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7th International Conference on Tethers in Space

Iterative Learning Control for Multiple Deployment and Retrieval of Tethered Satellite System with Input Saturation

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Contents

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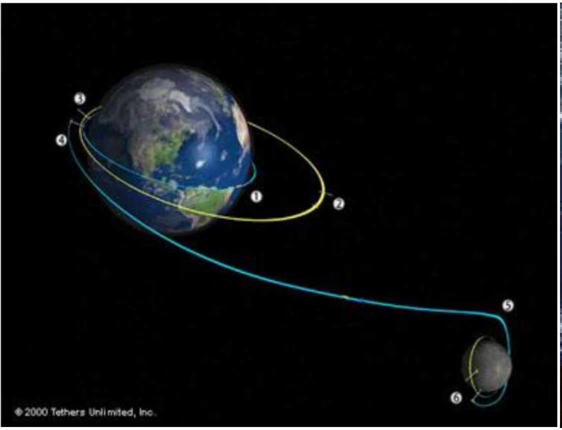
1. Introduction

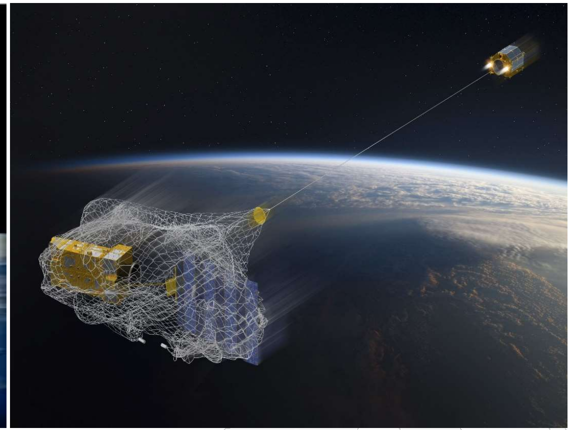
Tethered satellite system (TSS) refers to two or more end bodies connected by flexible tether flying in space.

- space debris capture and removal
- orbit transfer
- space elevator projects

With the advancement of TSS, these endeavors will most likely require systems capable of executing **multiple deployment and retrieval**.







Three dynamic phases:
- deployment
- station-keeping
- retrieval

- Coriolis acceleration.
- External disturbances and uncertainties.

How to achieve stable tether deployment and retrieval?

1. Introduction

How to apply the control action to the system?

- ➢ thrusters at the main/sub-satellites

(large amount propellent; pollution)

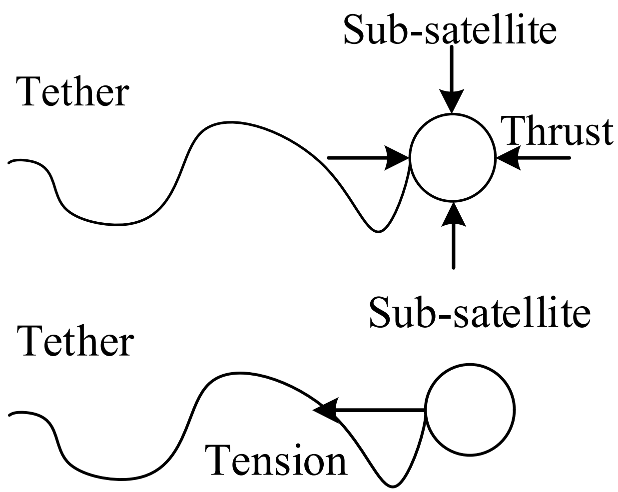
- ➢ the tension in the tether

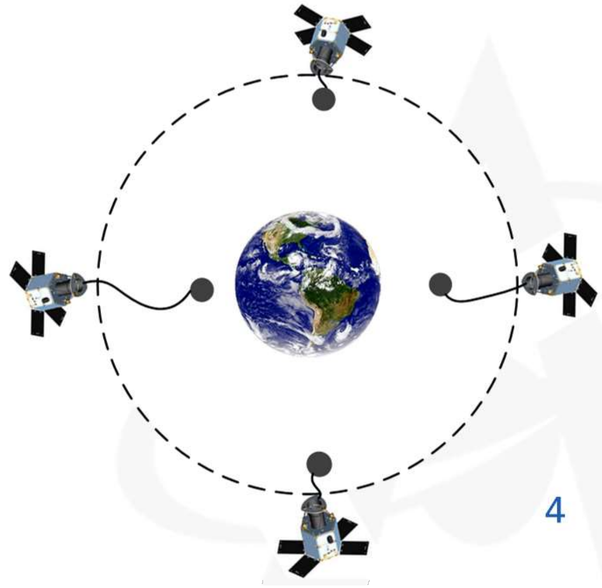
(**positive tension constraint**)

How to address the positive tension constraint?

- ➢ constrained optimization strategies
- ➢ analytical control laws:
- • fractional-order control
- • fuzzy-logic-based control
- • sliding mode control
- • …

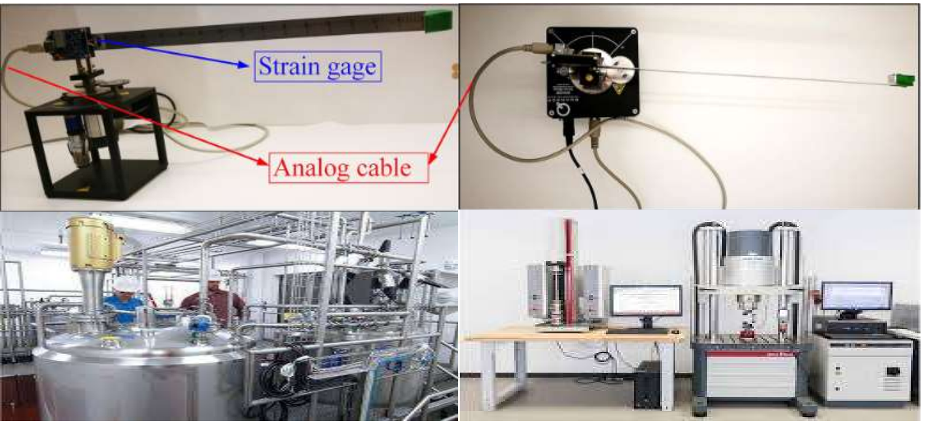
→ Concentrated on single instance of deployment or retrieval





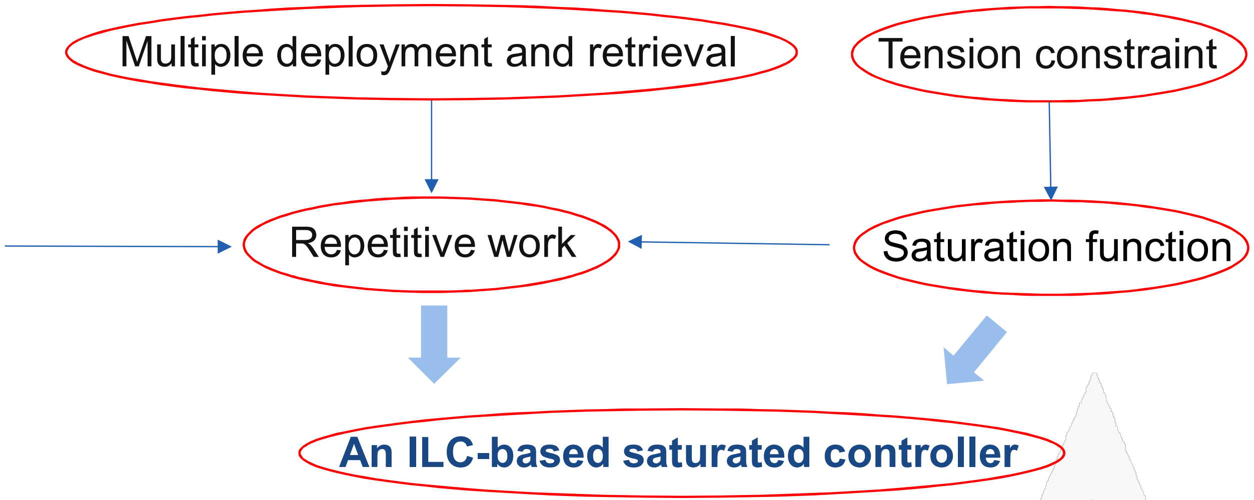
How to design control schemes for multiple deployment and retrieval by utilizing the task repeatability? ←

1. Introduction



Iterative learning control (ILC)

- simple structure
- model-free
- learning from the past control experience



Key contributions:

1) Based on the studies on ILC for systems with input saturation, an ILC-based saturated controller is developed for repetitive deployment and retrieval of a TSS, which is a underactuated system.
2) Compared with the past analytical control schemes for deployment or retrieval, the proposed controller can well deal with the problem of tension disturbances utilizing past control experiences.

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Contents

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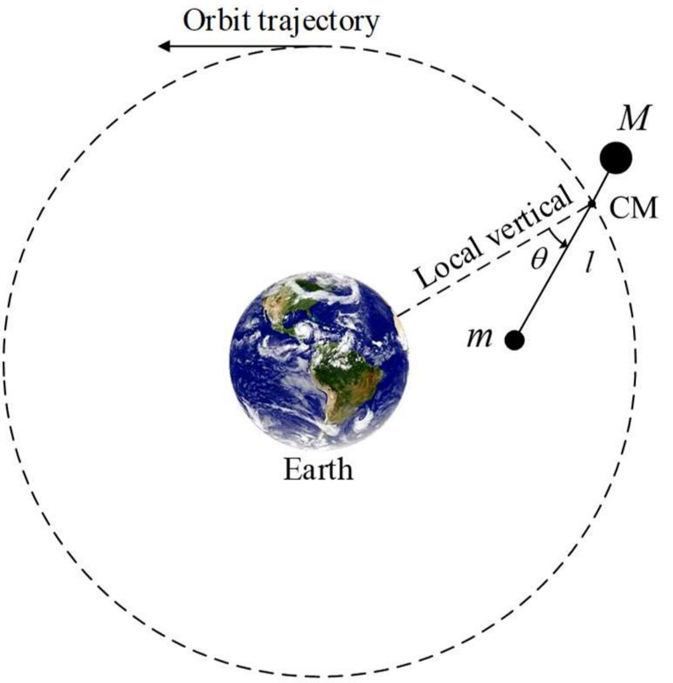
2. System description



The dynamic equations:

$$l'' - l[\theta'^2 + 2\theta'\omega_o + 3\omega_o^2\cos^2\theta] = -\frac{F_T + d}{m}$$

$$\theta'' + 2(\frac{l'}{l})(\omega_o + \theta') + \frac{3}{2}\omega_o^2\sin 2\theta = 0$$

d – disturbance

F_T – tension in the tether

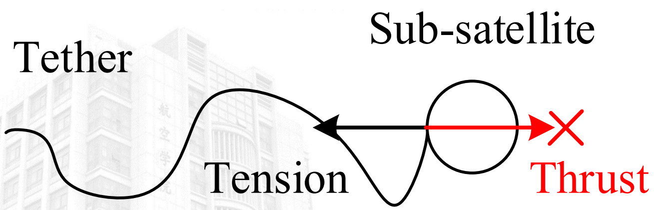
$$\xi = l / l_c,\ u = F_T / (m\omega_o^2 l_c),\ d(...)/dt = \omega_o d(...)/d\tau$$

The dimensionless form:

$$\ddot{\xi} - \xi[(1+\dot{\theta})^2 - 1 + 3\cos^2\theta] = -u + \tilde{d}$$

$$\ddot{\theta} + 2(\frac{\dot{\xi}}{\xi})(1+\dot{\theta}) + 3\sin\theta\cos\theta = 0$$

$\xi_{\min} > 0$ to avoid the singular problem



The control input should be satisfied the positive tension constraint

$$u \in (0, u_{\max})$$

$$\tilde{u} = \frac{u_{\max}}{2} - u$$

a symmetric domain

$$\tilde{u} \in (-\frac{1}{2}u_{\max}, \frac{1}{2}u_{\max})$$

which is equivalent to **a saturation requirement.**

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Contents

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3. ILC-based controller design

ILC-based tension control law: $\tilde{u}_j(\tau) = sat(\overline{\psi}(\xi_d(\tau)) - k_p(\overline{\xi}_j(\tau) + \zeta\overline{\xi}_{j-1}(\tau)) - k_v(\dot{\xi}_j(\tau) + \zeta\dot{\xi}_{j-1}(\tau)))$

$$\tilde{u}_j(\tau) = g_{1,j} + g_{2,j} + \overline{\psi}(\xi_d(\tau))$$

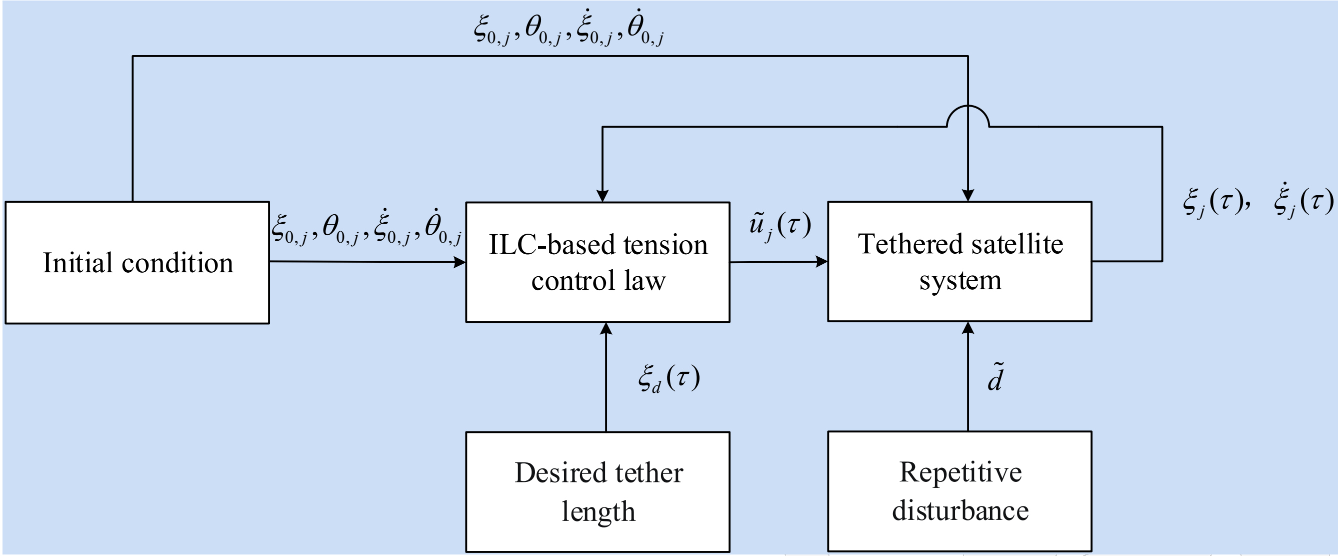
- $g_{1,j}$ the damping force
- $g_{2,j}$ the restoring force
- $\overline{\psi}(\xi_d(\tau))$ the constant force

$$g_{1,j} = sat(\overline{\psi}(\xi_d(\tau)) - k_p(\overline{\xi}_j(\tau) + \zeta\overline{\xi}_{j-1}(\tau)) - k_v(\dot{\xi}_j(\tau) + \zeta\dot{\xi}_{j-1}(\tau))) - sat(\overline{\psi}(\xi_d(\tau)) - k_p(\overline{\xi}_j(\tau) + \zeta\overline{\xi}_{j-1}(\tau)))$$

$$g_{2,j} = sat(\overline{\psi}(\xi_d(\tau)) - k_p(\overline{\xi}_j(\tau) + \zeta\overline{\xi}_{j-1}(\tau))) - sat(\overline{\psi}(\xi_d(\tau)))$$

$$sat(\lambda) = \begin{cases} -L + (M-L)\tanh\left(\dfrac{\lambda + L}{M-L}\right) & \lambda < -L \\ \lambda & -L \le \lambda \le L \\ L + (M-L)\tanh\left(\dfrac{\lambda - L}{M-L}\right) & \lambda > L \end{cases}$$

$sat(\lambda)$ is monotonically increasing with respect to λ



3. ILC-based controller design

Theorem 1. Under Assumption 1, the control command governed is stable in the sense of $L_2[0,T_0]$ norm.

Assumption 1. $\dot{\xi}_{j-1}(\tau)$ and $\dot{\xi}_j(\tau)$ have the same signs for $\forall\tau\in[0,T_0]$.

Proof.

$$E_j(\tau)=\frac{1}{2}\xi_j^2(\tau)\dot{\theta}_j^2(\tau)+\frac{1}{2}\dot{\xi}_j^2(\tau)-\frac{3}{2}\xi_j^2(\tau)\cos^2\theta_j(\tau)$$

$$U_{1,j}(\xi_j(\tau))=\int_0^{\xi_j(\tau)}3\delta d\delta$$

$$U_{2,j}(\bar{\xi}_j(\tau))=\int_0^{\bar{\xi}_j(\tau)+\zeta\bar{\xi}_{j-1}(\tau)}\{\bar{\psi}(\xi_d+\delta)-sat(\bar{\psi}(\xi_d)-k_p\delta)\}d\delta$$

$$V_j(\tau)=E_j(\tau)+U_{1,j}(\tau)+U_{2,j}(\tau)$$

$$\dot{V}_j(\tau)=\dot{E}_j(\tau)+\dot{U}_{1,j}(\xi_j(\tau))+\dot{U}_{2,j}(\bar{\xi}_j(\tau))$$

$$=\dot{\xi}_j(\tau)(\tilde{u}_j(\tau)-\frac{u_{\max}}{2}+\tilde{d})+3\dot{\xi}_j(\tau)\xi_j(\tau)+\ldots$$

$$\dot{\xi}_j(\tau)(\bar{\psi}(\bar{\xi}_j(\tau)+\zeta\bar{\xi}_{j-1}(\tau))-sat(\bar{\psi}(\xi_d)-k_p(\bar{\xi}_j(\tau)+\zeta\bar{\xi}_{j-1}(\tau))))$$

$$=\dot{\xi}_j(\tau)(g_{1,j}+\tilde{d}-3\zeta\bar{\xi}_{j-1}(\tau))$$

Assumption 1. There is no tether rebound in each iteration.

→ $\dot{\xi}_j(\tau)g_{1,j}\le 0$

When $\tilde{d}-3\zeta\bar{\xi}_{j-1}(\tau)$ has the opposite sign of $\dot{\xi}_j(\tau)$, we can obtain $\dot{V}_j(\tau)\le 0$.

3. ILC-based controller design

Proof.

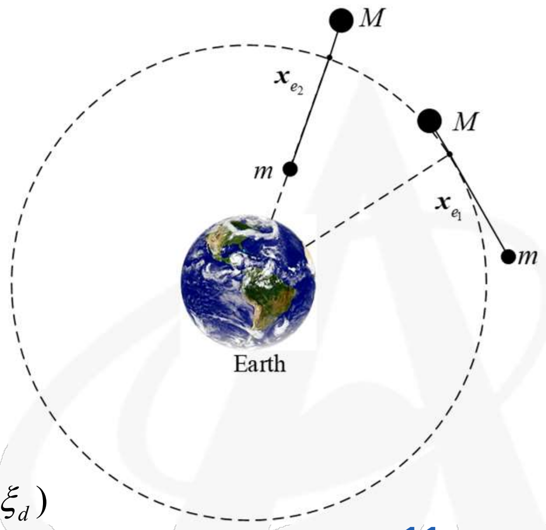
$$\dot{V}_j(\tau)\le 0 \Rightarrow \begin{cases} ①\dot{V}_j(\tau)=0 \quad , \quad \dot{\xi}_j \equiv 0 \\ ② V_{j,0}\ge V_j(\tau), \quad \text{otherwise} \end{cases}$$

① $\Rightarrow$ Let $\boldsymbol{W}_j$ be the largest invariance set within the set defined by $\dot{\xi}_j(\tau)$.

② $\Rightarrow$ $\xi_j(\tau)$, $\dot{\xi}_j(\tau)$ and $\dot{\theta}_j(\tau)$ are bounded. $\theta_j(\tau)$ is unbounded.

To obtain the set $\boldsymbol{W}_j$ ➡ 1)Dynamic equations; 2)Energy function and its differential; 3)ILC-based controller $\xrightarrow{\dot{\xi}_j \equiv 0}$ $\ddot{\theta}_j \approx 0,\ \dot{\theta}_j \approx 0,\ \theta_j \approx \frac{n\pi}{2}$ ➡ $\boldsymbol{x}_{e_1}=(\xi_{e_1}, n\pi, 0, 0)$, $\boldsymbol{x}_{e_2}=(\xi_{e_2}, (n+\frac{1}{2})\pi, 0, 0)$

Equilibrium points



$$③ \Rightarrow sat(\frac{1}{2}u_{\max} - 3\xi_d - k_p(\bar{\xi}_j(\tau)+\zeta\bar{\xi}_{j-1}(\tau))) + \tilde{d} = \psi(\xi_j,\theta_j)$$

$$\Rightarrow \frac{1}{2}u_{\max} - 3\xi_d - k_p(\bar{\xi}_j(\tau)+\zeta\bar{\xi}_{j-1}(\tau)) + \tilde{d} = \frac{1}{2}u_{\max} - 3\xi_j\cos^2\theta_j$$

$$\Rightarrow 3\xi_d + k_p(\bar{\xi}_j(\tau)+\zeta\bar{\xi}_{j-1}(\tau)) - \tilde{d} = 3\xi_j\cos^2\theta_j$$

If $\cos\theta_j = 0$, ③ does not hold. ➡ $\theta_e \equiv n\pi$ ➡ $k_p > 3,\ \xi_j \equiv \xi_d$

④ To limit the pitch angle within 90 deg: $\bar{V}_j(\xi_j) = U_{1,j}(\xi_j) + U_{2,j}(\bar{\xi}_j) \le V\big|_{\theta=\pm\frac{\pi}{2}}$

$$\frac{d\bar{V}_j(\xi_j)}{d\xi_j} = \frac{1}{2}u_{\max} - sat(\frac{1}{2}u_{\max} - 3\xi_d - k_p\bar{\xi}_j) > 0$$ ➡ $\bar{V}_j(\xi) > U_{2,j}(0-\xi_d)$ ➡ $-\frac{\pi}{2} < \theta_{j,0} < \frac{\pi}{2},\ V_{j,0} < U_{2,j}(-\xi_d)$

3. ILC-based controller design

Theorem 2. For tethered satellite system studied in this paper, by using the ILC-based tension control law (9), $\bar{\xi}_j$ will converges to 0 along the iteration axis.

Assumption 2. In each iteration, the initial conditions $\boldsymbol{x}_{0,j} = (\xi_{0,j}, \theta_{0,j}, \dot{\xi}_{0,j}, \dot{\theta}_{0,j})^T$ have the same value.

Proof. Step 1). Address the non-increasing property of the energy function along the iteration axis.

$\Delta E_j(T_0) \triangleq E_j(T_0) - E_{j-1}(T_0)$ Note: $E_j(T_0) = V_j(T_0) - U_{1,j}(T_0) - U_{2,j}(T_0)$

According to ***Assumption*** 2

$$① \Rightarrow \Delta E_j(T_0) = \int_0^{T_0} \dot{V}_j(\tau) d\tau - \int_0^{T_0} \dot{U}_j(\tau) d\tau$$

$$= \int_0^{T_0} \dot{\xi}_j(\tau)(g_{1,j} + g_{2,j}) d\tau - \int_0^{\xi_d(T_0)} 3\delta d\delta - \int_{\xi_d(T_0)}^{\xi_j(T_0)} \{3\delta + \bar{\psi}(\delta) - sat[\bar{\psi}(\xi_d) - k_p(\delta - \xi_d)]\} d\delta$$

$$\leq -\int_{\xi_d(T_0)}^{\xi_j(T_0)} \{\frac{u_{\max}}{2} - sat[\bar{\psi}(\xi_d) - k_p(\delta - \xi_d)]\} d\delta$$

$$\dot{V}_j(\tau) \leq 0, \ \frac{u_{\max}}{2} - sat[\bar{\psi}(\xi_d) - k_p(\delta - \xi_d)] \geq 0$$

$$② \Rightarrow \Delta E_j(T_0) \leq -\int_{\xi_d(T_0)}^{\xi_j(T_0)} \{\frac{u_{\max}}{2} - sat[\bar{\psi}(\xi_d) - k_p(\delta - \xi_d)]\} d\delta \leq 0$$

The energy function $E_j(T_0)$ is non-increasing along the iteration axis.

3. ILC-based controller design

Step 2). Prove the uniform convergence of $\bar{\xi}_j$.

$\dot{E}_0(\tau)$ has the opposite sign of $\dot{\xi}_0$.

ξ_0 is first increasing and then decreasing in the sense of $L_2[0,T_0]$ norm. Correspondingly, $E_0(\tau)$ is first decreasing and then increasing.

$E_0(T_0)$ is also lower bounded because $\xi_j(\tau), \dot{\xi}_j(\tau), \theta_j(\tau)$ and $\dot{\theta}_j(\tau)$ are bounded in the sense of $L_2[0,T_0]$ norm.

⬇ According to ***Assumption*** 1

$$① \Rightarrow E_j(T_0) = E_0(T_0) + \sum_{i=1}^{j} \Delta E_i(T_0) \le E_0(T_0) - \sum_{i=1}^{j} \int_{\xi_d(T_0)}^{\xi_i(T_0)} \{\frac{u_{\max}}{2} - \sigma[\bar{\psi}(\xi_d) - k_p(\delta - \xi_d)]\} d\delta$$

$$② \Rightarrow \lim_{j\to\infty} \sum_{i=1}^{j} \int_{\xi_d(T_0)}^{\xi_i(T_0)} \{\frac{u_{\max}}{2} - \sigma[\bar{\psi}(\xi_d) - k_p(\delta - \xi_d)]\} d\delta = 0$$

⬇

$\bar{\xi}_j(T_0)$ will converge to zero in the sense of $L_2[0,T_0]$ norm.

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Contents

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4. Numerical simulation

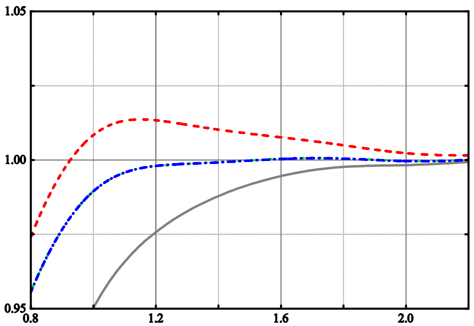
- Simulation parameters of TSS

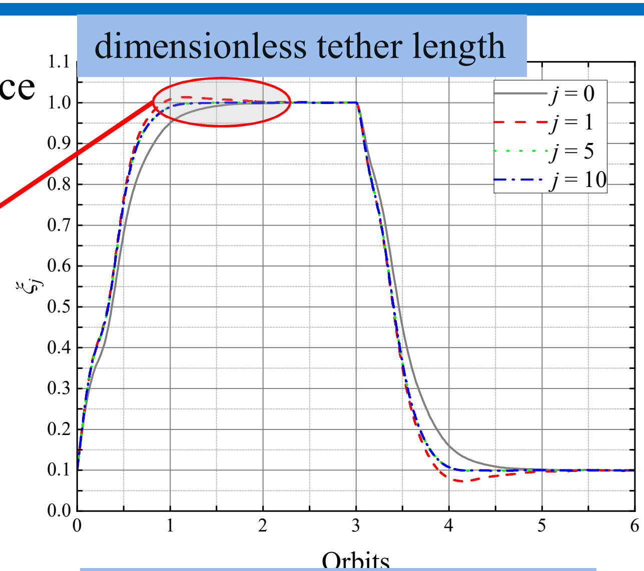
Parameters	Values
The dimensionless time domain of deployment phase	$\tau \in [0, 3\ \text{orbits})$
The dimensionless time domain of retrieval phase	$\tau \in [3\ \text{orbits}, 6\ \text{orbits}]$
Learning gain	$\zeta = 0.08$
Initial condition of the case	$\boldsymbol{x}_{0,j} = (0.1, 0, 0, 0)^T$
Parameters of saturation function	$M = 5,\ L = 4.99$
Velocity gain	$k_v = 4$
Length gain	$k_p = 3$

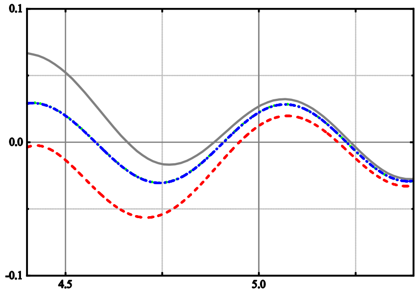
4. Numerical simulation

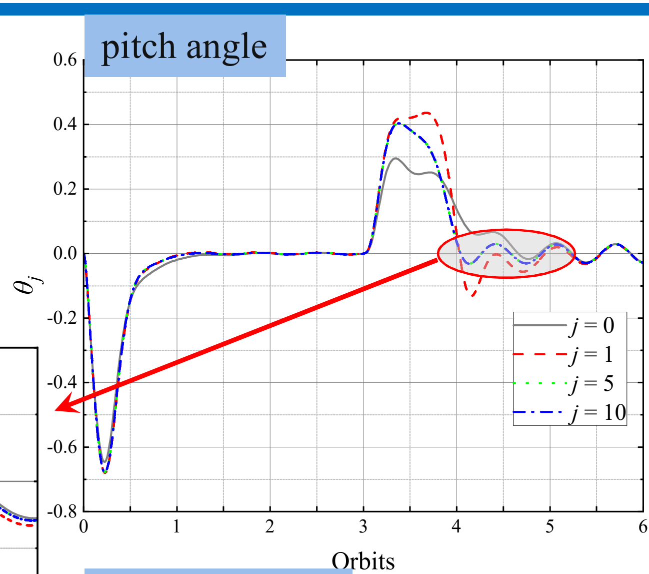
- Case with external disturbance

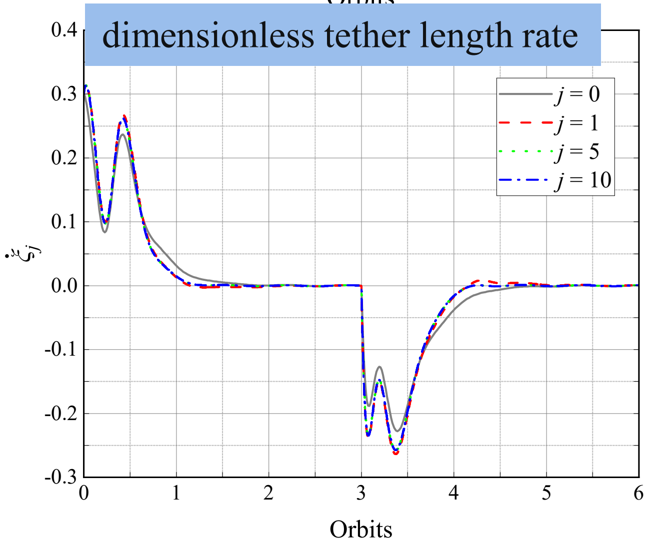
  $d = 0.01\sin(\pi\tau/2)$

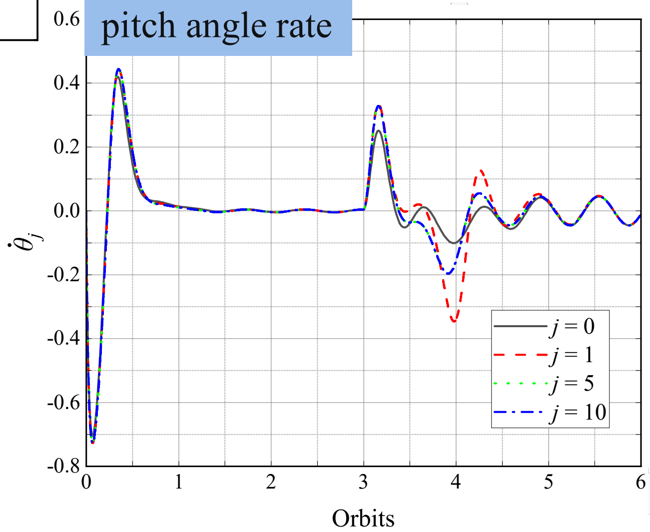










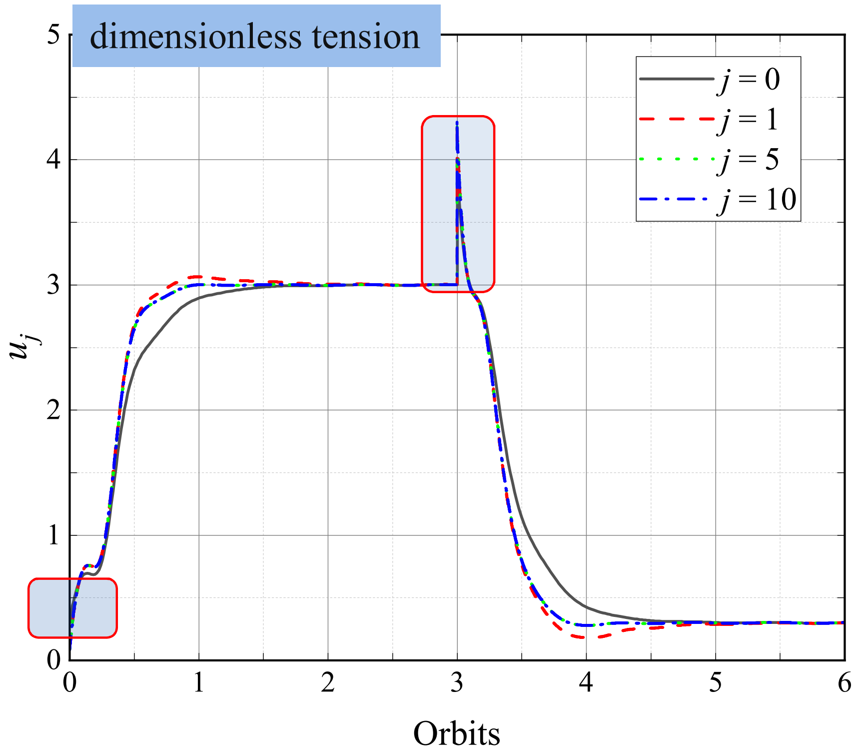


The proposed ILC-based controller achieves the desired length faster than the results in 0^{th} iteration.

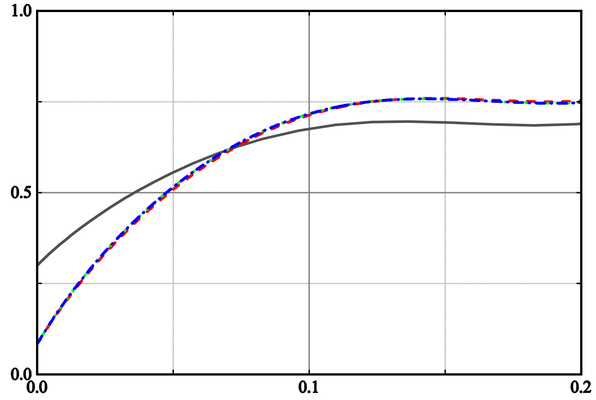
4. Numerical simulation

- Case with external disturbance

  $d = 0.01\sin(\pi\tau/2)$



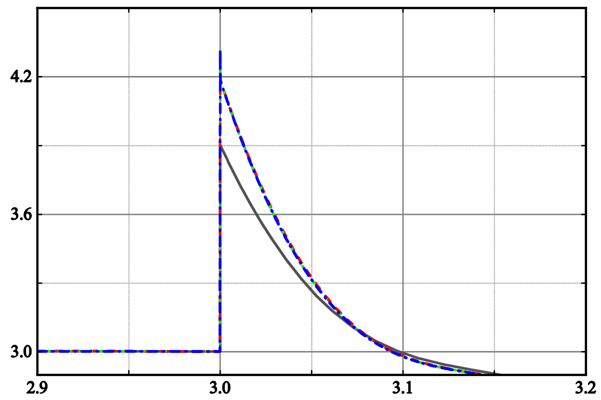
✓ ***Assumption 1.*** $\dot{\xi}_{j-1}(\tau)$ and $\dot{\xi}_j(\tau)$ have the same signs for $\forall \tau \in [0, T_0]$



There is no tether rebound in each iteration.

$u \in (0,10)$

✓ The tether can hold the tension mutation under the proposed controller.



$\tau = 3 : \xi_d = 1 \rightarrow \xi_d = 0.01$

$u \in (0,10)$

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Contents

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5. Conclusions

1) The ILC-based tension control law is proposed for the multiple deployment and retrieval of TSS with input saturation.
2) **Stability of the controller is validated** using Lyapunov function and LaSalle's invariance principle. **The learning convergence** of the closed-loop system **is proved** based on the system's energy function.
3) The control scheme can enhance the controller's performance during repetitive missions.

——7th International Conference on Tethers in Space——

Thanks!