu m_* l^2\sin(\alpha)\cos(\alpha)\cos^2(\beta)}{mr_c^{\ 5}}-\frac{2\dot{\delta}\dot{r}_c}{r_c}+\frac{Q_\delta}{mr_c^2}$$

$$\ddot{\alpha}=2\dot{\beta}\tan(\beta)\dot{\alpha}-\frac{3\mu m_* l^2\sin(\alpha)\cos(\alpha)\cos^2(\beta)}{mr_c^{\ 5}}$$

$$-\frac{3\mu\sin(\alpha)\cos(\alpha)}{r_c^{\ 3}}+\frac{2\dot{\delta}r_c}{r_c}+\frac{Q_\alpha}{mr_c^2}$$

$$\ddot{\beta}=-\frac{\sin(2\beta)\left(r_c^{\ 3}\left(\dot{\alpha}+\dot{\delta}\right)^2+3\mu\cos^2(\alpha)\right)}{2r_c^{\ 3}}+\frac{Q_\beta}{mr_c^2}$$

$$\ddot{r}_c=\frac{3l^2\mu\mathrm{m}_*\left(1-3(2q_1^{\ 2}+2q_2^{\ 2}-1)^2(2q_1^{\ 2}+2q_3^{\ 2}-1)^2\right)}{2mr_c^{\ 4}}-\frac{\mu}{r_c^{\ 2}}+r_c\dot{\delta}^2$$

$$\ddot{\delta}=\frac{3l^2\mu\mathrm{mx}(2q_1^{\ 2}+2q_2^{\ 2}-1)^2(-2q_1^{\ 2}-2q_3^{\ 2}+1)\sqrt{1-(2q_1^{\ 2}+2q_3^{\ 2}-1)^2}}{mr_c^{\ 5}}-\frac{2\dot{\delta}\dot{r}_c}{r_c}$$

$$\begin{aligned}\ddot{q}_1=&-\left(q_1^{\ 3}\left(\dot{q}_1^{\ 2}+\dot{q}_2^{\ 2}+\dot{q}_3^{\ 2}+\dot{q}_4^{\ 2}\right)\right)+2q_1^{\ 2}\dot{q}_1\left(q_2\dot{q}_2+q_3\dot{q}_3+q_4\dot{q}_4\right)\\&-q_1\left(q_2^{\ 2}\left(3\dot{q}_1^{\ 2}-\dot{q}_2^{\ 2}+\dot{q}_3^{\ 2}+\dot{q}_4^{\ 2}\right)+q_4^{\ 2}\left(3\dot{q}_1^{\ 2}+\dot{q}_2^{\ 2}+\dot{q}_3^{\ 2}-\dot{q}_4^{\ 2}\right)+q_3^{\ 2}\left(3\dot{q}_1^{\ 2}+\dot{q}_2^{\ 2}-\dot{q}_3^{\ 2}+\dot{q}_4^{\ 2}\right)-4q_2\dot{q}_2\left(q_3\dot{q}_3+q_4\dot{q}_4\right)-4q_3q_4\dot{q}_3\dot{q}_4\right)\\&-2\dot{q}_1\left(q_2^{\ 2}+q_3^{\ 2}+q_4^{\ 2}\right)\left(q_2\dot{q}_2+q_3\dot{q}_3+q_4\dot{q}_4\right)+\frac{Q_1}{2m_*l}\end{aligned}$$

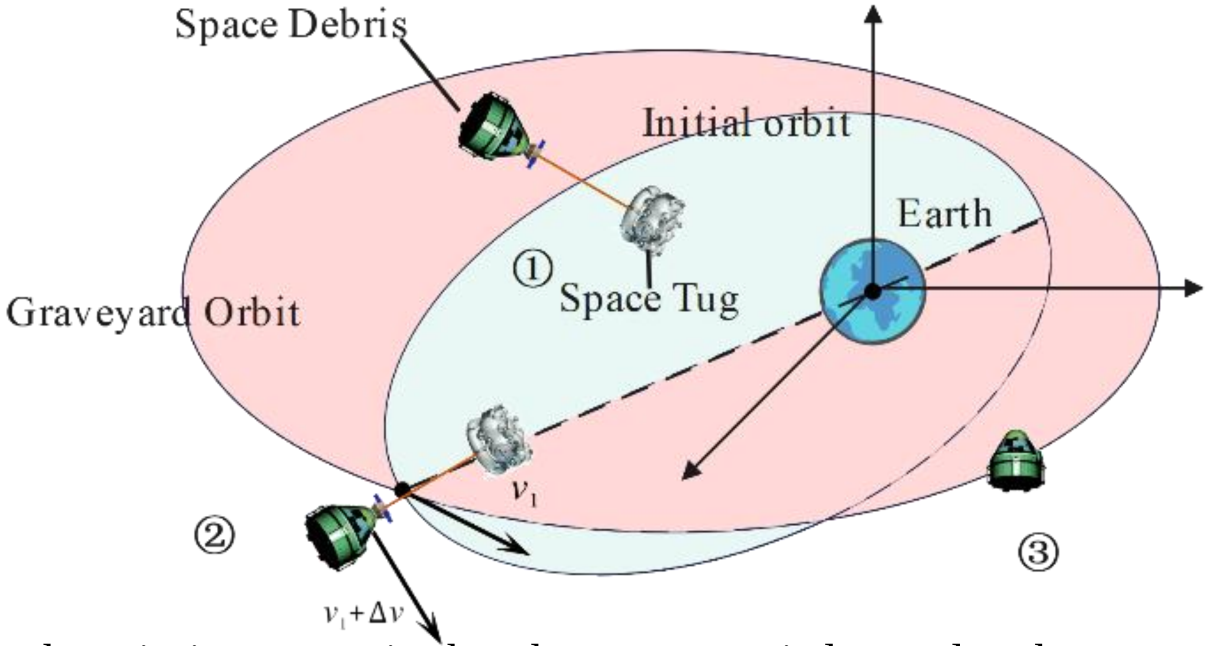
$$\begin{aligned}\dot{q}_2=&q_1^{\ 2}\left(q_2\left(\dot{q}_1^{\ 2}-3\dot{q}_2^{\ 2}-\dot{q}_3^{\ 2}-\dot{q}_4^{\ 2}\right)-2\dot{q}_2\left(q_3\dot{q}_3+q_4\dot{q}_4\right)\right)+2q_1\dot{q}_1\left(-\dot{q}_2\left(q_3^{\ 2}+q_4^{\ 2}\right)+q_2^{\ 2}\dot{q}_2+2q_2\left(q_3\dot{q}_3+q_4\dot{q}_4\right)\right)\\&-q_2^{\ 3}\left(\dot{q}_1^{\ 2}+\dot{q}_2^{\ 2}+\dot{q}_3^{\ 2}+\dot{q}_4^{\ 2}\right)-q_2\left(q_3^{\ 2}\left(\dot{q}_1^{\ 2}+3\dot{q}_2^{\ 2}-\dot{q}_3^{\ 2}+\dot{q}_4^{\ 2}\right)+q_4^{\ 2}\left(\dot{q}_1^{\ 2}+3\dot{q}_2^{\ 2}+\dot{q}_3^{\ 2}-\dot{q}_4^{\ 2}\right)-4q_3q_4\dot{q}_3\dot{q}_4\right)\\&-2q_1^{\ 3}\dot{q}_1\dot{q}_2+2q_2^{\ 2}\dot{q}_2\left(q_3\dot{q}_3+q_4\dot{q}_4\right)-2\dot{q}_2\left(q_3^{\ 2}+q_4^{\ 2}\right)\left(q_3\dot{q}_3+q_4\dot{q}_4\right)+\frac{Q_2}{2m_*l}\end{aligned}$$

$$\begin{aligned}\dot{q}_3=&q_1^{\ 2}\left(q_3\left(\dot{q}_1^{\ 2}-\dot{q}_2^{\ 2}-3\dot{q}_3^{\ 2}-\dot{q}_4^{\ 2}\right)-2\dot{q}_3\left(q_2\dot{q}_2+q_4\dot{q}_4\right)\right)+2q_1\dot{q}_1\left(2q_2q_3\dot{q}_2+q_2^{\ 2}(-\dot{q}_3)+(q_3^{\ 2}-q_4^{\ 2})\dot{q}_3+2q_3q_4\dot{q}_4\right)\\&-q_2^{\ 2}\left(q_3\left(\dot{q}_1^{\ 2}-\dot{q}_2^{\ 2}+3\dot{q}_3^{\ 2}+\dot{q}_4^{\ 2}\right)+2q_4\dot{q}_3\dot{q}_4\right)+q_3\left(-\left(q_3^{\ 2}+q_4^{\ 2}\right)\left(\dot{q}_1^{\ 2}+\dot{q}_2^{\ 2}\right)-\left(\left(q_3^{\ 2}+3q_4^{\ 2}\right)\dot{q}_3^{\ 2}\right)\right)\\&-2q_1^{\ 3}\dot{q}_1\dot{q}_3+2q_2\dot{q}_2\left(\left(q_3^{\ 2}-q_4^{\ 2}\right)\dot{q}_3+2q_3q_4\dot{q}_4\right)-2q_2^{\ 3}\dot{q}_2\dot{q}_3+2q_4\left(q_3^{\ 2}-q_4^{\ 2}\right)\dot{q}_3\dot{q}_4+q_3\left(q_4^{\ 2}-q_3^{\ 2}\right)\dot{q}_4^{\ 2}+\frac{Q_3}{2m_*l}\end{aligned}$$

$$\begin{aligned}q_4=&-2\dot{q}_4\left(q_1^{\ 2}+q_2^{\ 2}+q_3^{\ 2}-q_4^{\ 2}\right)\left(q_1\dot{q}_1+q_2\dot{q}_2+q_3\dot{q}_3\right)\\&+q_4\left(4q_3\dot{q}_3\left(q_1\dot{q}_1+q_2\dot{q}_2\right)+4q_1q_2\dot{q}_1\dot{q}_2-\left(\dot{q}_1^{\ 2}\left(-q_1^{\ 2}+q_2^{\ 2}+q_3^{\ 2}+q_4^{\ 2}\right)\right)-\dot{q}_2^{\ 2}\left(q_1^{\ 2}-q_2^{\ 2}+q_3^{\ 2}+q_4^{\ 2}\right)-\dot{q}_3^{\ 2}\left(q_1^{\ 2}+q_2^{\ 2}-q_3^{\ 2}+q_4^{\ 2}\right)\right)\\&-q_4\dot{q}_4^{\ 2}\left(3\left(q_1^{\ 2}+q_2^{\ 2}+q_3^{\ 2}\right)+q_4^{\ 2}\right)+\frac{Q_4}{2m_*l}\end{aligned}$$

2 Research Contents

Situation of the mission



In this paper, we consider the mission scenario that the space tug is located at the true anomaly of the Keplerian orbit at $\frac{4}{5}\pi$ after the completion of the capture, with an orbital eccentricity of 0.05 and a semi-long axis of 40489.5km, and is tangent to the graveyard orbit (orbital altitude of 42514km) at apogee, and the debris will be released from the apogee after the rotation from the completion of the capture. The velocity and flight path angle of the debris at the moment of release are obtained by superimposing the in-plane/out-of-plane rate and in-plane/out-of-plane angle on the velocity and flight path angle of the tether system at the moment of release, after which the debris enters the graveyard orbit to complete the removal.

Spin trajectory planning

fuel optimal cost function $J=\int_0^{t_f}\|\boldsymbol{U}\|dt$

boundary conditions

$$\boldsymbol{X}_1(0)=[r_0\ f_0\ q_{10}\ q_{20}\ q_{30}\ q_{40}]$$

$$\boldsymbol{X}_1(t_f)=[r_{t_f}\ f_{t_f}\ q_{1t_f}\ q_{2t_f}\ q_{3t_f}\ q_{4t_f}]$$

$$\dot{\boldsymbol{X}}_1(0)=\left[\dot r_0\ \dot f_0\ \dot q_{10}\ \dot q_{20}\ \dot q_{30}\ \dot q_{40}\right]$$

$$\dot{\boldsymbol{X}}_1(t_f)=\left[\dot r_{t_f}\ \dot f_{t_f}\ \dot q_{1t_f}\ \dot q_{2t_f}\ \dot q_{3t_f}\ \dot q_{4t_f}\right]$$

state constraint

$$\dot{\boldsymbol{X}}_1=\boldsymbol{X}_2$$

$$\dot{\boldsymbol{X}}_2=\boldsymbol{F}(\boldsymbol{X})+\boldsymbol{\rho G}(\boldsymbol{X})\boldsymbol{U}$$

Optimal control problems for continuous systems（OCP）

Infinite time transform

numerical integration of cost function

Dynamical equation fitting

$$\min\sum_{i=1}^{N}w_iT(\tau_i)g(\mathbf{X}_i,\mathbf{U}_i)$$

$$T(\tau_i)\mathbf{f}(\mathbf{X}_i,\mathbf{U}_i)-\mathbf{D}_i\mathbf{X}=0,\quad 1\le i\le N$$

$$\mathbf{X}_0-\mathbf{X}_{N+1}+\sum_{i=1}^{N}w_iT(\tau_i)\mathbf{f}(\mathbf{X}_i,\mathbf{U}_i)=0$$

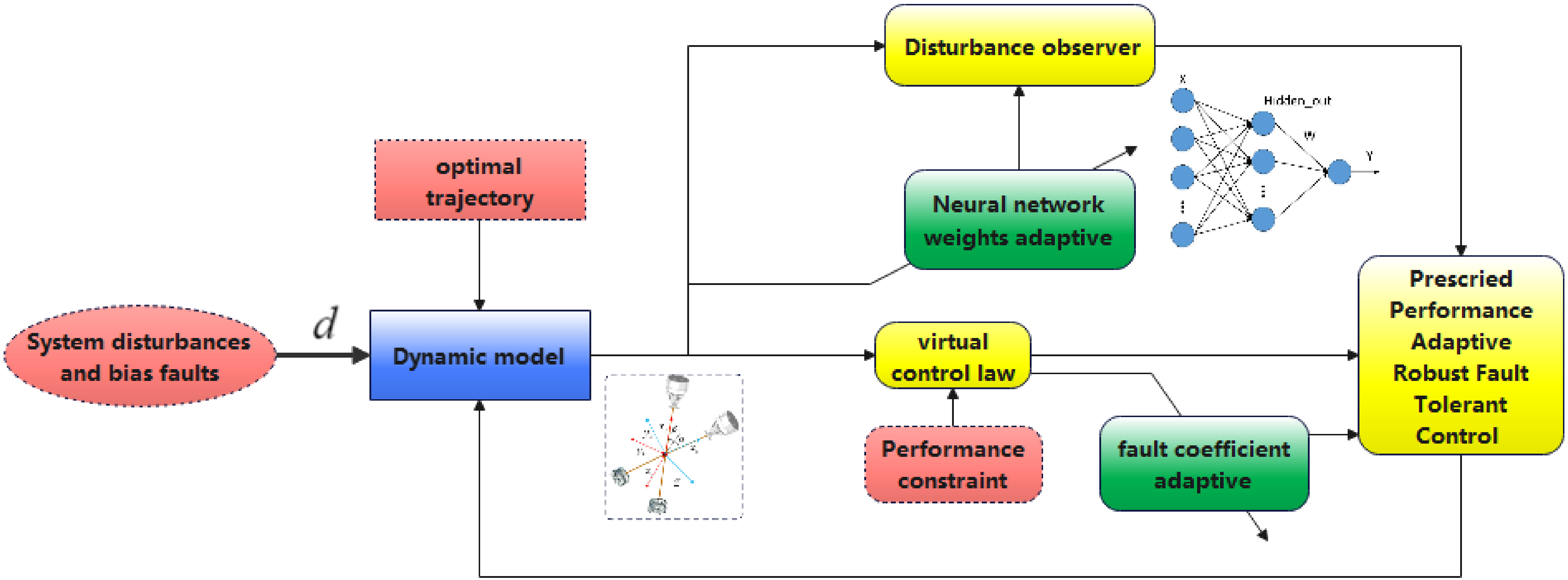
$$\mathbf{x}_0-\mathbf{X}_0=0.$$

Nonlinear program for discrete systems（NLP）

Optimal trajectory

Prescribed performance adaptive robust fault-tolerant control

Prescribed performance adaptive robust fault-tolerant control



2 Research Contents

Prescribed performance adaptive robust fault-tolerant control

Dynamic Model

$$\dot{\boldsymbol{X}}_1 = \boldsymbol{X}_2$$

$$\dot{\boldsymbol{X}}_2 = \boldsymbol{F}(\boldsymbol{X}) + \boldsymbol{G}(\boldsymbol{X})\boldsymbol{U}$$

Controller and Fault Coefficient Adaptive Law

$$\begin{cases} \boldsymbol{U} = -G(X)^{-1}\hat{\boldsymbol{K}}\boldsymbol{\eta} \\ \dot{\hat{k}}_i = \xi\hat{k}_i - s_i\eta_i, \quad i = 1, 2, \ldots, n \end{cases}$$

Virtual Control Law

$$\boldsymbol{\eta} = \dot{\boldsymbol{X}}_1 - \boldsymbol{X}_{2\mathrm{d}} - \dot{\boldsymbol{X}}_{2\mathrm{d}} + \boldsymbol{F}(\boldsymbol{X}) + \hat{\boldsymbol{W}}^{\boldsymbol{T}}\boldsymbol{R}(\boldsymbol{X}) + \hat{\boldsymbol{D}}(\mathrm{t}) + \kappa_1\boldsymbol{s} + \boldsymbol{\zeta}$$

Barrier Lyapunov function

$$\zeta_i == \frac{H(e_i)e_i(t)}{\underline{\gamma}_i^2(t) - e_i^2(t)} + \frac{(1 - H(e_i))e_i(t)}{\underline{\gamma}_i^2(t) - e_i^2(t)}$$

Adaptive Perturbation Observer

$$\hat{D} = \varPhi(-Z + X_2)$$

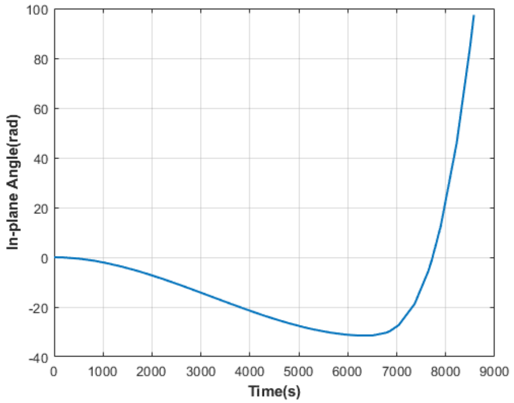
$$\dot{Z} = F(X) + G(X)U + \hat{W}^{\mathrm{T}}R(X) + \varepsilon + \hat{D} + \boldsymbol{s}$$

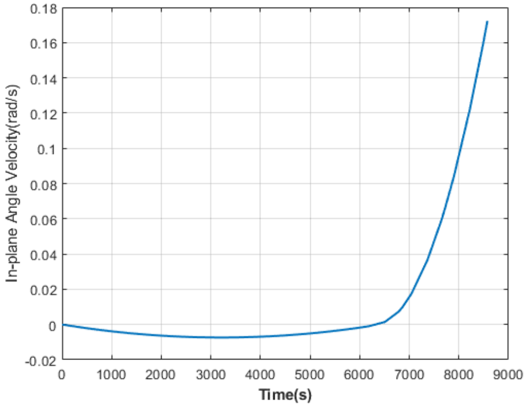
Neural network weights adaptive law

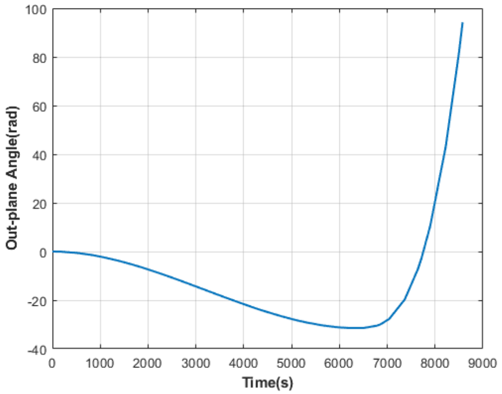
$$\dot{\hat{W}} = (\boldsymbol{s} + \boldsymbol{e}_n)\boldsymbol{R}(X) + \gamma\hat{W}, i = 1, 2, \ldots, n$$

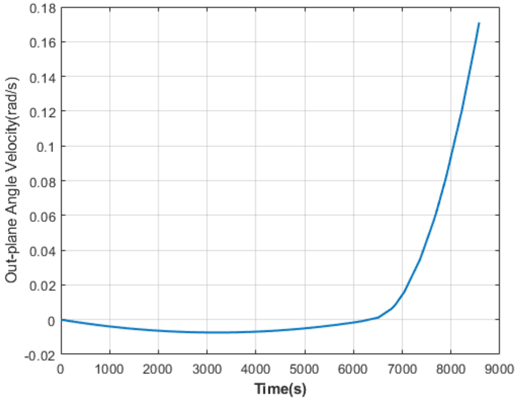
Research Contents

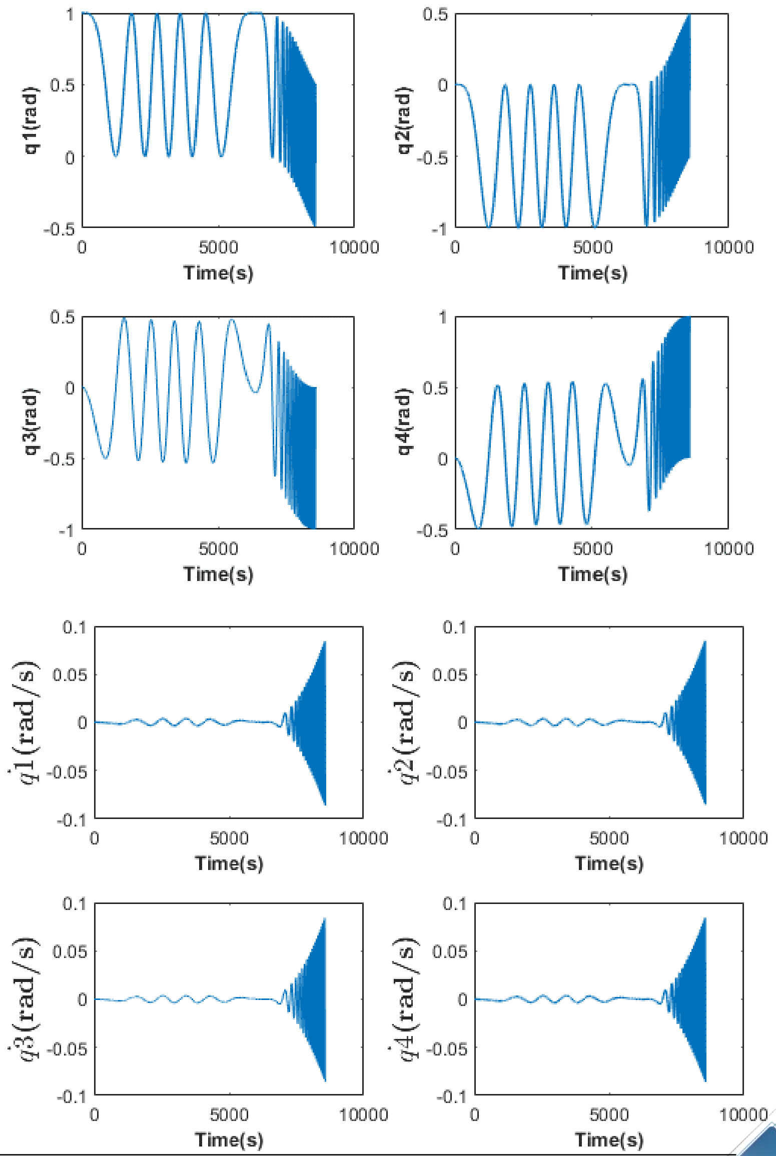
Optimal trajectory



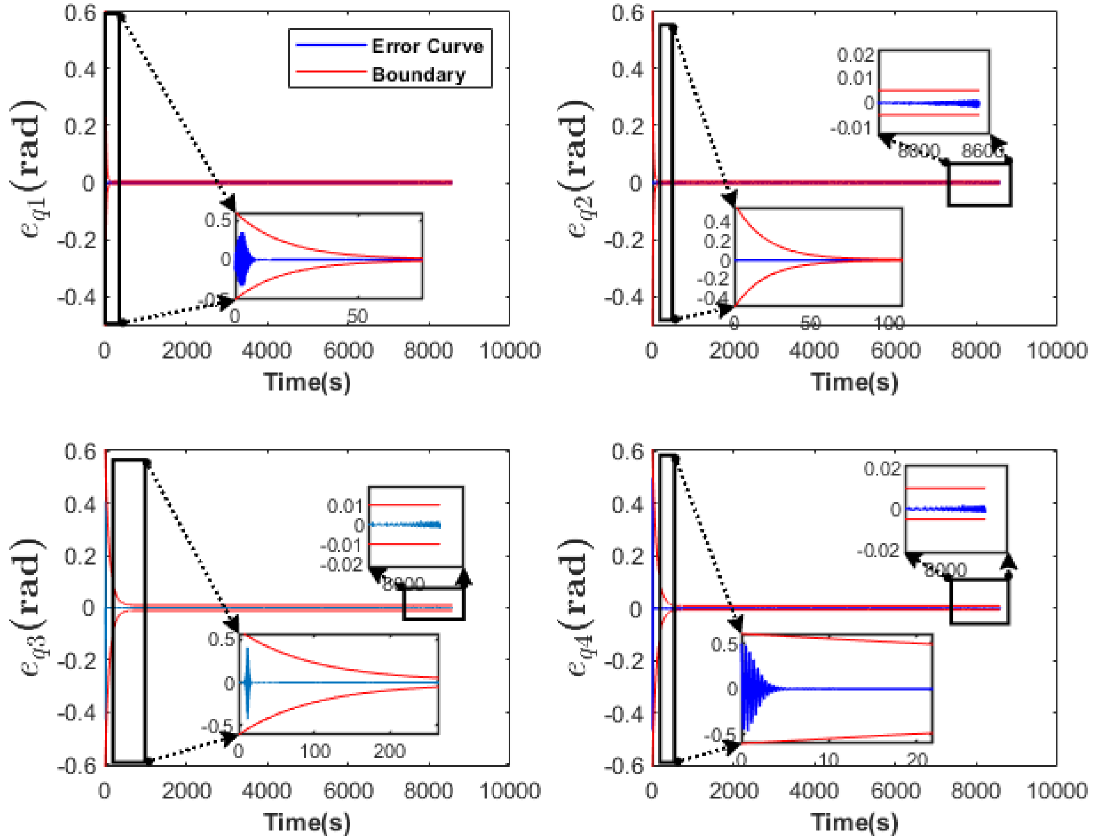


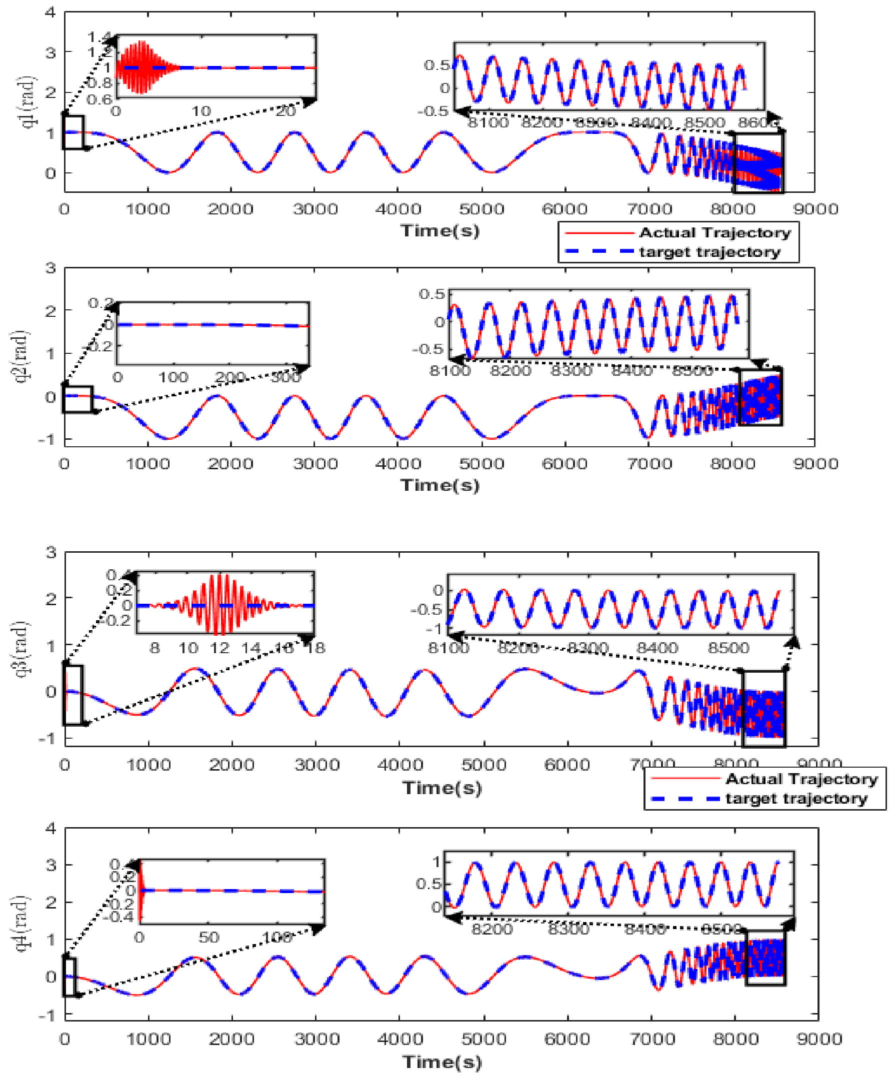






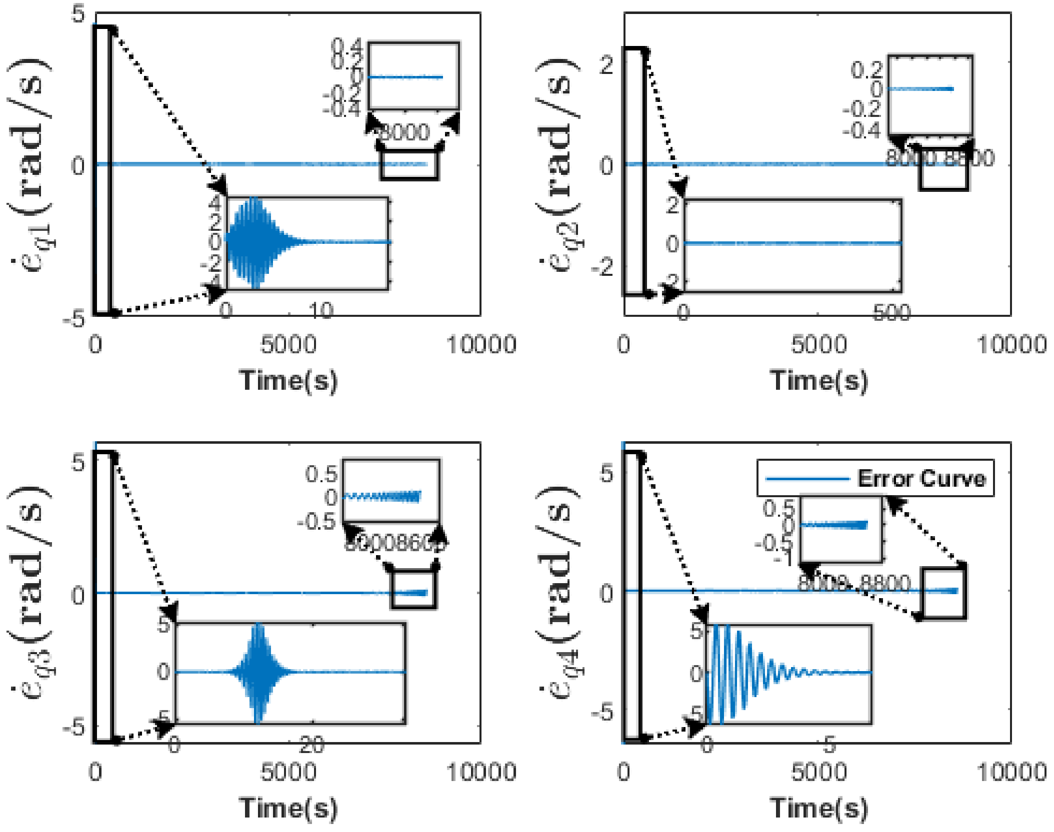
Controller Numerical Simulation

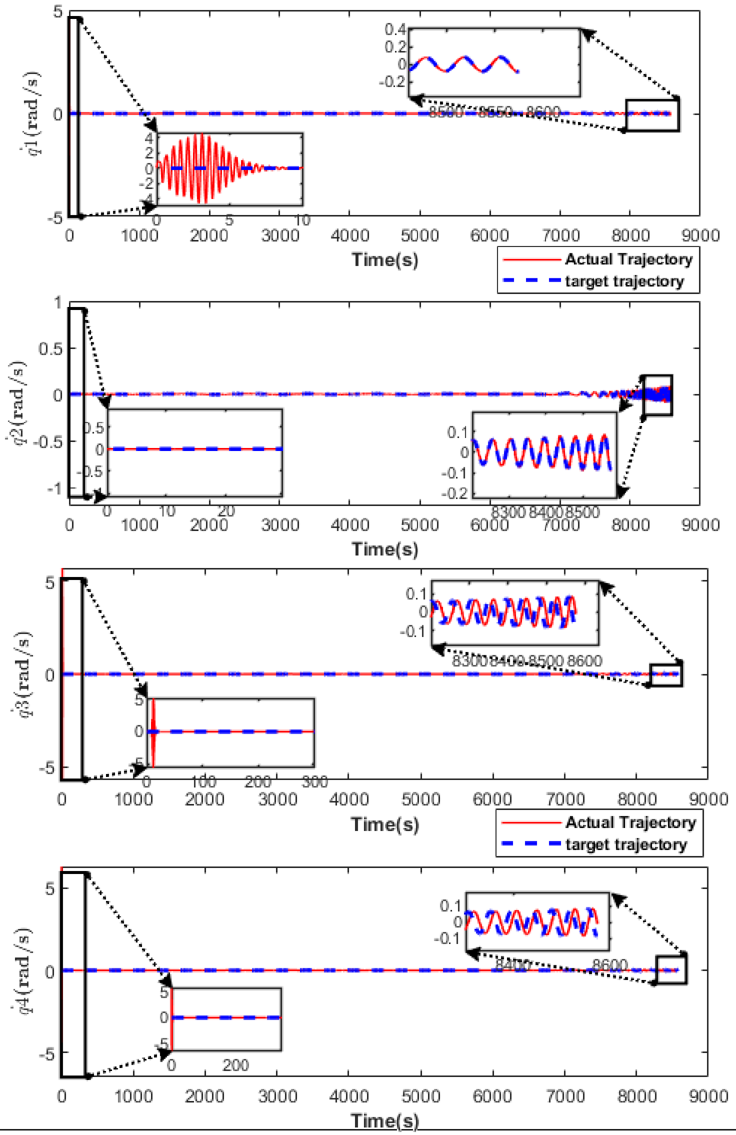




2 Research Contents

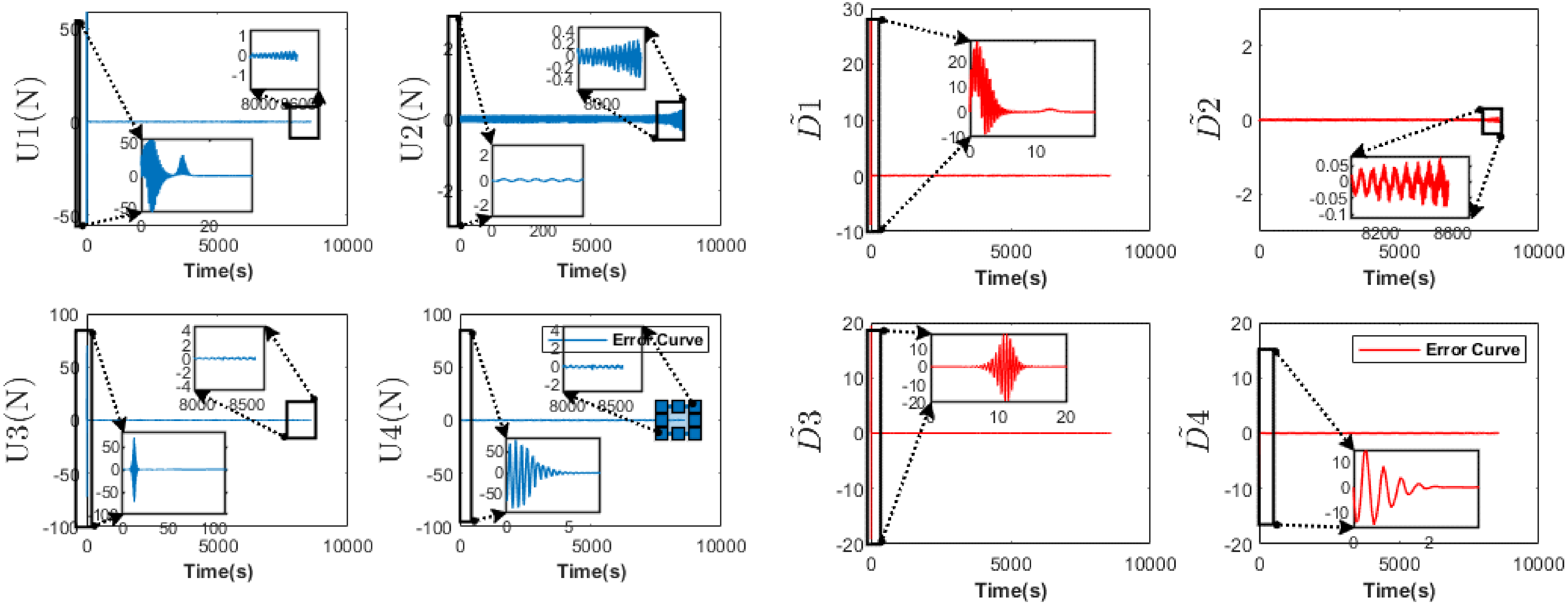
Controller Numerical Simulation





2 Research Contents

Control output and observer error



Thank you for your attention!