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Satellite Attitude Motion Analysis of Three-Body Tethered System during Its Deployment Process Using Method of Integral Manifolds

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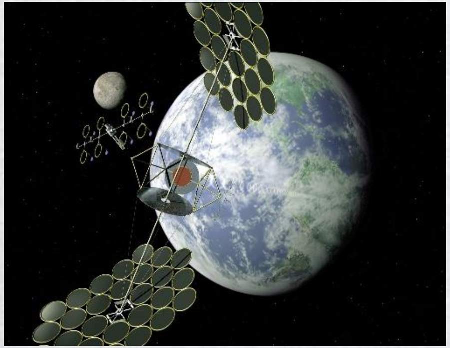
Section 01

Introduction

- Background
- LTBTS description

Background

Assembly of large-scale aerospace structures



Space-based solar power

Interferometric telescopes

Space station

Requirements

R 1	The lightweight design of the structure
R 2	The controllability design of the structure

LTBTS description

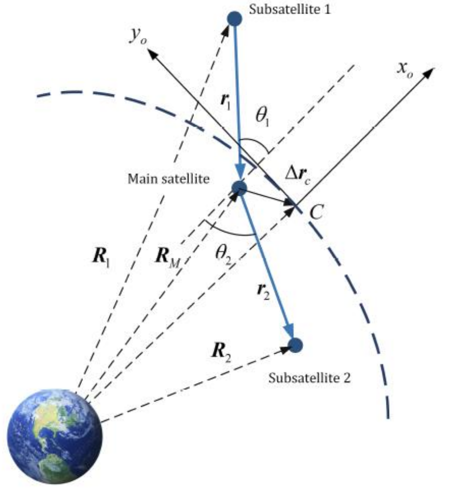


Fig. 1. The schematic diagram of LTBTS

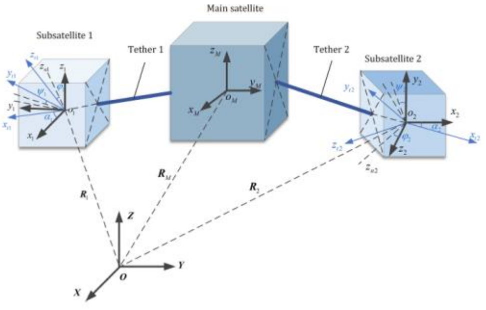


Fig. 2. Schematic of LTBTS satellite attitude

Based on the Lagrangian equations the LTBTS can be described as

$$A\ddot{\xi} = B$$

$$\ddot{\xi} = \left(\ddot{l}_1, \ddot{l}_2, \ddot{\theta}_1, \ddot{\theta}_2\right)$$

$$A = [a_{ij}]$$

$$\boldsymbol{B} = \boldsymbol{Q} + \boldsymbol{F} + \boldsymbol{R}$$

$$\begin{cases} a_{11} = m_1 (m_2 + M)/m_s , \\ a_{12} = a_{21} = m_1 m_2 \cos(\theta_1 - \theta_2)/m_s , \\ a_{13} = a_{31} = 0, \\ a_{14} = a_{41} = m_1 m_2 l_2 \sin(\theta_1 - \theta_2)/m_s , \\ a_{22} = m_2 (m_1 + M)/m_s , \\ a_{23} = a_{32} = m_1 m_2 l_1 \sin(\theta_2 - \theta_1)/m_s , \\ a_{24} = a_{42} = 0, \\ a_{33} = m_1 (m_2 + M)\, l_1^2/m_s , \\ a_{34} = a_{43} = m_1 m_2 l_1 l_2 \cos(\theta_1 - \theta_2)/m_s , \\ a_{44} = m_2 (m_1 + M)\, l_2^2/m_s \end{cases}$$

The end-body's attitude described by Eulerian angles

$$\boldsymbol{J}_k \frac{\mathrm{d}\boldsymbol{\omega}_k}{\mathrm{d}t} = \boldsymbol{M}_k - \boldsymbol{\omega}_k \times \boldsymbol{J}_k \times \boldsymbol{\omega}_k$$

$$\begin{cases} \dot{\alpha}_k = \omega_{zk} \cos\varphi_k + \omega_{yk} \sin\varphi_k + \Delta\dot{\alpha}_k \\ \dot{\psi}_k = (\omega_{zk} \sin\varphi_k - \omega_{yk} \cos\varphi_k) / \sin\alpha_k + \Delta\dot{\psi}_k \\ \dot{\varphi}_k = \omega_{xk} - \dot{\psi}_k \cos\alpha_k + \Delta\dot{\varphi}_k \end{cases}$$

Section 02

Application of Integral Manifold Method

- Equation transformation
- Application of integral manifold method

Equation transformation

Theoretical analysis of the satellite attitude dynamics using the method of integral manifolds. Before this, the dynamic equations need to be transformed into a form suitable for the application of the integral manifold method.

$$\boldsymbol{J}_k \frac{\mathrm{d}\boldsymbol{\omega}_k}{\mathrm{d}t} = \boldsymbol{M}_k - \boldsymbol{\omega}_k \times \boldsymbol{J}_k \times \boldsymbol{\omega}_k$$

$$\begin{cases} \dot{\alpha}_k = \omega_{zk}\cos\varphi_k + \omega_{yk}\sin\varphi_k + \Delta\dot{\alpha}_k \\ \dot{\psi}_k = \left(\omega_{zk}\sin\varphi_k - \omega_{yk}\cos\varphi_k\right)/\sin\alpha_k + \Delta\dot{\psi}_k \\ \dot{\varphi}_k = \omega_{xk} - \dot{\psi}_k\cos\alpha_k + \Delta\dot{\varphi}_k \end{cases}$$

$$\Rightarrow$$

$$\begin{cases} J_k^x \dfrac{\mathrm{d}\omega_k^x}{\mathrm{d}t} = M_k^x \\ \dfrac{\partial K_k^{xt}}{\partial t} = M_k^{xt} + \dfrac{K_k^x - K_k^{xt}\cos\alpha_k}{\sin\alpha_k}\omega_k^{z't} - J_k\omega_k^{zn}\omega_k^{y't} \\ \dfrac{\mathrm{d}\omega_k^{zn}}{\mathrm{d}t} = \dfrac{M_k^{zn}}{J_k} + \dfrac{\left(K_k^x\cos\alpha_k - K_k^{xt}\right)\left(K_k^x - K_k^{xt}\cos\alpha_k\right)}{{J_k}^2\sin^3\alpha_k} \\ \dfrac{\mathrm{d}\varphi_k}{\mathrm{d}t} = \dfrac{K_k^x}{J_k^x} + \dfrac{K_k^x\cos\alpha_k - K_k^{xt}}{J_k\sin^2\alpha_k}\cos\alpha_k + \Delta\dot{\varphi}_k \\ \dfrac{\mathrm{d}\psi_k}{\mathrm{d}t} = \dfrac{K_k^{xt} - K_k^x\cos\alpha_k}{J_k\sin^2\alpha_k} + \Delta\dot{\psi}_k \\ \dfrac{\mathrm{d}\alpha_k}{\mathrm{d}t} = \omega_k^{zn} + \Delta\dot{\alpha}_k \end{cases}$$

where K_k^x, K_k^{xt}, M_k^x, M_k^{xt} — the projection values of angular momentum and the total external force torque vector onto the axes $c_k x_k$ and $c_k x_{tk}$.

In this study, we assume that $J_k^y = J_k^z = J_k \neq J_k^x$

Application of integral manifold method

$$\begin{cases} \dfrac{\mathrm{d}\alpha_k}{\mathrm{d}t} = \omega_k^{zn} \\ \dfrac{\mathrm{d}\omega_k^{zn}}{\mathrm{d}t} + F_k(\alpha_k, \boldsymbol{x}_k) = \varepsilon_k f(\alpha_k, \varphi_k, \boldsymbol{x}_k) \\ \dfrac{\mathrm{d}\varphi_k}{\mathrm{d}t} = \omega_{\varphi_k}(\alpha_k, \boldsymbol{x}_k) + \varepsilon_k \Phi_k(\alpha_k, \varphi_k, \boldsymbol{x}_k) \\ \dfrac{\mathrm{d}\boldsymbol{x}_k}{\mathrm{d}t} = \varepsilon_k R(\alpha_k, \varphi_k, \boldsymbol{x}_k) \end{cases}$$

$$F_k(\alpha_k, \boldsymbol{x}_k) = -\frac{M_k^{zn}}{J_k} + \frac{(K_k^x \cos\alpha_k - K_k^{xt})(K_k^{xt}\cos\alpha_k - K_k^x)}{{J_k}^2 \sin^3\alpha_k}$$

$$\omega_{\varphi_k}(\alpha_k, \boldsymbol{x}_k) = \frac{K_k^x}{J_k^x} + \frac{K_k^x \cos\alpha_k - K_k^{xt}}{J_k \sin^2\alpha_k}\cos\alpha_k$$

Integral manifold

$$\begin{cases} \omega_k^{zn*} = 0 \\ F_k(\alpha_k^*, \boldsymbol{x}_k^*) = 0 \\ \dfrac{\mathrm{d}\varphi_k^*}{\mathrm{d}\tau} = \sigma_k(\alpha_k^*, \boldsymbol{x}_k^*) \\ \dfrac{\mathrm{d}\boldsymbol{x}_k^*}{\mathrm{d}\tau} = 0 \end{cases}$$

Mean value

$$\frac{\partial F_k}{\partial \alpha_k^*}\frac{\mathrm{d}\alpha_k^*}{\mathrm{d}t} = -\frac{M_k^{zn}}{J_k T_k}\frac{\mathrm{d}T_k}{\mathrm{d}t}$$

Section 03

Analysis of the unideal configuration of end-bodies and simulation results

Analysis of the unideal configuration of end-bodies and simulation results

Unideal configuration

1. **The impacts of different initial perturbations on dynamic behaviors of LTBTS**
2. **The impacts of different satellite structures on dynamic behaviors of LTBTS**
3. **The impacts of the tether connection point offset errors on dynamic behaviors of LTBTS**

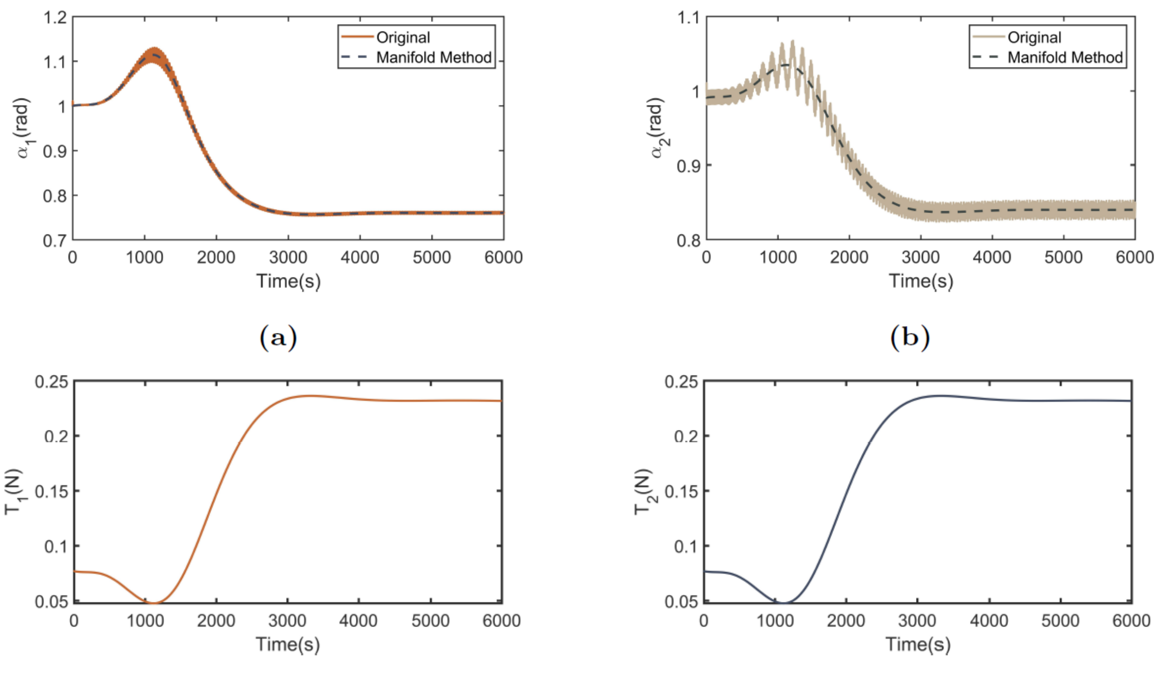
1 *The impacts of different initial perturbations on dynamic behaviors of LTBTS*

Initial angular velocity disturbances in the case of large angles of nutation

$$\alpha_{10} = \alpha_{20} = 1 \text{ rad}$$

The nutation angle's temporal curve precisely mirrors the tension's temporal curve.

$\omega_{1x0} = 0.01$ rad/s $\qquad \omega_{2x0} = 0.5$ rad/s

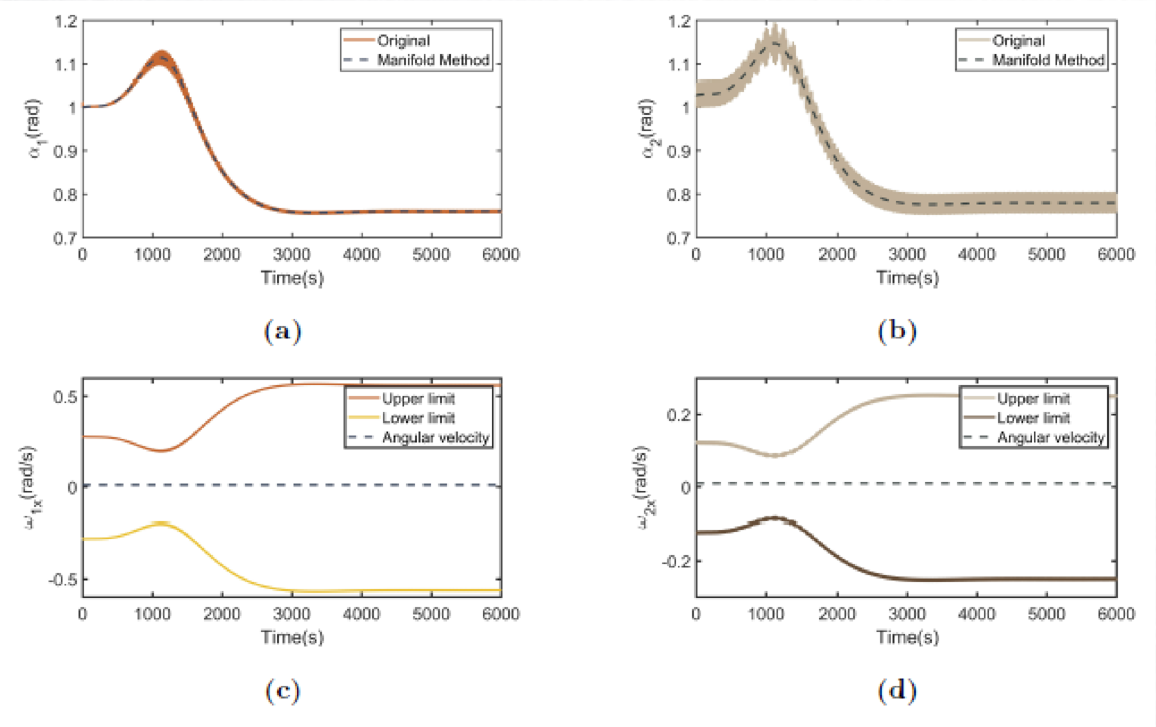


(a) (b)

(c) (d)

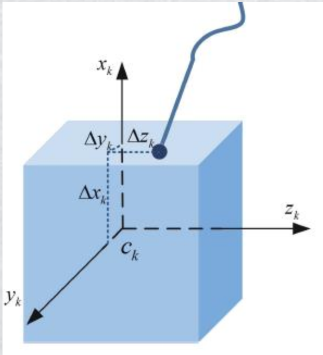
2 The impacts of different satellite structures on dynamic behaviors of LTBTS

$J_{x1}/J_1 = 0.8$ $J_{x2}/J_2 = 0.1$



The impacts of different satellite structures are reflected in the formula as the ratio of J_{xk}/J_k

3 *The impacts of the tether connection point errors on dynamic behaviors of LTBTS*

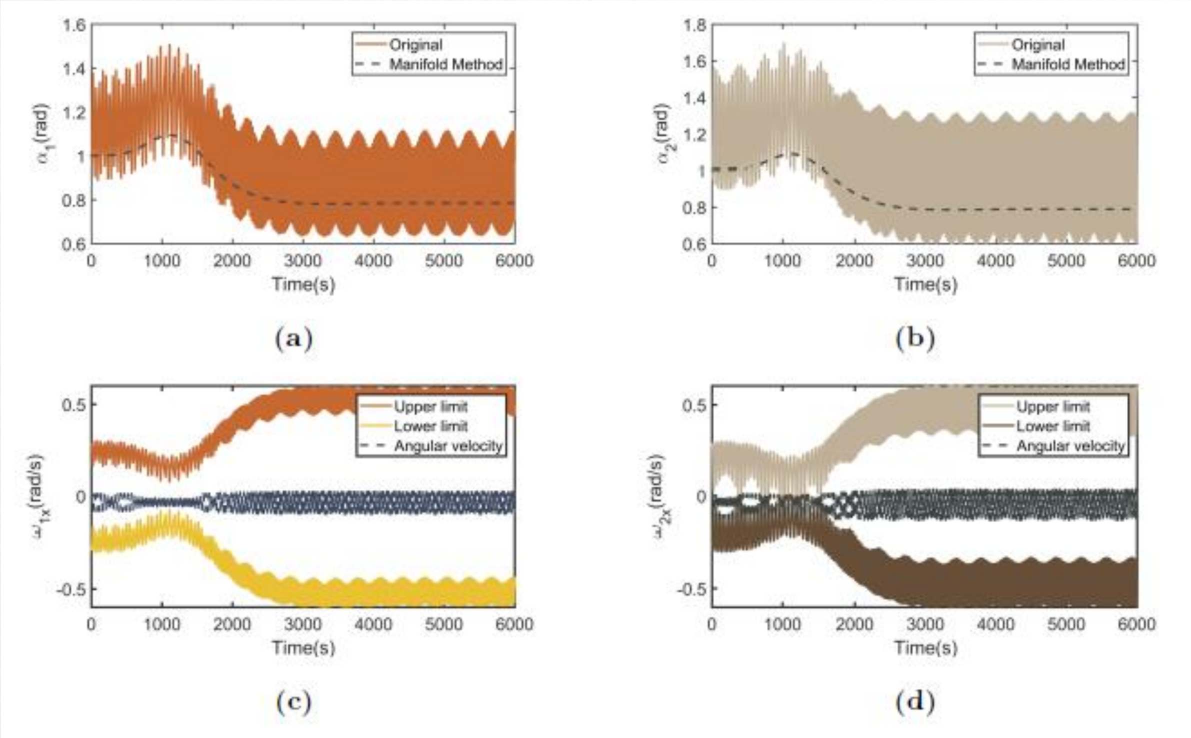


The offset ratios for the y-axis $\Delta y_k / \Delta r_k$

$$\Delta r_k = \sqrt{\Delta x_k^{\ 2} + \Delta y_k^{\ 2} + \Delta z_k^{\ 2}}$$

Resonance does not occur until offset ratios of 30 %

$\Delta y_1 = 0.04$ $\Delta y_2 = 0.06$

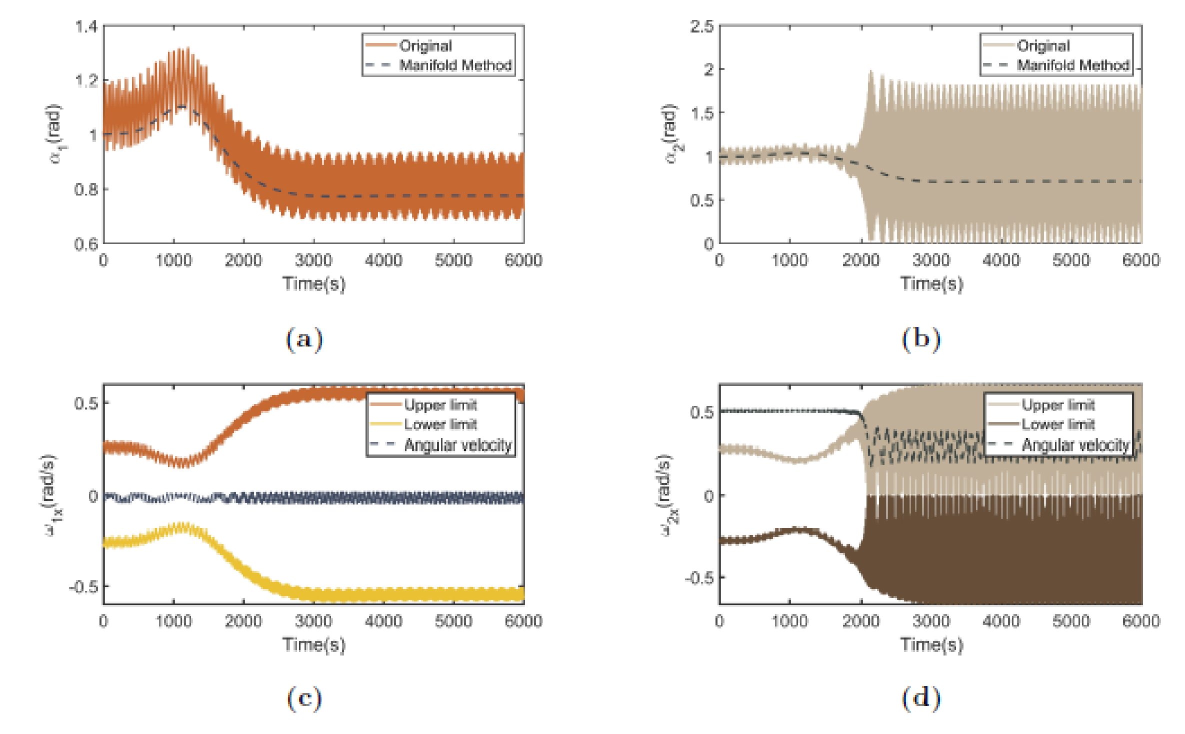


3 The impacts of the tether connection point errors on dynamic behaviors of LTBTS

$\Delta y_1 = 0.02 \quad \omega_{1x0} = 0.01 \ \mathrm{rad/s}$ $\Delta y_2 = 0.02 \quad \omega_{2x0} = 0.5 \ \mathrm{rad/s}$

Considering the superimposed effects of connection point errors and initial angular velocity disturbances

Resonance does not occur until offset ratios < 10 %



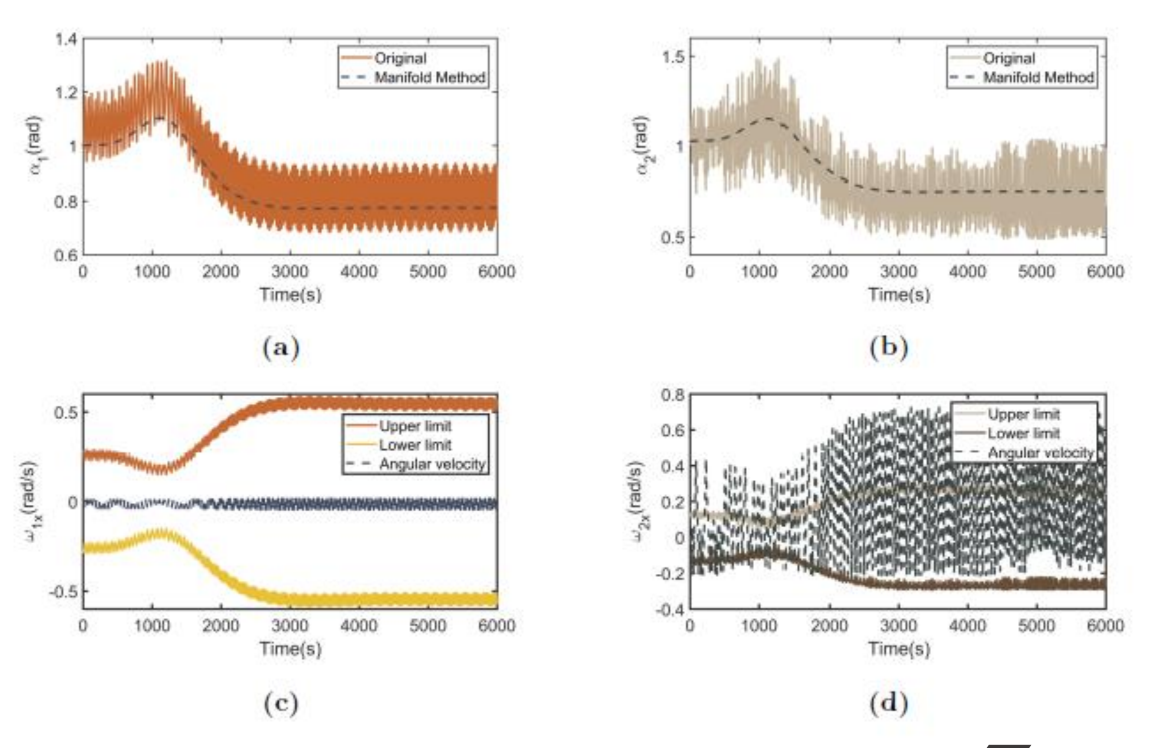
The impacts of the tether connection point errors on dynamic behaviors of LTBTS

Considering the superimposed effects of connection point errors and different satellite structures

Resonance does not occur until offset ratios of 10 %

$\Delta y_1 = 0.02 \quad J_{x1}/J_1 = 0.8$

$\Delta y_2 = 0.02 \quad J_{x2}/J_2 = 0.1$



Section 04

Conclusions & Expectation

Conclusions & Expectation

1. The method of integral manifolds has proven to be effective in estimating end-body oscillations and can provide an analytical expression through averaging.

2. In situations where the external torque is solely tension, the nutation angle's temporal curve precisely mirrors the tension's temporal curve. As tension diminishes, the oscillation amplitude of the nutation angle escalates, whereas an increase in tension causes a decrease in the oscillation amplitude of the nutation angle.

3. Subsatellites manifest diverse dynamical properties contingent on their structural design and the positioning of their connection points.

Thanks for listening

Question & Answer