50 Years of Modeling and Simulation of Space Tether Dynamics

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OUTLINE

- Introduction
- Dynamics of Tethered Satellites
- Dynamics of Very Long Tethers
- Dynamics of Tethered Systems at Lagrangian Points
- Space Debris Towing Dynamics
- Conclusion

INTRODUCTION

- Tethered satellite systems:
  - **Two or more** orbiting bodies connected by **one or more** cables/tethers.
  - Tethers can be several meters to thousands kilometers long.

- Space tethers can have many potential applications:
  - Upper atmospheric research
  - Removal of space debris
  - Orbital transfer
  - Formation flying
  - Planetary and astronomical research
  - Asteroid diversion
  - Partial space elevator

INTRODUCTION

HISTORY

- Feasibility of using tether systems in space was first established during two Gemini missions: Gemini XI and Gemini XII (1966).

- The manned vehicle Gemini XI was tethered to an unmanned Agena vehicle. A rotating configuration could be maintained by the Gemini thruster control system. Gemini XII demonstrated a gravity gradient configuration.

- Interest in space tethers took a big jump when the proposal of **Skyhook** made by Harvard-Smithsonian Institution was accepted by NASA. This was 1974. This led to numerous investigations of dynamics and control of tethered space systems.

INTRODUCTION

HISTORY

Gemini XI- Agena

Image courtesy of
https://www.nasa.gov

Gemini XII - Agena

Image courtesy of
https://www.nasa.gov

INTRODUCTION
HISTORY

- **Space elevator**
  - Tsiolkovsky proposed the concept of a space elevator in 1895.
  - Subsequent proposals (e.g., Artsutanov, 1959) involved a base, ribbon/tether, and a counterweight located above the geostationary altitude.
  - Tether / ribbon is expected to be a few millimeters wide and 100,000 km long.
  - A partial space elevator is more likely to be realized in the near term.

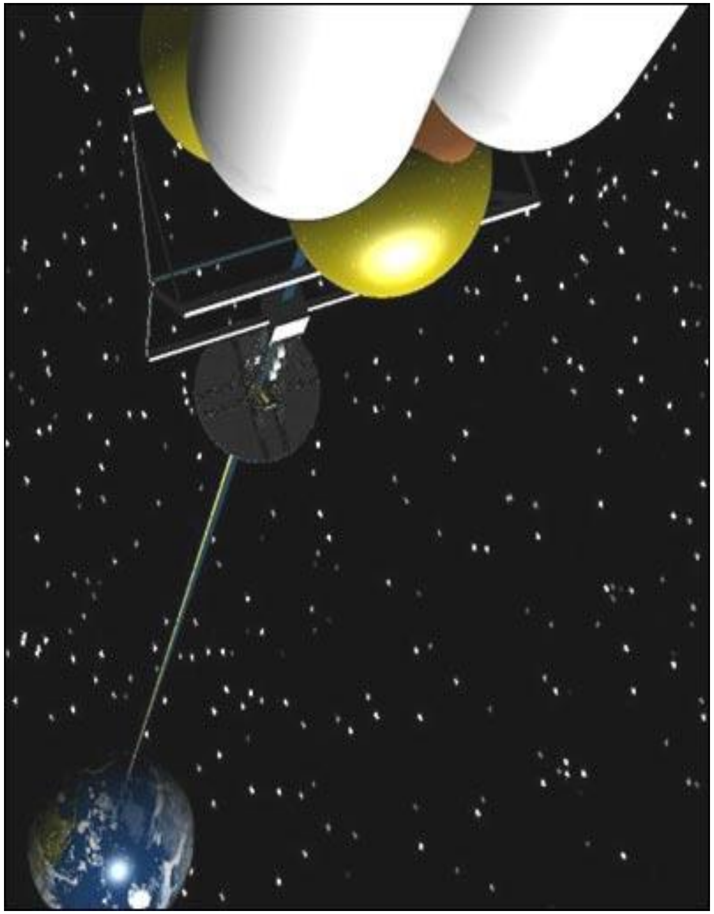
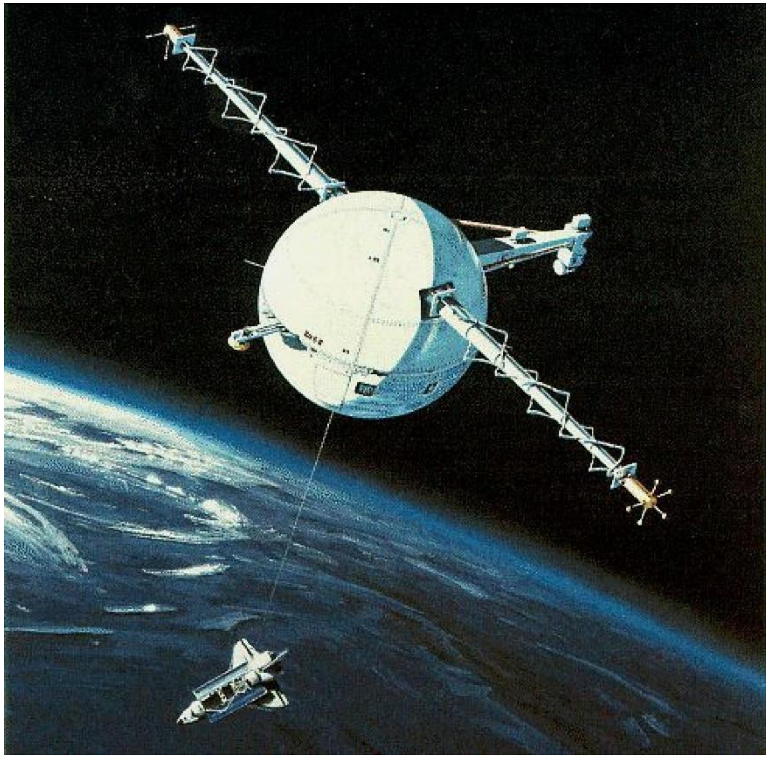


Image courtesy of www.isr.us

INTRODUCTION

HISTORY



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http://science.nasa.gov

The era of tethers in space really started when Columbo et al. proposed a “Shuttle-borne Skyhook”. This gave rise to the Tethered Satellite System-1 (TSS-1) mission.

TSS-1 flew in 1992. TSS-1R flew in 1996 and deployed a 19.7 km tether.

INTRODUCTION

HISTORY

TiPS and SEDS

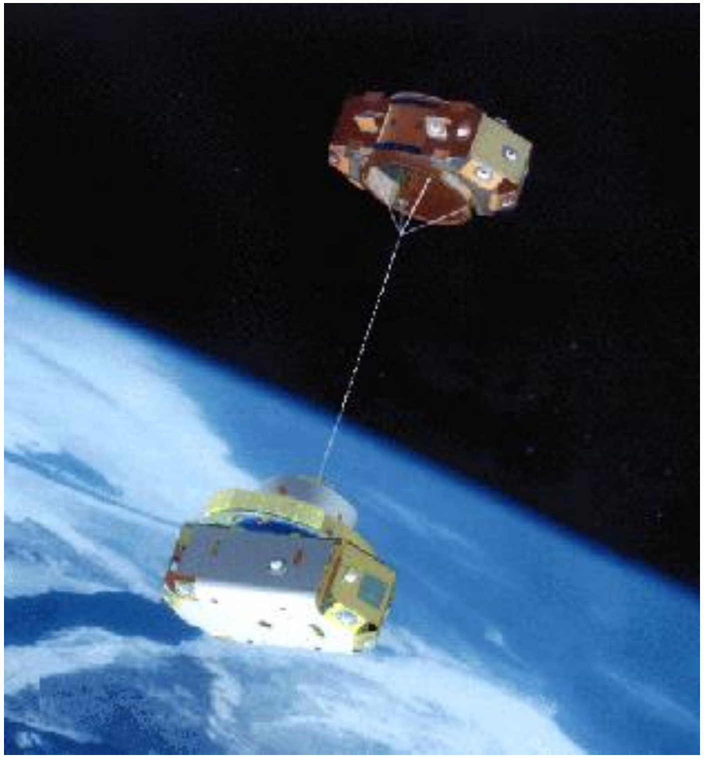


Image courtesy of http://projects.nrl.navy.mil

- Tether Physics and Survivability Experiment (TiPS) was an NRL mission that was launched in 1996. It had been planned as a 2-year mission, but it lasted until 2006.
- Small Expendable Deployer System (SEDS) was a 20-km tether satellite mission which was deployed in 1993.
- SEDS 2 flew in 1994 and sent several hours of data.

INTRODUCTION

SUMMARY OF SPACE TETHER MISSIONS

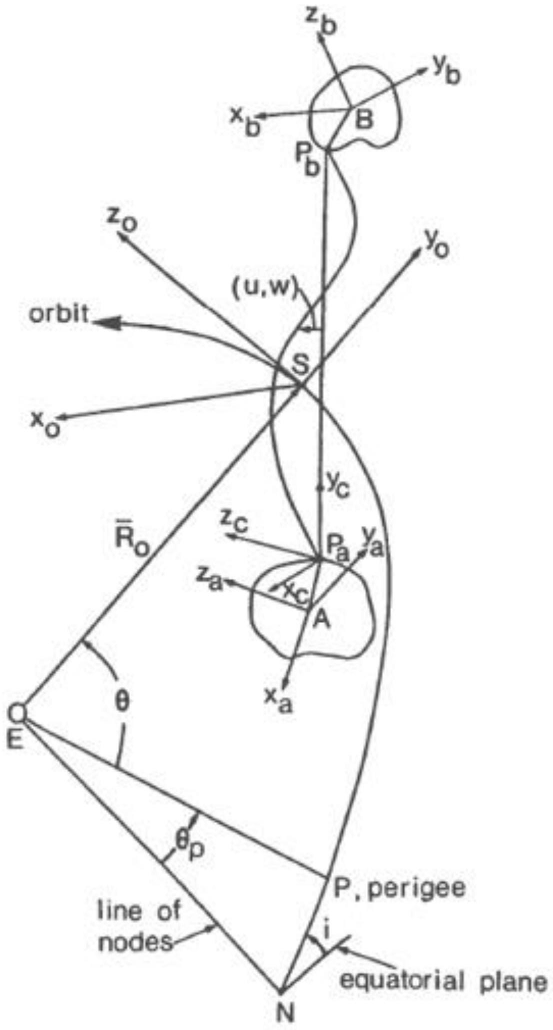
Name	Date	Orbit	Length	EDT	Agency	Comments
Gemini 11	1966	LEO	30m		NASA	Spin stable 0.15rpm
Gemini 12	1966	LEO	30m		NASA	Local vertical, stable swing
H-9M-69	1980	Suborbital	500m	✓	NASA	Partially deployed
S-520-2	1981	Suborbital	500m	✓	NASA/ISAS	Partially deployed
Charge-1	1983	Suborbital	500m	✓	NASA/ISAS	Fully deployed
Charge-2	1984	Suborbital	500m	✓	NASA/ISAS	Fully deployed
Oedipus-A	1989	Suborbital	958m		CSA/NASA	Spin stable @ 5rpm, magnetic field aligned
Charge-2B	1992	Suborbital	500m	✓	NASA	Fully deployed
TSS-1	1992	LEO	260m	✓	NASA/ISA	Partially deployed, retrieved
SEDS-1	1993	LEO	20km		NASA	Downward fully deployed, swing, and cut
PMG	1993	LEO	500m	✓	NASA	Upward deployed
SEDS-2	1994	LEO	20km		NASA	Fully deployed, local vertical stable
Oedipus-C	1995	Suborbital	1170m		CSA/NASA	Spin stable @ 5rpm, magnetic field aligned
TSS-1R	1996	LEO	19.6km	✓	NASA/ISA	Close to full deployment, severed by arcing

INTRODUCTION

SUMMARY OF SPACE TETHER MISSIONS

Name	Date	Orbit	Length	EDT	Agency	Comments
TiPS	1996	LEO	4km		NRO/NRL	Long-life tether on-orbit (survived 12 years)
ATEx	1999	LEO	6km		NRL	Partially deployed
ProSEDS	2003	LEO	15km	✓	NASA	Hardware built but not flown
DTUsat-1	2003	LEO	450m	✓	TUD	Nano-Satellite failed to work in space
MAST	2007	LEO	1km		NASA	Tether failed to deploy
YES2	2007	LEO	30 km		ESA	Fully deployed but lost
Cute-1.7+APDII	2008	LEO	10m	✓	Tokyo Tech	Cube-Satellite worked, Tether failed to deploy
STARS	2009	LEO	10m		Kagawa U	Space tethered robot mission, tether deployed
T-Rex	2010	Suborbital	300m	✓	JAXA	Tether deployed, current not measured
STARS-II	2014	LEO	300m		JAXA	Electro-dynamic tether
KITE	2016	LEO	700m		JAXA	Deployment failure
STARS-C	2016	LEO	100m		JAXA	15 months in orbit
TEPCE	2019	LEO	1km	/	NRL	Electro-dynamic Triple Cubesat
DESCENT	2020	LEO	100m	/	CSA	Electro-dynamic Tether Cubesat
MiTEE	2021	LEO	30m	/	NASA	Electro-dynamic Triple Cubesat

DYNAMICS OF TETHERED SPACE SYSTEMS



DYNAMICS OF TETHERED SATELLITE SYSTEMS

- Dynamics models of two-body tethered satellite systems have ranged from very simple to very complex over the years.

- The simplest model is two orbiting point masses connected by a straight massless rigid tether.

- This model can be extended to:
  - Two orbiting point masses connected by a straight rigid tether with mass;
  - Two orbiting point masses connected by a straight elastic tether;
  - Two orbiting point masses connected by an elastic tether undergoing both longitudinal and transverse elastic oscillations;

- Various environmental forces, such as electrodynamic, aerodynamic, gravitational perturbations have been added.

DYNAMICS OF TETHERED SATELLITE SYSTEMS

- Tether has been modelled as a:
  - Collection of point masses or beads
  - Continuum

- Beads may be connected by springs and dashpots representing tether stiffness and damping.

- Various continuum approaches used are:
  - Galerkin and Assumed Modes Method
  - Finite Difference Method
  - Finite Element Method (ANCFEM, NPFEM)

DYNAMICS OF TETHERED SPACE SYSTEMS

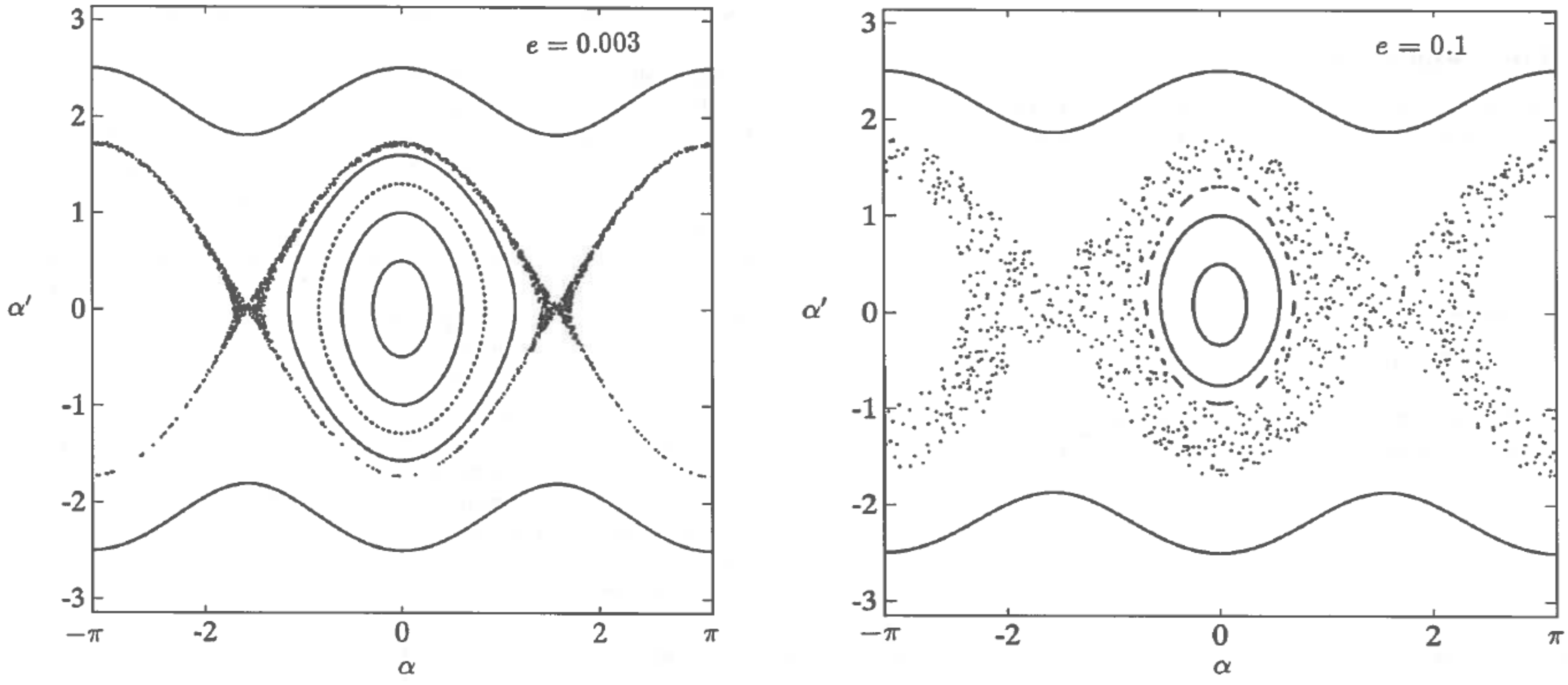
- For small pitch and roll motions, in the absence of generalized forces and for a circular orbit,

$$\ddot{\alpha} + 3n^2\alpha = 0$$
$$\ddot{\gamma} + 4n^2\gamma = 0$$

- Pitch frequency is $\sqrt{3}$ times the orbital frequency.
- Roll frequency is 2 times the orbital frequency.
- Coupled pitch and roll motions are quasi-periodic in the general case.
- Under certain conditions, nonlinear motion can become chaotic.

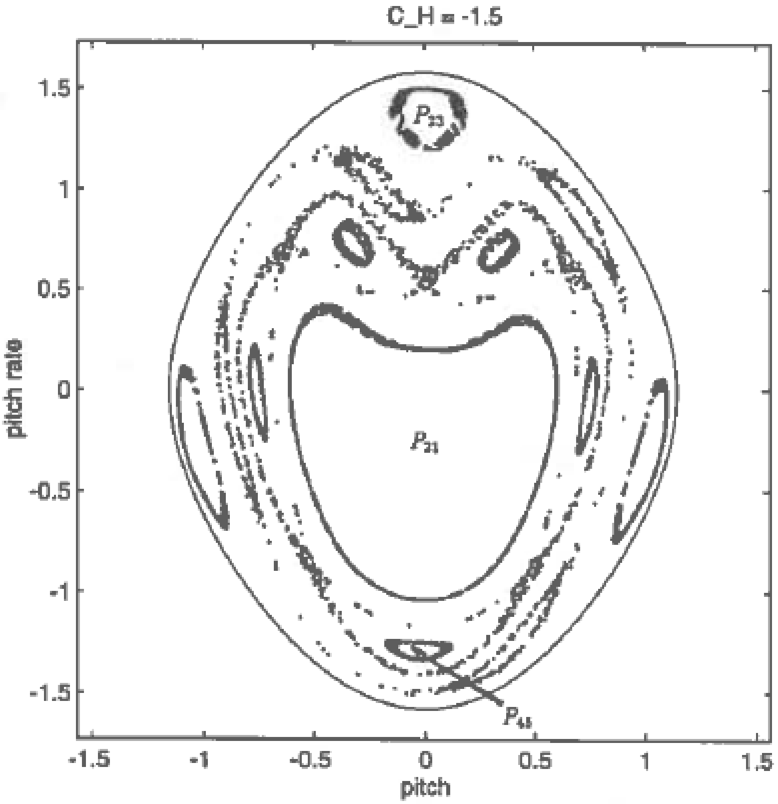
DYNAMICS OF TETHERED SPACE SYSTEMS

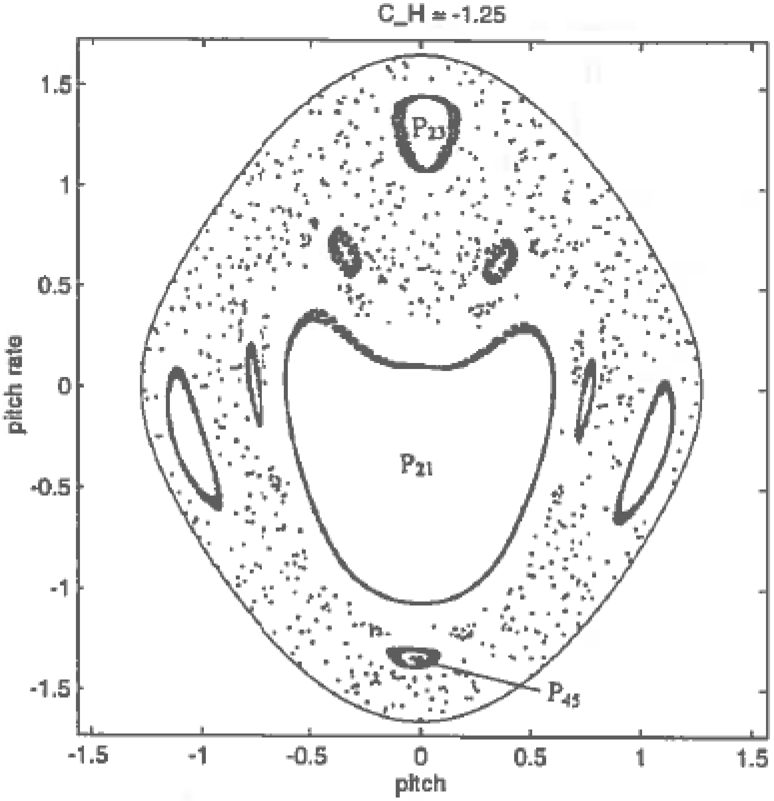
- Motion is regular for $e = 0$, but can become chaotic for non-zero e.



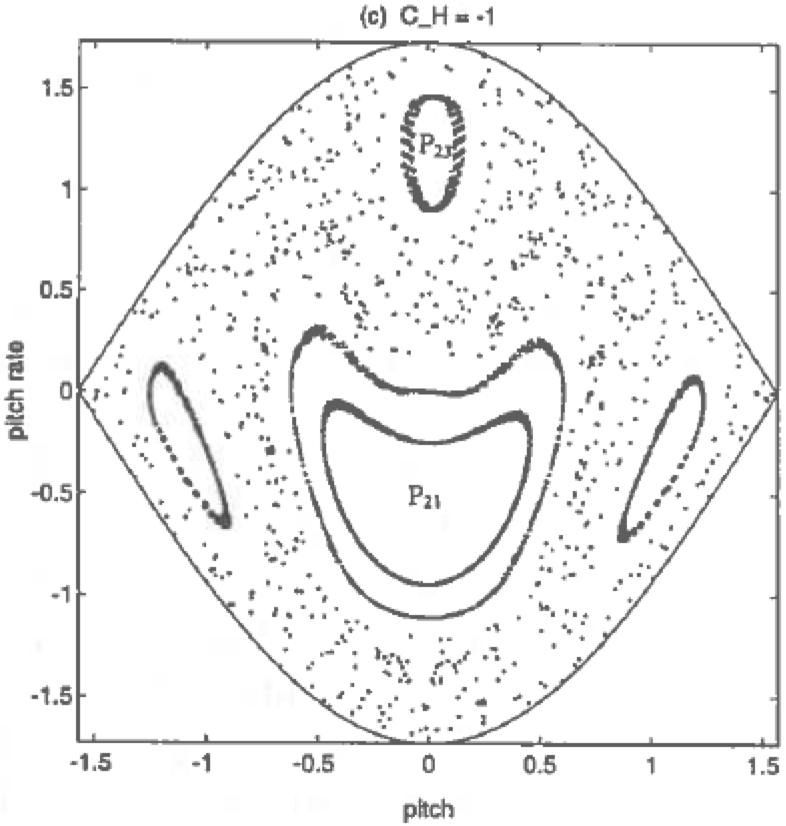
DYNAMICS OF TETHERED SPACE SYSTEMS

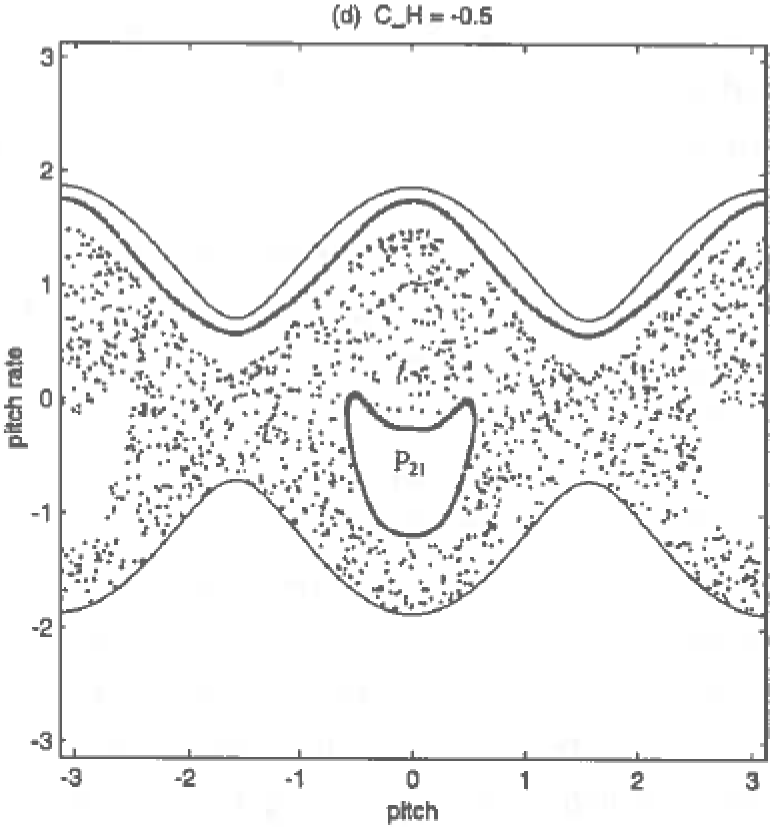
- Coupled pitch-roll motion can become very irregular even for a circular orbit





DYNAMICS OF TETHERED SPACE SYSTEMS





DYNAMICS OF TETHERED SPACE SYSTEMS

- When the tether length is changing (during deployment and retrieval), motion can be described by:

$$\ddot{\alpha} + 2\frac{\dot{l}}{l}(\dot{\alpha} + n) + 3n^2\alpha = \hat{Q}_\alpha$$

$$\ddot{\gamma} + 2\frac{\dot{l}}{l}\dot{\gamma} + 4n^2\gamma = \hat{Q}_\gamma$$

- Pitch and roll motions grow during retrieval. A stabilization scheme is needed.

- Various control schemes have been proposed:
  - ❖ Tension control
  - ❖ Reel rate control or length rate control
  - ❖ Thruster control
  - ❖ Offset control

DYNAMICS OF TETHERED SPACE SYSTEMS

- Tension in the tether arises due to the combined gravity gradient and centrifugal gradient effects.
- Steady state tension for a vertical tether is approximately given by $T_o = 3m_2\Omega^2\ell_o[1 + (1/2)(m_t/m_2)(1 - s^2/\ell_o^2)]$
- Clearly, tension can be very small for short tethers.
- The smallest elastic longitudinal frequency varies as $l_0^{-1/2}$, while the others vary as l_0^{-1}.
- The in-plane transverse frequencies are given by

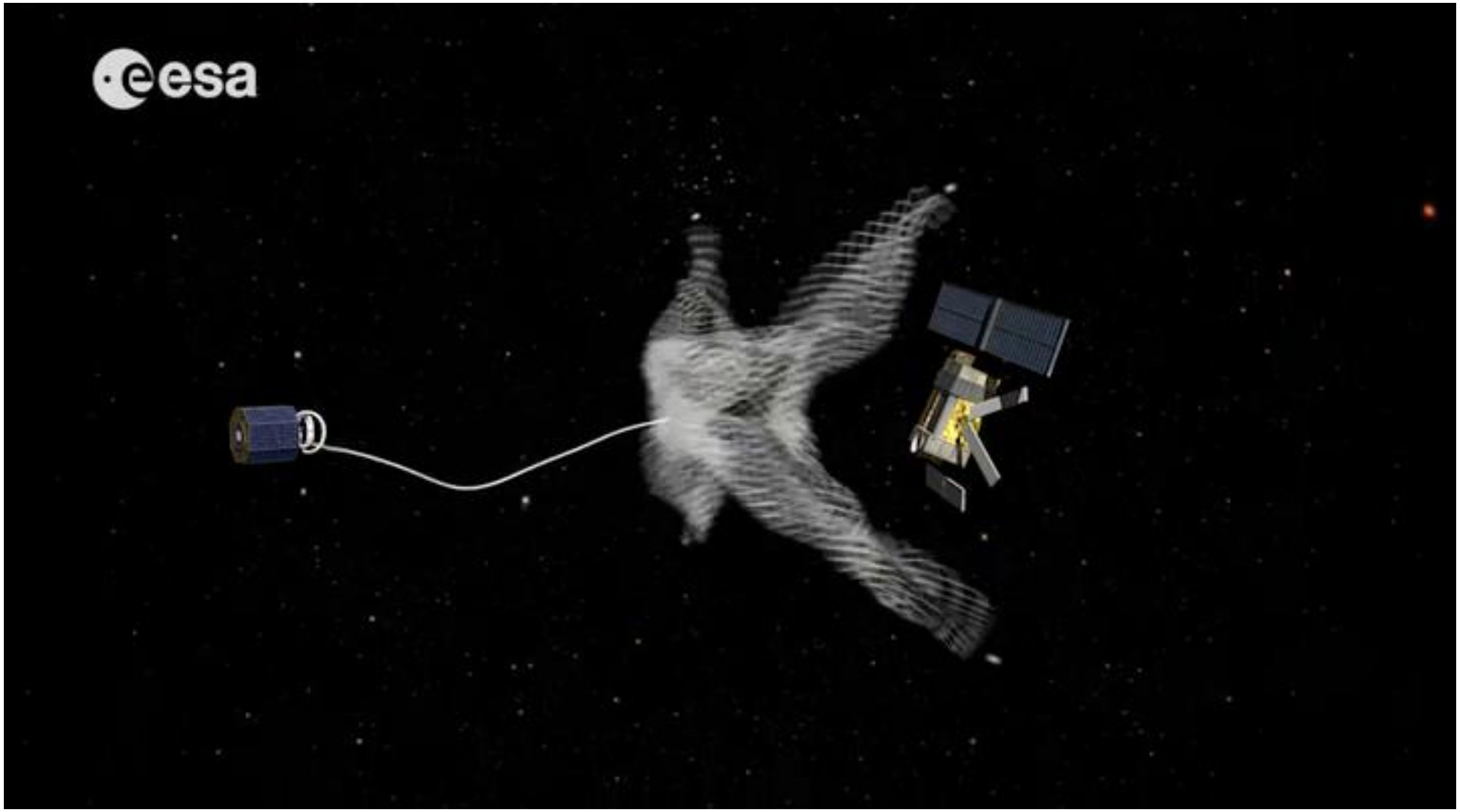
$$\omega_{n_I} = \beta_n(3m_2/m_t)^{1/2}\Omega, \quad \beta_n^2 = n^2\pi^2[1 + (m_t/3m_2)] - (m_t/4m_2)$$

- The out-of-plane transverse frequencies are given by

$$\omega_{n_o}^2 = \omega_{n_I}^2 + \Omega^2$$

USE OF TETHERS FOR DEBRIS REMOVAL

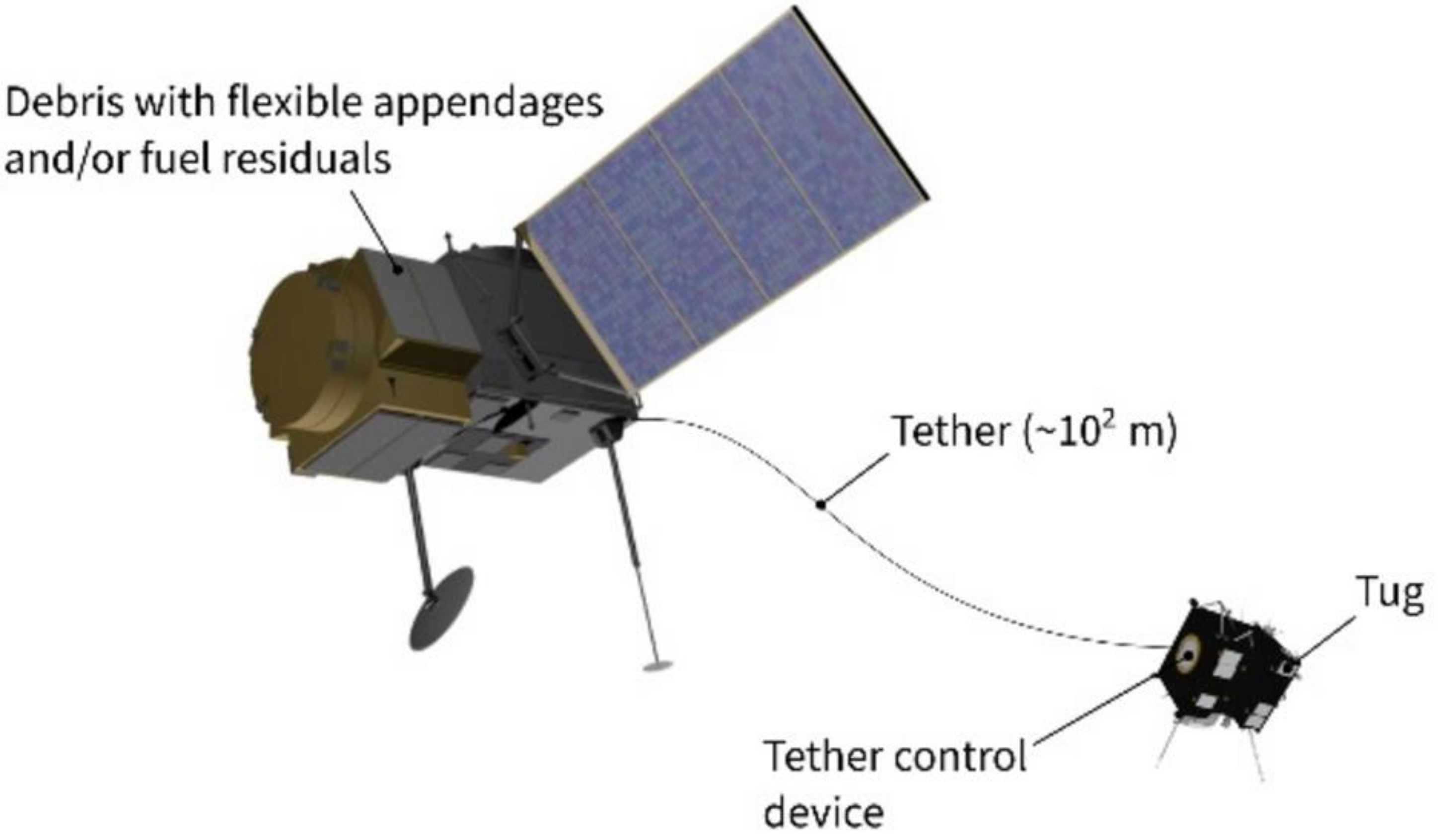
TETHER-NET CAPTURE



ESA (http://www.esa.int/Our_Activities/Space_Engineering_Technology/Clean_Space/e.Deorbit)

USE OF TETHERS FOR DEBRIS REMOVAL

SPACE TUG

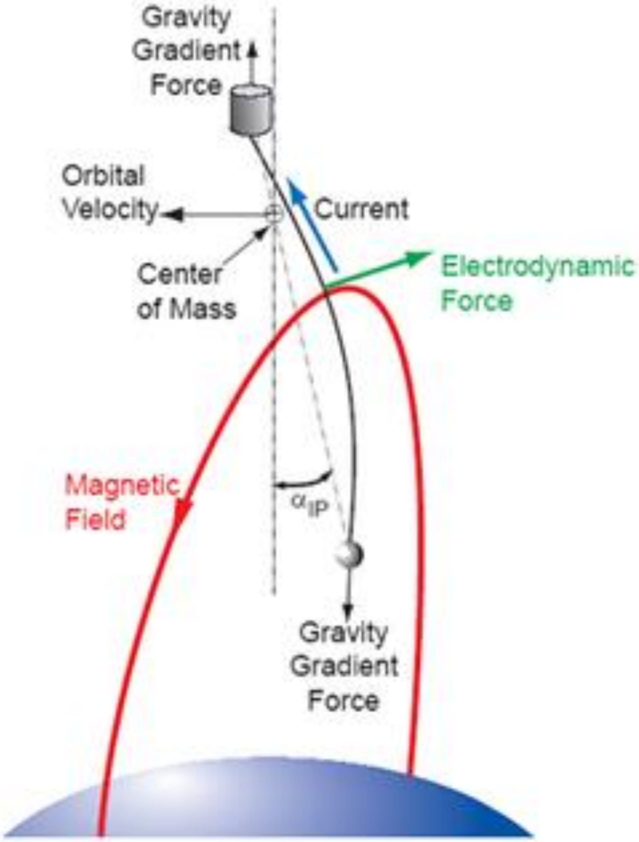


(Aslanov, 2015)

USE OF TETHERS FOR DEBRIS REMOVAL

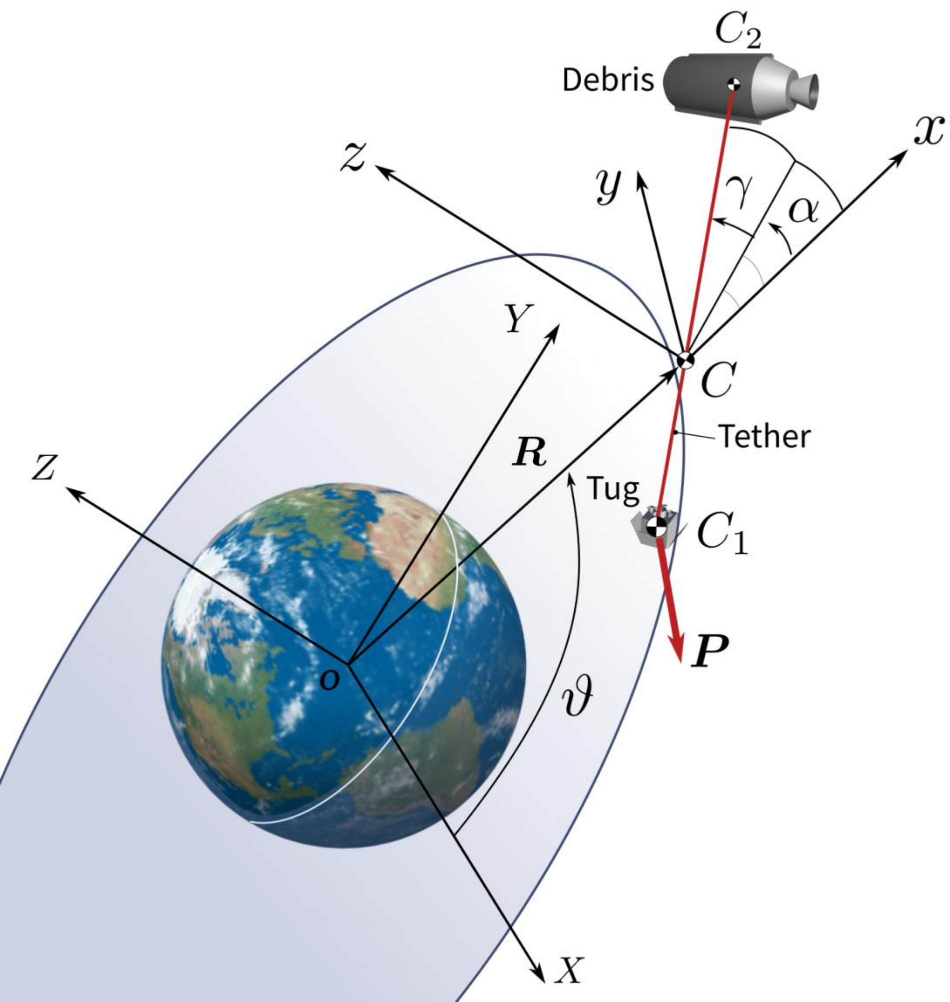
ELECTRODYNAMIC TETHERS

- Motion of a conductive wire across the magnetic field of the Earth induces an emf and Lorentz force.
- This so-called “electrodynamic propulsion” can be used to raise or lower the orbit.
- This effect can be utilized for removal of space debris.



USE OF TETHERS FOR DEBRIS REMOVAL

TOWING OF SPACE DEBRIS



- The tug and debris are point masses
- Thrust applied to tug is low
- The orbit change is slow
- Tether length is constant
  - $C_1C_2 = l = \text{const}$

USE OF TETHERS FOR DEBRIS REMOVAL

TOWING OF SPACE DEBRIS

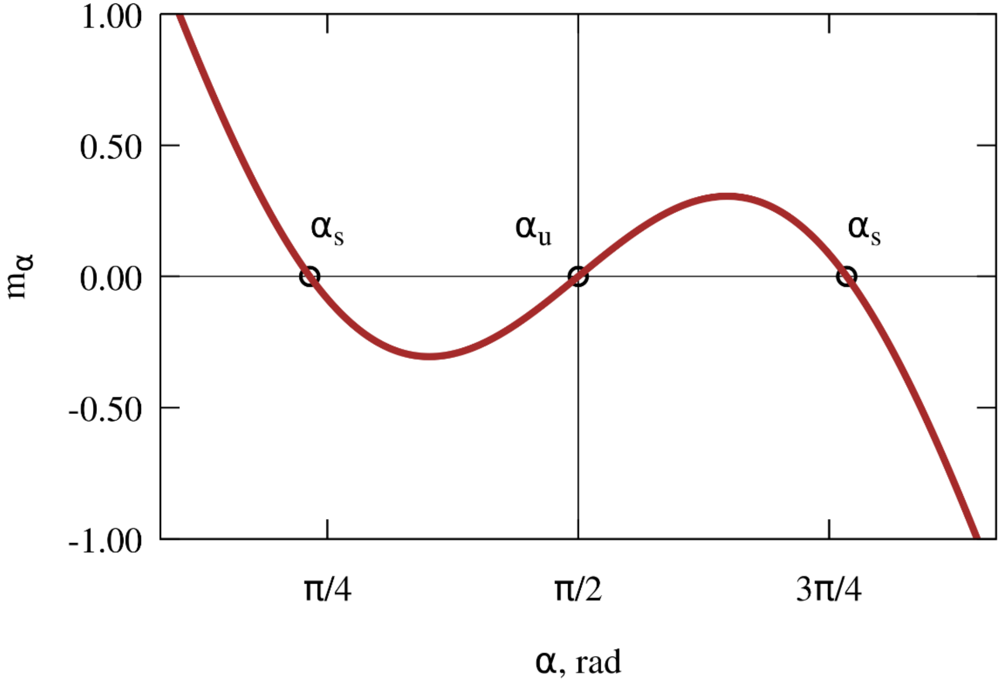
- For in-plane motion in a circular orbit (e=0, γ =0)

$$\alpha'' = \frac{P}{m_1 l_0 \omega^2}\cos\alpha - \frac{3}{2}\sin 2\alpha = m_\alpha$$

- First integral

$$\frac{(\alpha')^2}{2} + W(\alpha) = E$$

Torque m_α



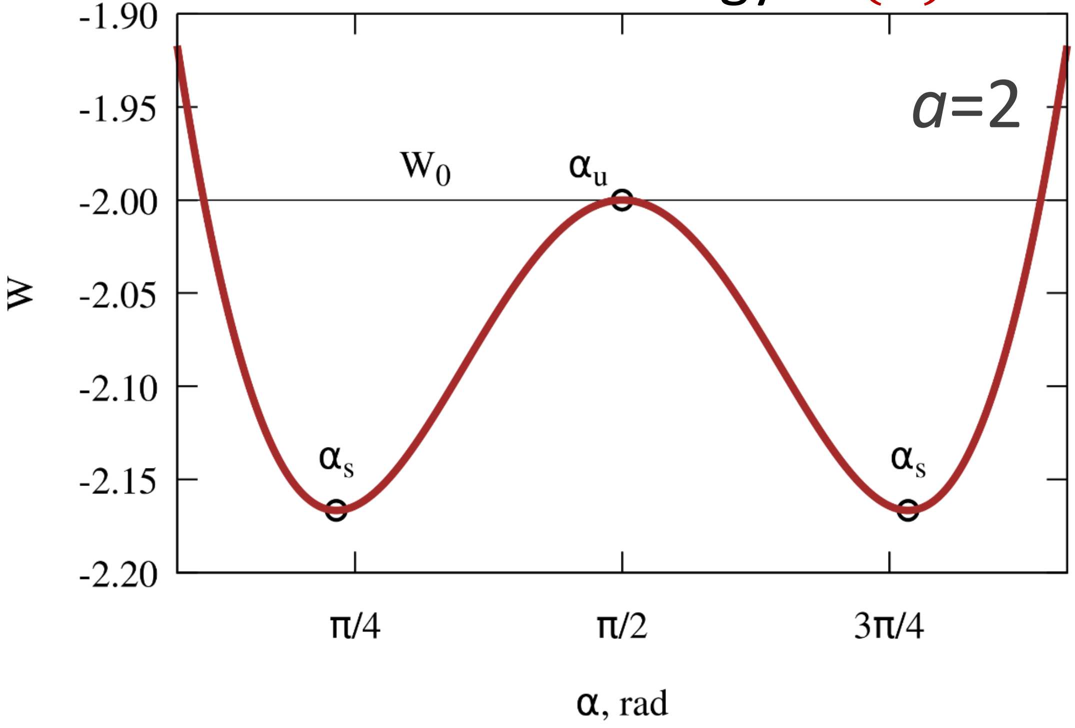
USE OF TETHERS FOR DEBRIS REMOVAL

TOWING OF SPACE DEBRIS

- First Integral

$$\frac{(\alpha')^2}{2} + W(\alpha) = E$$

Potential energy $W(\alpha)$



- Potential energy

$$W(\alpha) = -a\sin\alpha - b\cos^2\alpha$$

where

$$a = \frac{P}{m_1 l_0 \omega^2}, \qquad b = \frac{3}{2}, \qquad \omega = \sqrt{\frac{\mu}{R^3}}$$

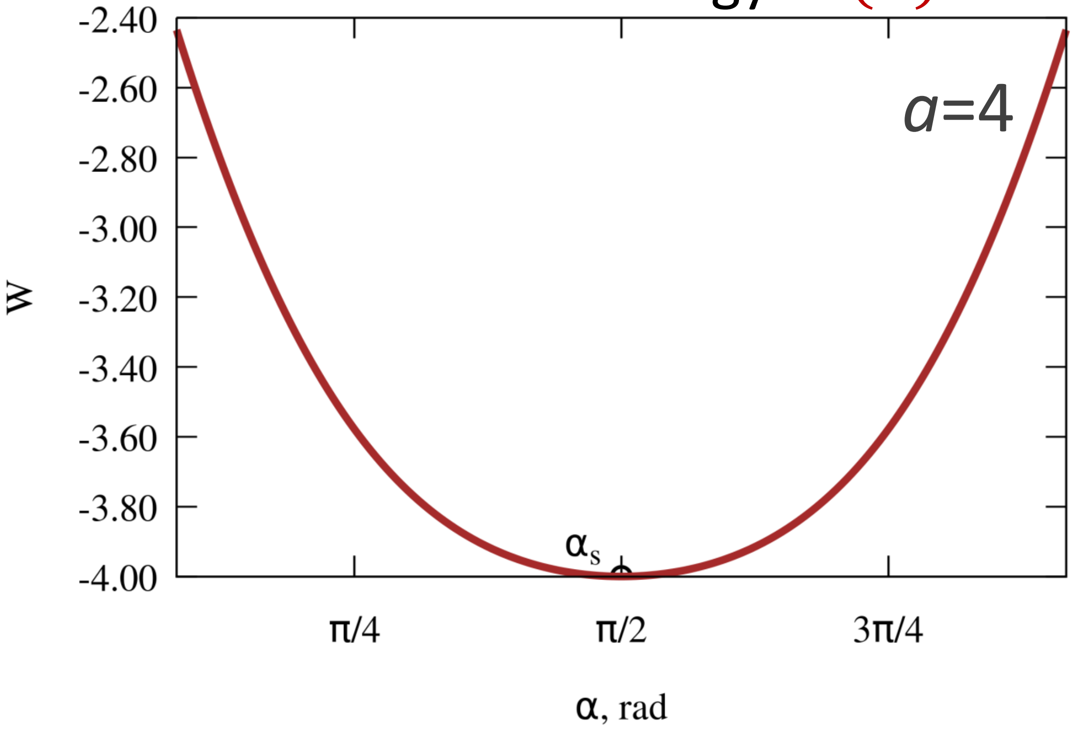
USE OF TETHERS FOR DEBRIS REMOVAL

TOWING OF SPACE DEBRIS

- First Integral

$$\frac{(\alpha')^2}{2} + W(\alpha) = E$$

Potential energy $W(\alpha)$



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USE OF TETHERS FOR DEBRIS REMOVAL

TOWING OF SPACE DEBRIS

- If

$$\frac{P}{m_1 l_0 \omega^2} < 3$$

- Stable positions

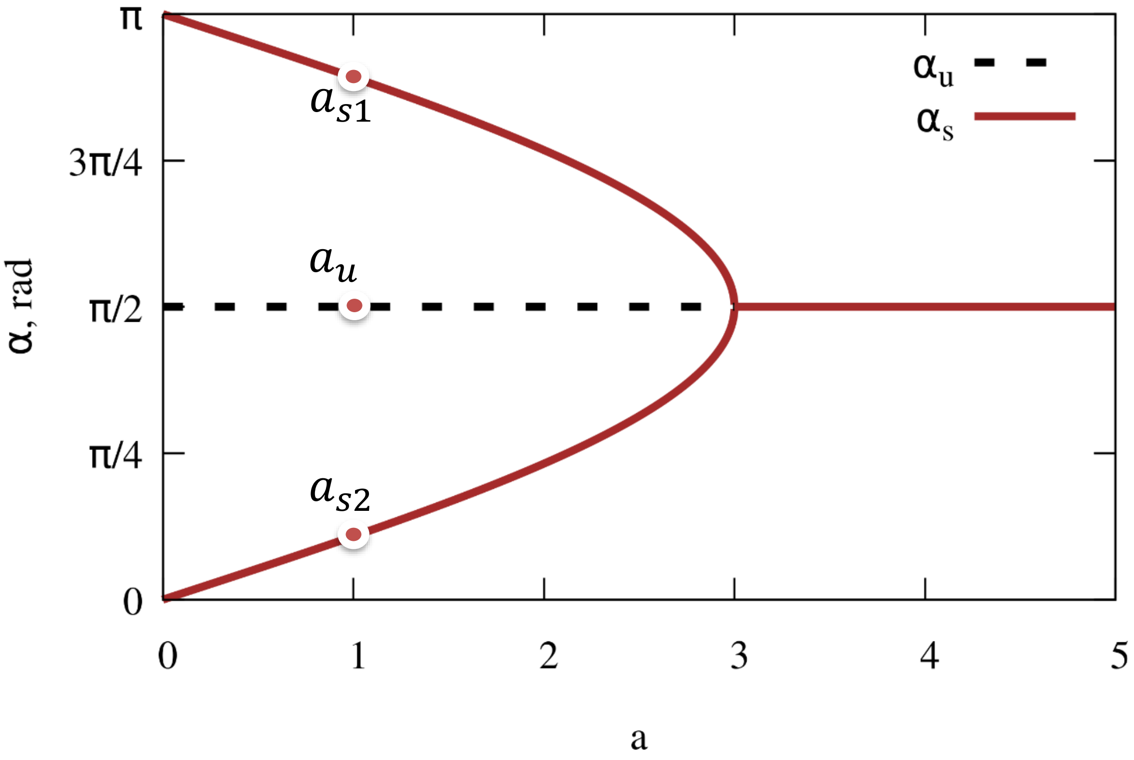
$$a_{s1} = \operatorname{asin}\frac{a}{2b}$$

$$a_{s2} = \pi - \operatorname{asin}\frac{a}{2b}$$

- Unstable position

$$a_u = \pi/2$$

Bifurcation diagram



VERY LONG TETHERS

- There is currently a fair amount of interest in the space elevator.
- The technology required for the space elevator is not likely to be available for several decades.
- Partial space elevators may be feasible in the near term.
- Partial space elevators are very long tethers or ribbons.

VERY LONG TETHERS

CENTRE OF ORBIT

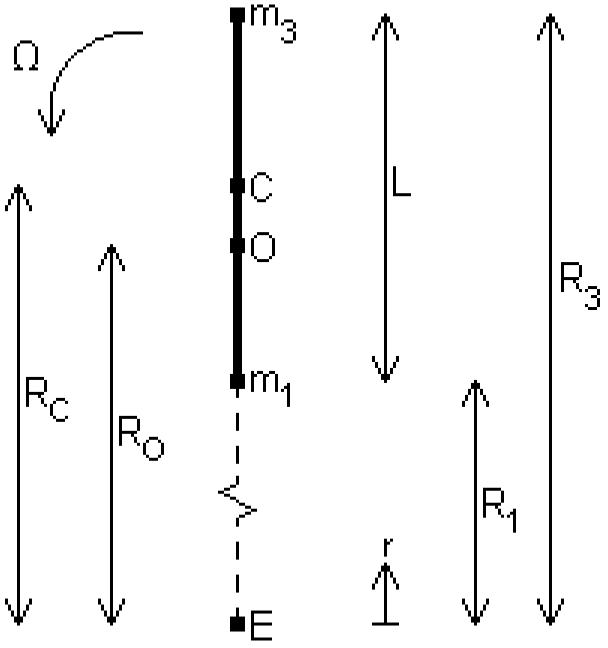
- Centrifugal force and gravitational force do not balance at *C*. Hence,

$$\Omega^2 \neq \frac{GM_E}{R_C{}^3}$$

- Location of *C* does not determine the orbital angular velocity.

- Centrifugal force and gravitational force balance at another point *O* called the centre of orbit.

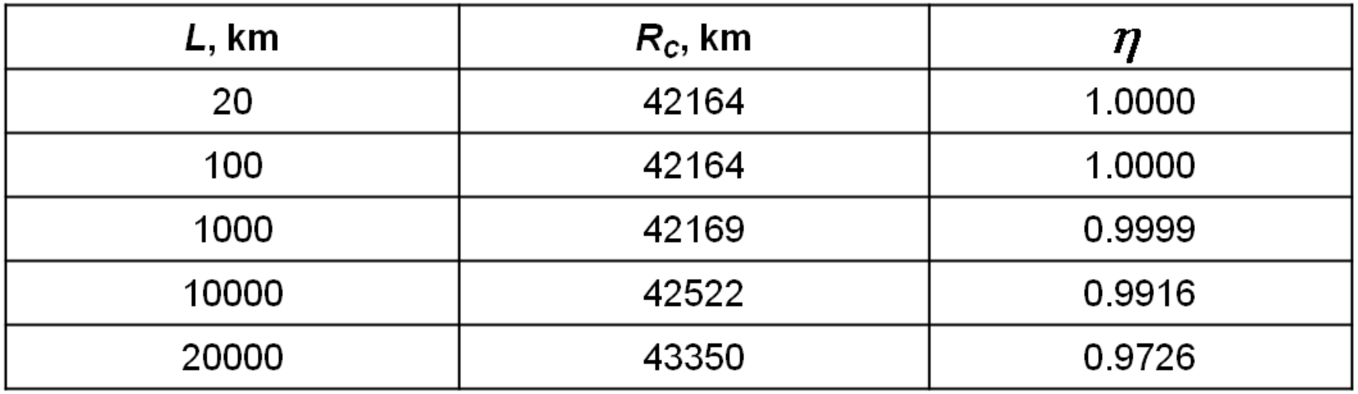
$$\Omega^2 = \frac{GM_E}{R_O{}^3}$$



VERY LONG TETHERS

CENTRE OF ORBIT

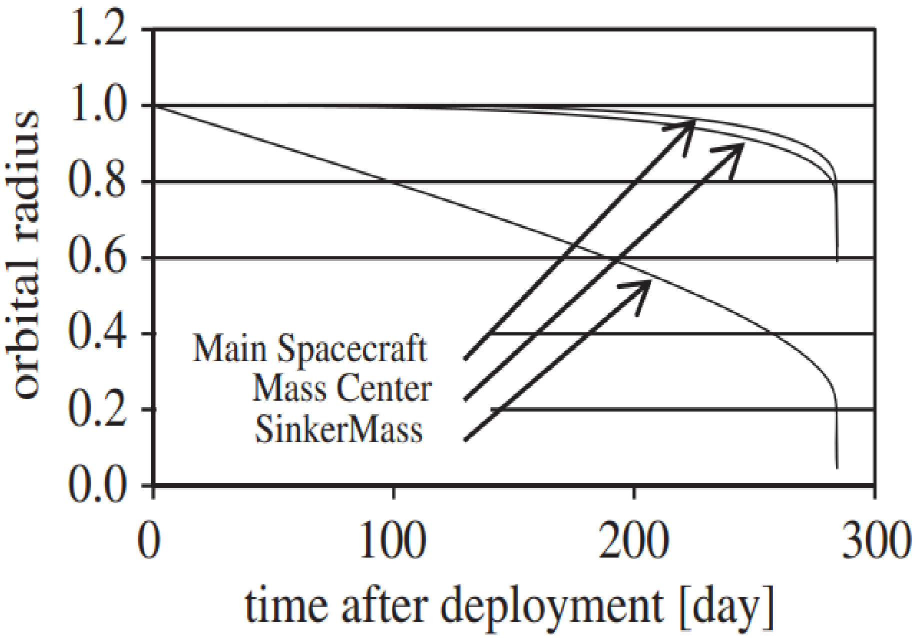
- Geosynchronous orbit: $R_O = 42164$ km

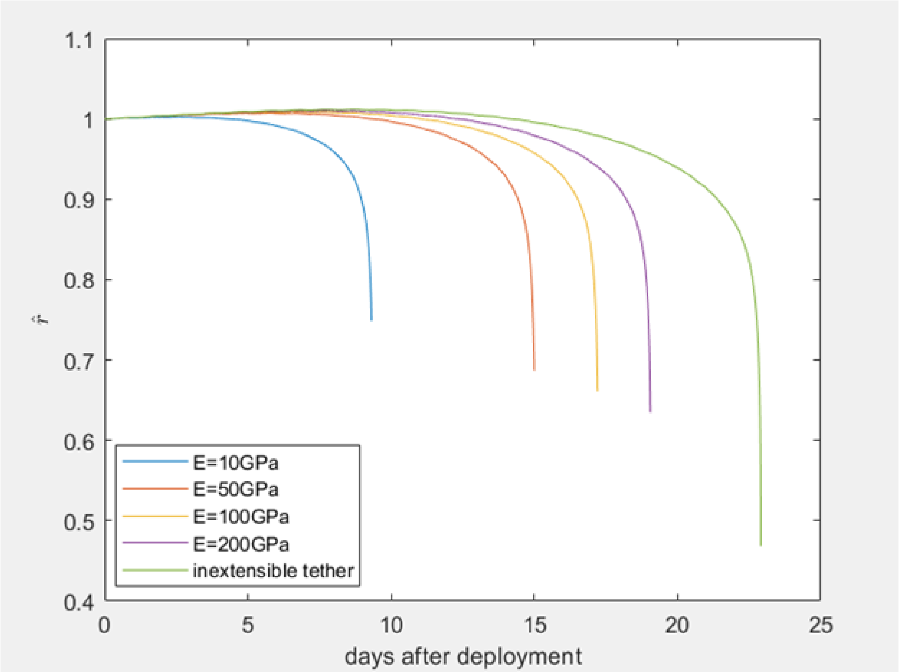


L, km	R_C, km	η
20	42164	1.0000
100	42164	1.0000
1000	42169	0.9999
10000	42522	0.9916
20000	43350	0.9726

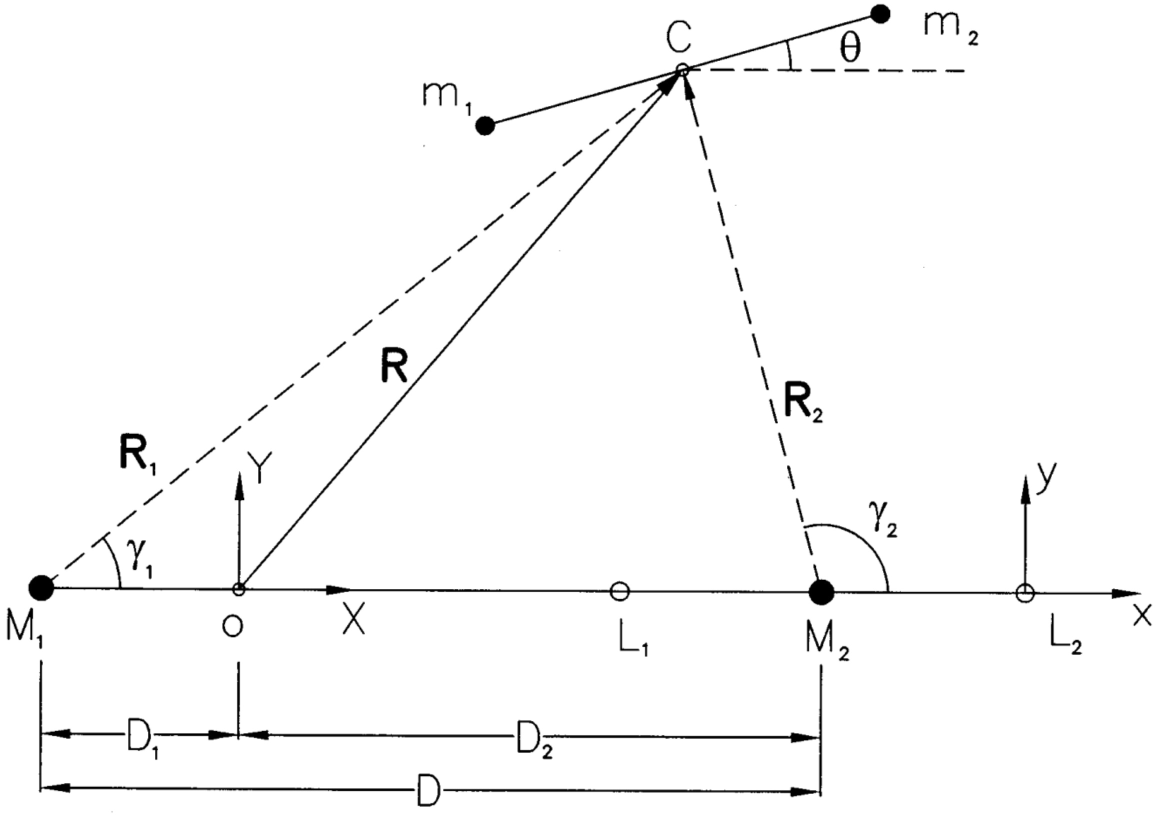
- $\eta = Ro / Rc \approx 1$ for short tethers

VERY LONG TETHERS





DYNAMICS OF TETHERED SYSTEMS AT COLLINEAR LAGRANGIAN POINTS



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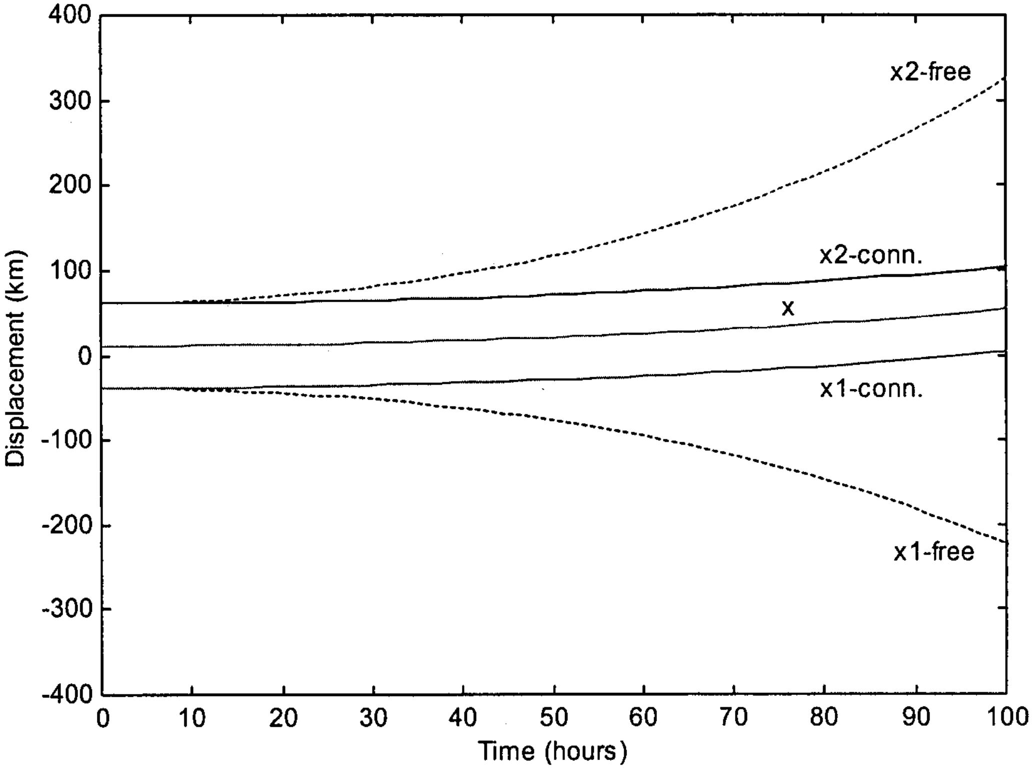
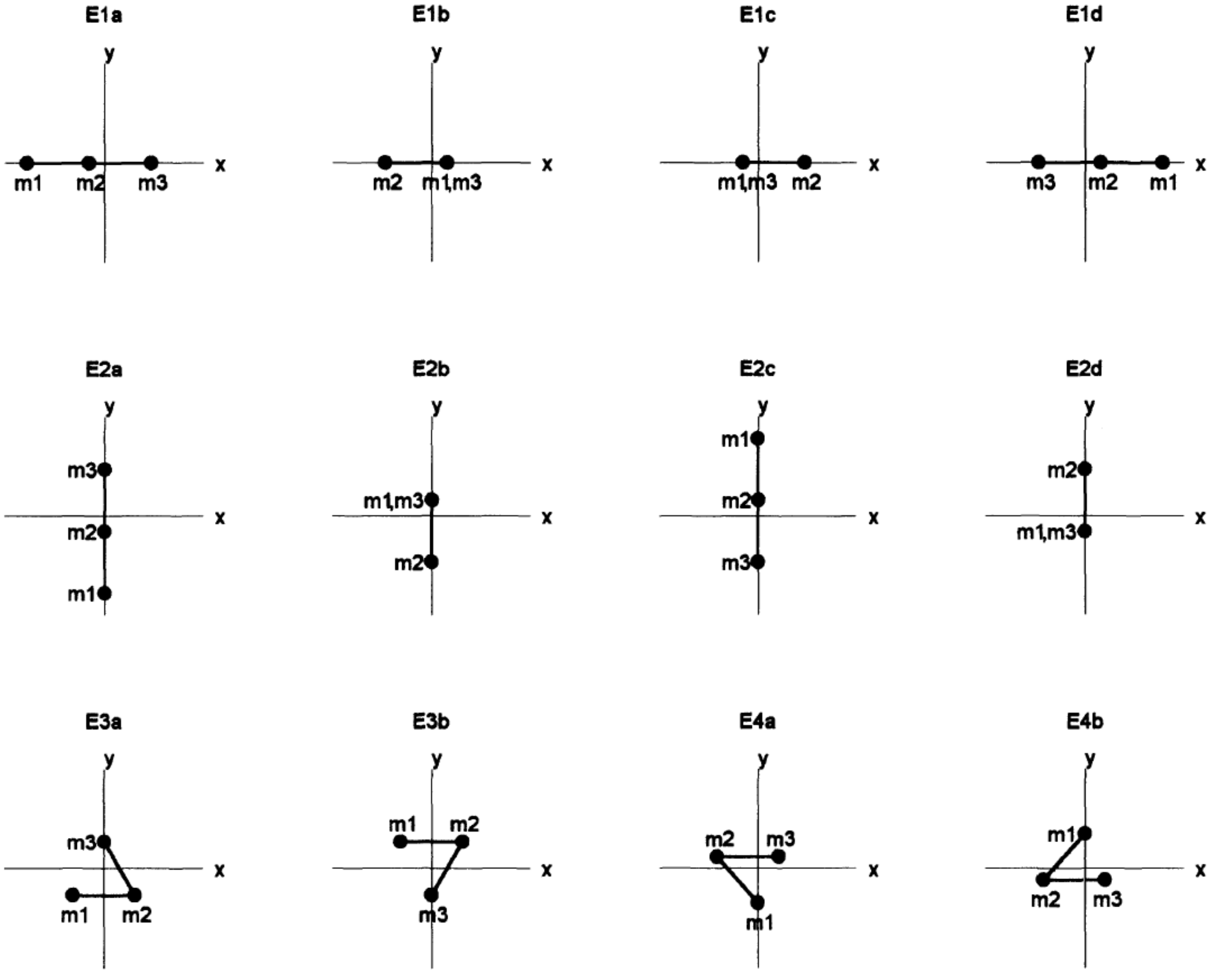


Comparison of x displacements of two masses in free and tether-connected states; $l = 100$ km.

THREE-BODY TETHERED SYSTEMS

EQUILIBRIUM CONFIGURATIONS

- E1



CONCLUSIONS

- A rich literature on dynamics and control of space tethers exists.

- Mathematical models for tether dynamics analysis can be quite different for different applications. Even for the same application, investigators have used many different approaches to study tether dynamics.

- In the near term, application of tethers for space debris removal are likely to be the major drivers for study of space tether dynamics.

QUESTIONS ?